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Á. Debreczeni, CSc, Miskolc, Hungary

REMARKS ON THE FAILURE CURVES OF ROCKS

А. ДЕБРЕЦЕНІ

ЗАУВАЖЕННЯ ЩОДО КРИВИХ ВІДМОВИ ПРИРОДНОГО КАМЕНЮ

Гіперболічні криві відмови природного каменю застосовувалися й розвивалися Институтом Гірської промисловості й Геотехнічної Розробки, Університету Мишкольца (Венг-рия) представлені в роботі. При використанні цих кривих і представленого методу хоро-шая оцінка може бути отримана для параметрів сили зламаного каменю. Одне з найважливіших переваг цього методу - те, що всі його елементи є заснованими на зважених особливостях тому, це можна порахувати гарною альтернативою широко исполь-зуемому методу.

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Ключові слова: обмеження умови, крива, гіпербола, блок каменю, маса каменю

Гиперболические кривые отказа природного камня применялись и развивались Институтом Горной промышленности и Геотехнической Разработки, Университета Мишкольца (Венгрия) представлены в работе. При использовании этих кривых и представленного метода хорошая оценка может быть получена для параметров силы взломанного камня. Одно из самых важных преимуществ этого метода – то, что все его элементы являются основанными на взвешенных особенностях поэтому, это можно счесть хорошей альтернативой широко используемому методу.

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Ключевые слова: ограничение условия, кривая, гипербола, блок камня, масса камня

Hyperbolic failure curves of rocks applied and developed by the Institute of Mining and Geotechnical Engineering, University of Miskolc (Hungary) are presented in the paper. By using these curves and a so-called ME-method, developed in the Institute, a good estimation can be obtained for the strength parameters of cracked rocks. One of the most important advantages of this method is that all of its elements are based on measured characteristics therefore it can be considered a good alternative to the widely used Hoek-Brown method.

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Keywords: limiting condition, curve, hyperbole, rock block, rock mass

1. INTRODUCTION

In the beginning, the failure states of rocks were described according to the criteria applied for metals such as the Mohr, Murrel, Huber-Mises or Huber-Mises-Hencky criteria. Practice has testified that the Mohr limiting condition can well be applied to rocks. It is a disadvantage of the Mohr condition that it leaves the impact of the middle principal stress out of consideration but approximation occurs in favour of greater safety.

The Mohr limiting condition may be investigated with the application of several failure curves such as the Mohr-Coulomb yield line as well as parabolic and hyperbolic curves. The Hoek-Brown curve, now widely used to describe cracked rock bodies, is also based on the Mohr condition.

There also exist more generally formulated criteria like the Mogi, Drucker-Prager, Wiebols-Cook and Lade criteria. These more complex criteria are successfully applied in certain special fields. (For example, the Lade criterion is applied in the stability studies of boreholes.) In other cases, however, despite their complexity, they are not more useful that the Mohr-Coulomb linear limiting condition. (For example, the Drucker-Prager criterion overestimates the role of middle principal stress, which may lead to an approximation at the expense of safety.)

In my talk, I will present the hyperbolic failure curves applied in the Institute of Mining and Geotechnical Engineering at the University of Miskolc. On the basis of these curves and the method (ME method) developed in the institute, the strength properties of cracked rock bodies can well be estimated. The advantage of the method is that every element of the process is based on measurement results, and thus it is a realistic alternative to the Hoek-Brown method, widely applied nowadays.

2. GENERAL REMARKS ON FAILURE LIMITING CONDITIONS

Limiting conditions can mostly be expressed as the invariants of the stress tensor or those of the deviator tensor that can be derived from it. The reason for this is that while stress tensor elements depend on the position of the coordinate system, limiting conditions should be true independently from the chosen directions. The so called octaeder stresses, which are normal (σ_{oct}) and tangent stresses (τ_{oct}) formed on the sheets of an octaeder of a specific position (body diagonals are in the directions of the three principal stresses). On every sheet of the octaeder, the absolute value of stress components is identical (Figure 1).

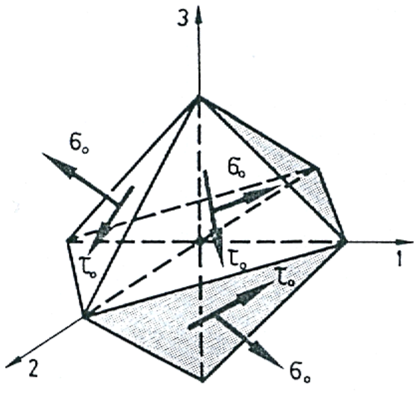

Figure 1 – Octaeder stresses

Octaeder normal stress is:

$$\sigma_{oct} = \frac{1}{3} \cdot I_1 = \frac{\sigma_1 + \sigma_2 + \sigma_3}{3} \text{ , and}$$

octaeder tangential stress is:

$$\tau_{oct} = \sqrt{\frac{2}{3} J_2} = \frac{1}{3} \sqrt{(\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} \text{ ,}$$

where:

I_1 – first invariant of stress tensor

$$I_1 = \sigma_1 + \sigma_2 + \sigma_3$$

J_2 – second invariant of deviator tensor

$$J_2 = \frac{1}{6}[(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]$$

σ_1; σ_2; σ_3 – principal stresses ($\sigma_1 \geq \sigma_2 \geq \sigma_3$).

For example, with octaeder stresses, the Murell limiting condition can be written up in the following way:

$$\tau_{oct}^2 = \frac{2}{3} \cdot \sigma_c \cdot \sigma_{oct}$$

The limits of this paper do not allow a detailed discussion of limiting conditions or the curves and interfaces interpreted with their help so I only have the opportunity to present the most important principles.

The following figure (Figure 2) shows the measurement results for unconfined compression (σ_1>0; σ_2=σ_3=0), biaxial (σ_1>0; σ_2>0; σ_3=0), conventional triaxial (σ_1>σ_2=σ_3>0) and polyaxial ($\sigma_1 \geq \sigma_2 \geq \sigma_3$) experiments.

The figure shows it well that failure is not independent from middle principal stress (σ_2) so failure criteria based on the Mohr theory (leaving the impact of σ_2 out of consideration) can only be limitedly applied under general stress conditions.

In some cases, a sufficiently accurate calculation of failure stresses requires the application of relatively complicated correlations. For example, the modified Lade criterion is preferred in the description of the stress conditions formed around boreholes [1]:

$$\frac{[(\sigma_1 + S) + (\sigma_2 + S) + (\sigma_3 + S)]^3}{(\sigma_1 + S) \cdot (\sigma_2 + S) \cdot (\sigma_3 + S)} = 27 + \frac{4 \cdot (tg^2\phi) \cdot (9 - 7 \cdot \sin\phi)}{1 - \sin\phi} \text{ ,}$$

where: $S = \frac{c}{tg\phi}$

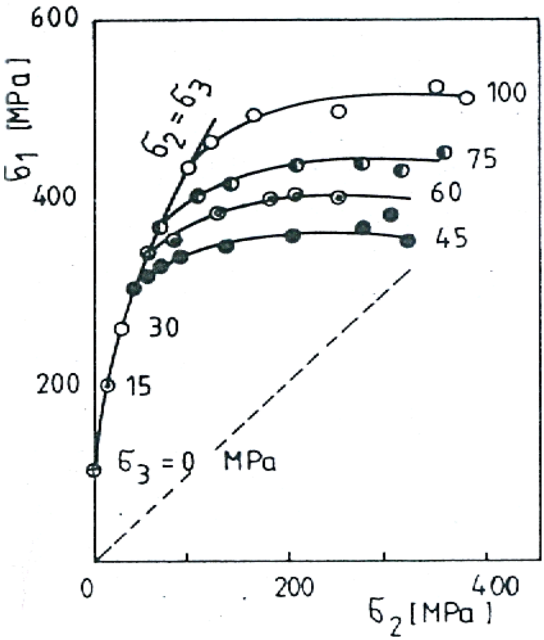

Figure 2 – Measurement results for dolomite sample bodies [3]

As it is well-known, in the case of deep bores, both the supporting of borehole wall and the determination of the pressure necessary for rock cracking requires enhanced circumspection and precise calculations. In such a case, it is not possible to counterbalance the inaccuracy of calculations by choosing high safety parameters.

However, in the overwhelming majority of rock mechanics problems, it is unsuitable to use a complex criterion similar to the modified Lade criterion. The most important reason for this is that laboratory tests are performed on intact pieces of rock, the stress parameters of which are significantly better than those of the cracked rock body. The crackedness of the rock body and the parameters of the cracks have an impact by orders larger than that of the middle principal stress. Accordingly, in practice, it is most typical to use criteria based on the Mohr failure condition. (Let me remark here that due to the ignorance of the impact of σ_2, rock strength is underestimated, which works in the direction of higher safety in the case of cave stability studies.) Such criteria based on the Mohr failure condition are the Mohr-Coulomb yield line, parabolic and hyperbolic curves and the Hoek-Brown curve, as well.

Nowadays, there is a preference for the application of the Hoek-Brown curve, which may be directly used to describe cracked rock bodies [2]:

$$\sigma_1 = \sigma_3 + \sigma_c \cdot \sqrt{m \cdot \frac{\sigma_3}{\sigma_c} + s}\ ,$$

where *m* and *s* are functions of rock quality and rock structure (GSI).

In addition to its simplicity, such a widespread application of the Hoek-Brown limiting condition is due to the fact that the authors processed a large number of empirical data. However, when it is applied to a new site, the question always arises: to what extent a fundamentally empirical formula based on data from another site may be adopted for a site with different rock parameters.

As an alternative process, the method developed in the Institute of Mining and Geotechnical Engineering of the University of Miskolc is suitable to decide this question.

3. A HYPERBOLIC CURVE BASED ON LABORATORY MEASUREMENTS AND THE 'ME' METHOD

It is a well-known fact that no linear curves may be applied in the whole range of stresses (compressive and pull stresses). In the range of pull stresses and small compressive stresses, failure may well be described with parabolic curves but such curves yield no good approximation in the case of large compressive stresses. In this range, the correlation between normal and shear stresses is much more linear. All this led to the idea that it is suitable to apply a hyperbolic failure curve across the whole range of stresses.

The main advantage of the process proposed by us is that every element is supported by measurement results. The conventional triaxial compressive strength of the rock is measured for uniaxial pull, uniaxial compression and different side pressures as many times as possible (at least 3 times each). The average values of the measurement results obtained in this way are plotted on the σ–τ plane (Mohr plane). Then, according to the well-known principles of function approximation, the hyperbolic failure curve best accommodated to measurement results is determined (Figure 3).

The more measurement results the curve thus constructed is based on, the more accurate it is. It should not be forgotten, however, that in laboratories, sample bodies formed from intact pieces of rock are examined. Therefore, the hyperbolic curve thus constructed characterises intact rock blocks instead of cracked rock bodies. The ME method offers an opportunity to convert the strength parameters obtained in laboratory into parameters of cracked rock bodies.

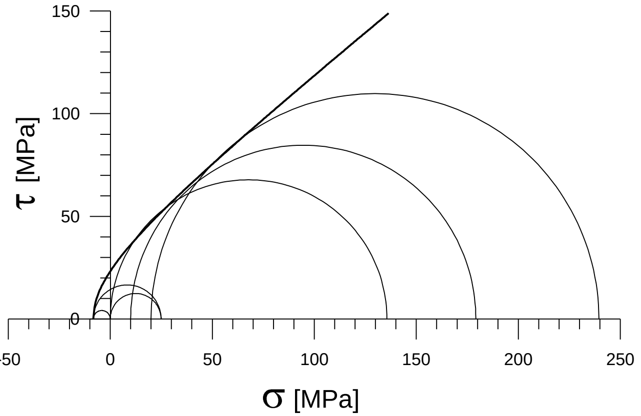

Figure 3 – Hyperbolic curve best accommodated to measurement results

The limits of this paper do not allow the detailed description of the ME method, elaborated at the University of Miskolc under the direction of Professor Somosvári. The essence of the process is that the so called reduction factor with the help of which the strength parameters of the cracked rock mass can be inferred from laboratory data is determined on the basis of measurements. The elasticity modulus may be measured in laboratory (both statically and dynamically), and with seismic methods directly in the cracked rock body, too. The reduction factor is specified as the rate of elasticity moduluses measured in laboratory and ’in situ’. For a detailed description of the method, see reference [4].

4. EVALUATION

We had an opportunity to apply the method in the rock mechanics studies of the dumping place of radioactive wastes of small and medium activity, under construction in Hungary. So far, the best hyperbolic failure curves have been specified for 13 rock groups. The results testify that the process proposed by us well supplements the application of other currently widespread methods, primarily that of the Hoek-Brown curves.

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