UDK 621.91

L. Kalmar, Dr., Associate Professor, Miskolc, Hungary

CALCULATION OF THE REAL PERFORMANCE CURVE OF RADIAL FLOW FAN IMPELLER

Головна мета цієї статті полягає в тому, щоб увести числові процедури, обчислити реальну криву роботи циліндричного планкового колеса радіального потоку. Особливості потоку, що належить різним пунктам обов'язку шанувальника також, були визначені числовою процедурою. У цьому обчисленні ефекти леза так само як ефекти рідкого тертя й завихрення були враховані окремо. Ефекти леза були представлені гідродинамічно обмеженою областю сили. Фрикційний ефект рідини й завихрення потоку були відбиті аналогічно між потоком у прямокутному каналі й потоком у планковому місці робочого колеса. Обчислено розподіли відносної швидкості, тиски й втрати енергії. Визначаючи втрату енергії, що належить різним нормам обсягу, приблизна реальна крива роботи робочого колеса може також бути визначена, належачи різної брутальності внутрішніх поверхонь робочого колеса.

Главная цель этой статьи состоит в том, чтобы ввести числовые процедуры, вычислить реальную кривую работы цилиндрического планочного рабочего колеса радиального потока. Особенности потока, принадлежащего различным пунктам обязанности поклонника также, были определены числовой процедурой. В этом вычислении эффекты лезвия так же как эффекты жидкого трения и завихрения были учтены отдельно. Эффекты лезвия были представлены гидродинамически ограниченной областью силы. Фрикционный эффект жидкости и завихрение потока были отражены аналогично между потоком в прямоугольном канале и потоком в планочном месте рабочего колеса. Вычислены распределения относительной скорости, давления и потери энергии. Определяя потерю энергии, принадлежащую различным нормам объема, приблизительная реальная кривая работы рабочего колеса может также быть определена, принадлежа различной грубости внутренних поверхностей рабочего колеса.

The main aim of this paper is to introduce numerical procedures to calculate the real performance curve of a cylindrically bladed radial-flow fan-impeller. The characteristics of the flow belonging to different duty points of the fan also were determined by the numerical procedure. In this calculation, the blade effects as well as the effects of the fluid friction and the turbulence were taken into consideration separately. The effects of the blade were represented hydro-dynamically by a constrain force field. The frictional effect of the fluid and the turbulence of the flow were reflected by the analogy between the flow in a rectangular channel and the flow in the bladed space of the impeller. Distributions of the relative velocity, pressure and energy loss are calculated. By determining the energy loss belonging to different volume rates an approximate real performance curve of the impeller can also be determined belonging to different roughness of the inner surfaces of the impeller.[2].

1. Introduction

The first main step of the calculation is to determine the change of the moment of momentum of the absolute non-viscous flow needed to determine the constrain force field. Next to them it is also possible to calculate the volume rate

Q_o at the optimal state of the fan impeller and the theoretical performance curve of the impeller. The second main step of the numerical procedure is to solve the system of the ordinary differential equation system based on the governing equations (equations of continuity, motion and energy) of the viscous relative flow on the main stream surface (F) of the fan impeller (Fig.1.). Applying the calculating results given by this way all the important characteristics of the flow can be determined. By using the calculated specific energy loss arisen in the impeller and theoretical performance curves of the impeller, the approximate real performance curves of the impeller can also be determined by subtracting from each other.

2. Theoretical Investigation

We have prepared only the short summary of the numerical method here. The additional and detailed information about the total numerical procedure can be found in [1, 2]. The applied numerical method is really an extension of the hydro-dynamical cascade theory for incompressible and non-viscous fluid flow. The basic equations of the calculation method are formulated in cylindrical co-ordinate system rotated together with the fan-impeller, where the co-ordinates in cyclical order are r, φ, z.

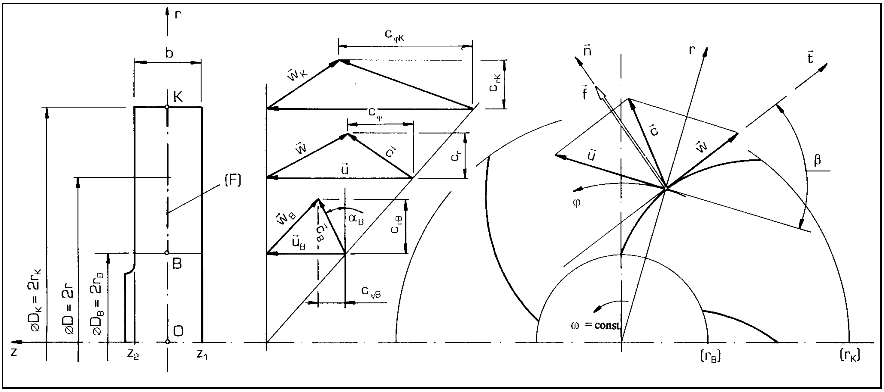

Figure 1 – Drawing and velocity triangles of the radial flow fan-impeller with backward curved blades

The Fig.1. – on left-hand side – shows the meridional cross-section of the fan-impeller, on right-hand side the shape of the blades viewed from the direction of the rotational axis of the impeller the directions of the co-ordinates and the vectors of the absolute velocity $\mathbf{c}$, the relative velocity $\mathbf{w}$, the peripheral velocity $\mathbf{u}$ and the specific constrain force $\mathbf{f}$ at an arbitrary point of the blade surface. In the middle of Fig.1. the velocity triangles can be seen at inlet-, outlet- and any arbitrary sections of the impeller. The blade angle β which can be measured between the tangents of the co-ordinate line $r =$constant and blade surface at the same point is also shown in Fig.1. The blade angle β uniquely determines the normal unit vector $\mathbf{n}$ of the blade surface.

2.1. The field of constrain forces in the bladed space of the impeller

The first step of the numerical procedure is to determine the components of the constrain force $\mathbf{f}$ needed to calculate the main characteristics of the viscous flow in the blade channel of the impeller.

Let us summarise the significant assumptions applied in determining the field of specific constrain force $\mathbf{f}$:

- The specific constrain force $\mathbf{f}$ - similarly to every mechanical constrain forces - expresses a friction-proof effect, in this way the specific constrain force $\mathbf{f}$ is parallel to the normal unit vector $\mathbf{n}$ of the blade surface.
- The frictional force is parallel to the wall near to it, so the specific constrain force and the friction force are perpendicular to each other.
- Since it is supposed that in the determination of the constrain force the fluid is non-viscous, so the specific constrain force and the relative velocity vectors are perpendicular to each other.
- The mean surface (F) of the meridional channel is a stream surface of the relative flow consequently the component w_z of relative velocity is equal to zero. At the same time, along this stream surface, the stress vector and the relative velocity vector are parallel to each other.

The equation of the motion relating to the absolute non-viscous steady fluid flow is as follows [1,2]:

$$rot\,\mathbf{c}\times\mathbf{w} = \mathbf{f} - \nabla\left(\frac{p}{\rho} + \frac{\mathrm{w}^2}{2} - \frac{u^2}{2}\right). \tag{1}$$

Multiplying Equ. (1) by the co-ordinate unit vector $\mathbf{e}_\varphi$, we can get the relationship to determine the component f_φ of the constrain force :

$$\mathrm{f}_\varphi = \mathrm{w}_r \;\; rot\,\mathbf{c}\;\;_z = \frac{\mathrm{w}_r}{r}\frac{\partial}{\partial r}\;\; r\,\mathrm{c}_\varphi \;\; .$$

The peripheral component f_φ of the constrain force is uniquely determined by the meridional velocity w_r and the change of the moment of momentum $r\,\mathrm{c}_\varphi$ in the absolute frictionless fluid flow which can be also expressed by a specific circulation γ :

$$\mathrm{f}_{\varphi} = \frac{\mathrm{w}_r}{r}\frac{\gamma}{2\pi} \,. \tag{2}$$

The meridional velocity component w_r and the specific circulation γ depend on the state of the operation and so consequently the constrain force also depends on the operating conditions of the fan. By using the conditions for the specific constrain forces $\mathbf{f}$ mentioned above we come to the expression between the two components of the specific constrain force as follows :

$$\mathrm{f}_r = \mathrm{f}_{\varphi}\frac{\mathrm{n}_r}{\mathrm{n}_{\varphi}} = \mathrm{f}_{\varphi}\, ctg\,\beta \,. \tag{3}$$

It is easy to realize that the radial component f_r of the constrain force can be determined by the geometrical data of the blades from Equ. (3) if the peripheral component f_{φ} of the constrain force is known. The component f_{φ} of the constrain force can be determined very simply way by the solution of inverse problem of hydro-dynamical cascade theory [1,2]. The constrain force field can be determined by the change of the moment of momentum in the absolute non-viscous fluid flow [1, 2]. To do this the determination of the specific vortex distribution γ is necessary to determine by the solution of the inverse task of the hydro-dynamical cascade theory.

2.2. Governing equations of the numerical procedure

For the relative viscous fluid flow in the bladed space of the impeller the equation of continuity is

$$div\ \ \mu\mathbf{w}\ \ = 0 \,. \tag{4}$$

The equation of motion in relative system can be written as follows :

$$\mathbf{w}\cdot\nabla\ \ \mathbf{w} + 2\omega\times\mathbf{w} = \mathbf{f} - \nabla\left(\frac{p}{\rho} - \frac{u^2}{2}\right) + \frac{1}{\rho} Div\ \sigma \,. \tag{5}$$

The form of the equation of energy for the relative flow is

$$\mathbf{w}\cdot\nabla\left(\frac{p}{\rho} + \frac{\mathrm{w}^2}{2} - \frac{u^2}{2}\right) = \frac{\mathbf{w}}{\rho}\cdot Div\,\sigma \,. \tag{6}$$

Let us introduce the following notations for the slope of the relative velocity

$$t = \frac{\mathrm{w}_{\varphi}}{\mathrm{w}_r} \,, \tag{7}$$

for the relative pressure potential

$$P = \frac{p}{\rho} - \frac{u^2}{2} \tag{8}$$

and for the specific relative energy

$$E = \frac{p}{\rho} + \frac{w^2}{2} - \frac{u^2}{2}\,. \tag{9}$$

the slope of the relative velocity. Considering the relations $t = w_\varphi / w_r = \tau_{\varphi z} / \tau_{rz}$ and $\tau = \sqrt{\tau_{rz}^2 + \tau_{\varphi z}^2}$ between the components of the shear stress, the energy equation can be written down as:

$$\frac{\partial E}{\partial r} = \frac{\sqrt{1+t^2}}{\rho}\frac{\partial \tau}{\partial z}\,.$$

The energy equation is directly available to determine the distribution of the shear stress τ in the relative flow. Similarly to the turbulent flow in the circular pipe we suppose the satisfaction of the next conditions for the turbulent flow in the rectangular channel bounded by the two neighbouring blades, the front plate and the back shroud of the impeller:

- The specific energy loss e'_S caused by the viscosity of the fluid and the turbulence of the flow concerning to length ΔL of the rectangular channel can be calculated by following well-know expression as:

$$\Delta e'_S = \lambda \frac{\Delta L}{D_H}\frac{\tilde{w}^2}{2} = -\Delta E\ ,$$

where D_H is the hydraulic diameter of the blade channel is expressed in the following form:

$$D_H = 4\frac{b\left(\frac{2\pi r}{N} - \frac{d}{\sin\beta}\right)}{2\left(b + \frac{2\pi r}{N} - \frac{d}{\sin\beta}\right)} = 2b\frac{\mu}{\mu + \frac{Nb}{2\pi r}}\,,$$

$\tilde{w}$ is the average relative velocity referring to the cross-section of the blade channel.

- In this case the shear velocity w^* can also be interpreted. It is supposed that connection between the shear velocity w^*, the average relative velocity $\tilde{w}$ and the dimensionless friction coefficient λ is similar to that of the circular pipe: $\lambda = 8\ \mathrm{w}^*/\tilde{\mathrm{w}}\ ^{2}$. The value of the friction coefficient λ varies along the blade channel.

Applying the conditions mentioned above in the energy equation we get a common differential equation for the shear stress. The shear stress τ can be also expressed by Karman's and Prandtl's formula where we applied an second order function to approximate the distribution of the mixing length [1,2]. By the solution of the given differential equation the distribution of the average relative velocity $\tilde{\mathrm{w}}$ can be determined. Similarly to the flow in circular pipe we can develop the expressions for calculation of the dimensionless friction factor λ in the cases when the inner walls of the impeller wetted by the fluid are hydraulically smooth or rough. The value of the friction coefficient λ varies along the blade channel. Knowing the distribution of $\tilde{\mathrm{w}}$ - by using the analogy to the flow in a pipe - it is possible to determine an expression for calculation the value of the dimensionless friction factor λ if the inner walls of the impeller wetted by the fluid are hydraulically smooth walls as follows :

$$\frac{1}{\sqrt{\lambda}} = 1.199;\sqrt{\frac{b}{D_H}}\left[\ln\ \mathrm{Re}\sqrt{\lambda}\ + \ln\left(\frac{b}{D_H}\right) + 6.861\sqrt{\frac{b}{D_H}} - 5.230\right]$$

where Re is the Reynolds number relating to the hydraulic diameter D_H of the cross-section of the blade channel which will have the form :

$$\mathrm{Re} = \frac{\tilde{w}\,D_H}{\nu}\ .$$

When the inner walls of the impeller wetted by the fluid are considered hydraulically rough walls – similarly to the formula developed by Colebrook – can be used to calculate the value of λ [1,2]. In this cases of course the values of friction factor λ is depend on the roughness k of inner wetted walls of the impeller.

Omitting the details the energy equation is available to determine the derivative of the specific frictional energy loss with respect to radius r:

$$\frac{d\,e'_S}{d\,r} = \frac{\lambda}{D_H}\frac{\tilde{\mathrm{w}}_r^2}{2}\sqrt{1+t^2} \qquad (10)$$

By rearranging the two component equations of the motion yields :

$$\frac{dP_F}{dr} = \mathrm{f}_{\varphi}\tan^{-1}\beta + \frac{\tilde{\mathrm{w}}^2}{1+t^2}\frac{1}{\mu}\frac{d\mu}{d\,r} + \frac{\tilde{\mathrm{w}}}{r}\left(\tilde{w} + \frac{2ut}{\sqrt{1+t^2}}\right) - \frac{\lambda}{2D_H}\frac{\tilde{\mathrm{w}}^2}{\sqrt{1+t^2}} \qquad (11)$$

$$\frac{dt}{dr}=\frac{1+t^2}{\tilde{w}^2}f_{\varphi}+\frac{t}{\mu}\frac{d\mu}{dr}-\frac{2u}{\tilde{w}r}\sqrt{1+t^2}-\frac{\lambda}{D_H}\frac{t}{2}\sqrt{1+t^2} \tag{12}$$

where $\tilde{w}$ is the average relative velocity, u is the peripheral velocity, $f_{\varphi}=\frac{\gamma}{2\pi r}\frac{\tilde{w}}{\sqrt{1+t^2}}$ is the peripheral component of the constrain force, β is the blade angle and μ is a factor to express the narrowing effect of the peripheral thickness of the blade.

The Eqs. (10)-(12) form a system of ordinary differential equations which can be solved by Runge-Kutta method on the main stream surface of the impeller by knowing the distribution of specific vortex γ before. The solution given by this way serves the slope of the average relative velocity $t\ r$, the distribution of the pressure $p\ r$ and the specific energy loss $e'_S\ r$ with respect to r. Next to them by knowing the distribution of the average relative velocity $\tilde{w}(r)$ and the slope of the average relative velocity $t\ r$ the radial and peripheral components of $\tilde{w}$ also can be calculated as:

$$w_r=\frac{\tilde{w}}{\sqrt{1+t^2}} \quad \text{and} \quad w_{\varphi}=t\,w_r\,.$$

The average relative velocity $\tilde{w}$ can be obtained by solving the equation of the continuity. The connection between the average relative velocity $\tilde{w}$ and its components can be written as Equ.(7) :

$$\tilde{w}=\sqrt{\tilde{w}_r^2+\tilde{w}_{\varphi}^2}=\tilde{w}_r\sqrt{1+t^2}\,. \tag{13}$$

Equ. (4) of the continuity and Equ. (13) are available to get an expression to calculate the value of $\tilde{w}$ as follows :

$$\tilde{w}=\frac{Q}{2\pi r b\mu}\sqrt{1+t^2} \tag{14}$$

where :

Q is the volume rate of the flow,

$\mu=1-\frac{N d_{\varphi}}{2\pi r}$ is the factor considered the effect of the blade thickness, (15)

N is the blade number of the impeller,

d_{φ} is the blade thickness in the direction of the co-ordinate φ .

The frictional energy loss from the inlet to the outlet of the impeller is $e_S^{'}\ r_K\ = \dfrac{\Delta\, p_S^{'}}{\rho}$ determined by solution of Eqs. (10-12). In this way the pressure losses from the inlet to the outlet of the impeller

$$\Delta\, p_S^{'}\ \ Q, r_K\ \ = \rho\, e_S^{'}\ \ Q, r_K \tag{16}$$

can be calculated.

After that it is possible to calculate the volume rate Q_o at the optimal state of the fan impeller and also the theoretical performance, so called "the theoretical total pressure difference — volume rate" curve of the fan impeller can be determined [1,2]. In that cases when the volume rate is not equal to the optimal volume rate Q_o of flow at shockless upstream the additional energy loss should be added to the previous one. The energy loss arises from the unsmooth upstream to the blades can be calculated as [2]:

$$e_T^{'}\ \ Q\ \ = \frac{\Delta\, p_T^{\ '}}{\rho}\ \ Q\ \ = 0.192\left(\frac{\omega\, r_K}{Q_o}\right)^2\ \ Q - Q_o\ \ ^2 \tag{17}$$

By using the values of the energy losses belonging to different volume rates and pre-wirl the real performance curves of the impeller can be determined [3, 4].

3. Application of the calculation procedure

Computerised solution as a numerical application of the calculation procedure introduced above is illustrated for a radial flow fan impeller which blades formed by straight cylindrical vane with constant thickness (see Fig.2.).

The initial data of the investigated fan impeller were the following :

- the diameters of the fan impeller at inlet was $D_B = 2r_B =$ 0.14 m and at outlet was $D_K = 2r_K =$ 0.3 m.
- the width of the meridian channel was constant b = 0.023 m, the thickness of the blades was also constant d =0.003 m,
- the number of the blades was N =17, the rotation speed of the impeller was n =3000 rpm.
- the density of the air was ρ =1.2 kg/m^3, the kinetic viscosity of the air was ν =1.0 10^{-6} m^2/s.

in our numerical applications. Computer code of the numerical procedure was developed by applying FORTRAN programming language to solve the flow problem numerically introduced above. This computer code was used to analyse and determine several characteristics the turbulent flow in the bladed space of the radial-flow fan impeller given by the initial data introduced above.

The Fig.3. shows the theoretical $\Delta p_{th}\;\;Q$ and real $\Delta p\;\;Q$ total pressure difference-volume rate curves of the fan impeller FAN-300/140-17. On the right hand side of Fig.3. the data of the operating points of the fan impeller are printed belonging to different states of the flow in the fan impeller: first for non-viscous fluid and later than for viscous fluids with different roughness of the inner walls of the fan impeller. The first column of the table contains the values of optimal volume rates Q_o, in the second column the values of the total pressure difference $\Delta p\;\;Q_o$ are printed and in the third column the values of roughness k can be seen.

The operating point number 1 in Fig.3. is belonging to the theoretical case, when it is supposed that the fluid is non-viscous and the characteristic cure is a straight line. The operating point number 2 is belonging to hydraulically smooth surfaces of the impeller. The operating points numbers 2-6 are belonging to hydraulically rough surfaces of the impeller with different roughness k. These characteristic curves for the fan impeller can be determined by subtracting of the theoretical characteristic curve $\Delta p_{th}\;\;Q$ and pressure losses $\Delta p_S^{'}\;\;Q, r_K$ and $\Delta p_T^{'}\;\;Q$ calculated by Eqs. (16-17).

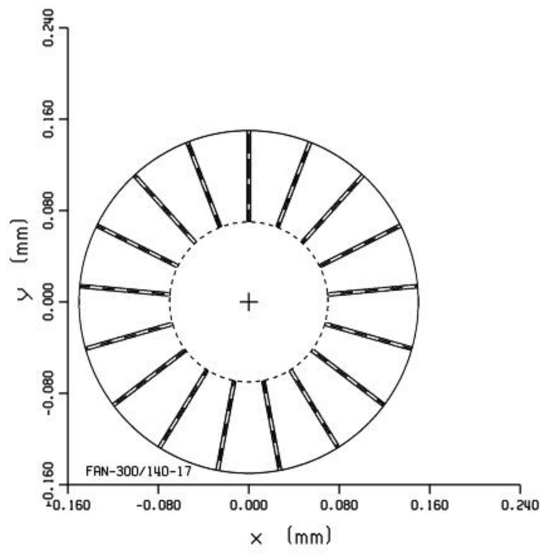

Figure 2 – Drawing of the radial flow fan-impeller with straight cylindrical blades viewing from rotational axes

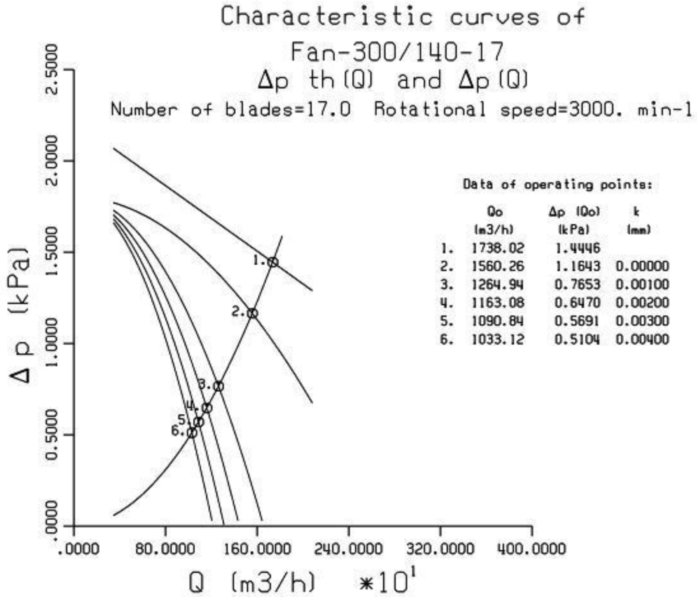

Figure 3 – The real total pressure difference-discharge characteristics
of the radial flow fan-impeller

The operating points are connected in Fig.3. by affine parabola. In this way an approximate real performance curves of the fan impeller can be determined to get information about the performance of fan impeller right after the design process and before manufacturing of the fan impeller.

References: 1. *Czibere, T.* Solution of the Two Main Problems of the Hydrodynamic Cascade Theory by the Theory of Potentials (in Hungarian), Thesis for D.Sc., Miskolc, 1965. **2.** *Kalmár, L.* Numerical Method for Viscous Flow in Radial-Flow Pumps (in Hungarian), Thesis for Ph.D., Miskolc, 1997. **3.** *Kalmár, L.* Numerical Analysis of the Flow in Radial Pump Impeller Acta Mechanica Slovaca 4/2004, pp.109-118., Kosice, 2004. **4.** *Kalmár, L.* Numerical Investigaton of the Flow int he Bladed Spaces of Radial-Flow Fan-Impeller RECENT ADVANCES in MATHEMATICAL and COMPUTATITONAL METHODS in SCIENCE and ENGINEERING, Proceedings of the 10th WSEAS International Conference on MATHEMATICAL and ENGINEERING (MACMESE'08), Part II., (ISSN: 1790-2769, ISBN: 978-960-474-019-2), pp. 328-333, Bucharest, Romania, 2008.

Поступила в редколлегию 15.04.2010