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MATHEMATICAL DESCRIPTION OF MACHINING EXTERNAL CYLINDRICAL SURFACE IN CASE OF ROTATIONAL TURNING

Rotational turning is one of the newly used methods in hard turning operations. Its complex kinematical relations and helical edged tool makes the study of this method more difficult than ordinary turning. The aim of this paper is to describe a case of turning with a mathematical model.

1. Introduction

The opportunity of machining hardened surfaces (where hardness is above 45 HRC) with defined edge geometry tools has been proved to be a significant advance. These machining methods have become available with the appearance of super hard CBN tools. Besides their hardness, which is near that of diamond, their chemical properties allow them to cut steel. This is the two main reasons wherefore that became the alternative for machining hardened surfaces. In this methods a typical saw-tooth like chip formation mechanism is can be seen [1]. There are several opportunities to carry out research in this mechanism, for example FEM analysis. However, in case of FEM we can see that at high speed the morphology of the chip formation mechanism changes [2].

Hardening heat treatments are applied mostly for the increase of toughness and wear resistance values. Therefore one of the main scopes of hard turning is the machining of tooth wheels. Previously precision machining of these parts could only be solved by grinding [3]. The reachable machining accuracy in hard turning is suitable to skip the grinding process [4]. For a better understanding of these methods there are research studies to determinate the processes in the surface [5]

The main disadvantages of hard turning are the followings: the passive force is very high during the process due to the application of a tool with negative rake angle, furthermore a micro-thread is generated on the surface of the workpiece because of the kinematic relations during the method. One opportunity to solve these difficulties is the application of the combined process. Another option is to change the kinematic relation by using other turning methods. One of these can be rotational turning [6] which belongs to the group of tangential feeds [7].

In rotational turning the cutting edge of the tool is a helical curve. The axis of this helix and the axis of the workpiece must be parallel. The chip formation is caused because the tool rotates with a very slow speed during the ordinary rotation speed of the workpiece [8].

The study of the rotational turning method is not easy when we use the helical edged tool. However the chip formation mechanism is very similar to the oblique cutting [9]. The chip flow angle depends on technological and material structural parameters [10]. Furthermore, in regard of the cutting force, the normal rake angle of the tool's cutting edge is essential [11]. Two dimensional mathematical equations [12] or elastic-plastic finite element models [13] can be used to describe the cutting process.

To describe the cutting process a properly chosen and defined kinematic model can be applied. For example such method is able to define the geometrical relations of toothwheels [14] or worm gear drives [15]. There is a research to apply the Monge projection for the description of the kinematic model [16]. The method presented by Perepelica [17,18] is able to define simple geometrical relations [19] during cutting without using of homogeneous coordinates. The model we described earlier [20] has proved promising. This paper describes the kinematic relations in a special case of the model: machining external cylindrical surface with rotational turning.

2. DEFINING THE TRANSFORMATIONAL EQUATIONS AND THE COORDINATE-SYSTEMS

The essence of the method described by Perepelica [17] is to define the equation of the machined surface with the transformed equation of the cutting edge and the relative movements of the workpiece and the tool. To do this the coordinate systems must be properly chosen and the equations describing the relations between these system must be written down.

Determination of the coordinate systems

Figure 1. shows the relative position of the workpiece and the tool's cutting edge, the main movements to be described and the chosen coordinate systems. The model contains the case with no feed in axial direction ($\mathbf{v_t = 0}$ condition must be fulfilled). One of the main aspects in the definition of the coordinate systems was to make the systems' **z** axes parallel to the axis of the machine's spindle. The remaining axes were drawn on order that the three axes would make the coordinate system right-handed. Another condition is that both the workpiece and the tool must have one fixed and one moving coordinate system. In general it can be said that the moving systems describe the main movements and the fixed systems describe the side motions of the tool and the workpiece. For a better distinction the axes of the moving systems are marked with Greek letters.

The symbols of Figure 1. are the followings:

K_t – fixed coordinate system (t.)
K_{tm} – moving coordinate system (t.)
K_w – fixed coordinate system (w.)
K_{wm} – moving coordinate system (w.)
1 – intersection point of the edge and

ω_w – angular frequency (w.)
ω_t – angular frequency (t.)
v_t – axial feed rate of the tool
a_w – centre distance
l_m – initial distance between moving

the surface and standing coordinate systems

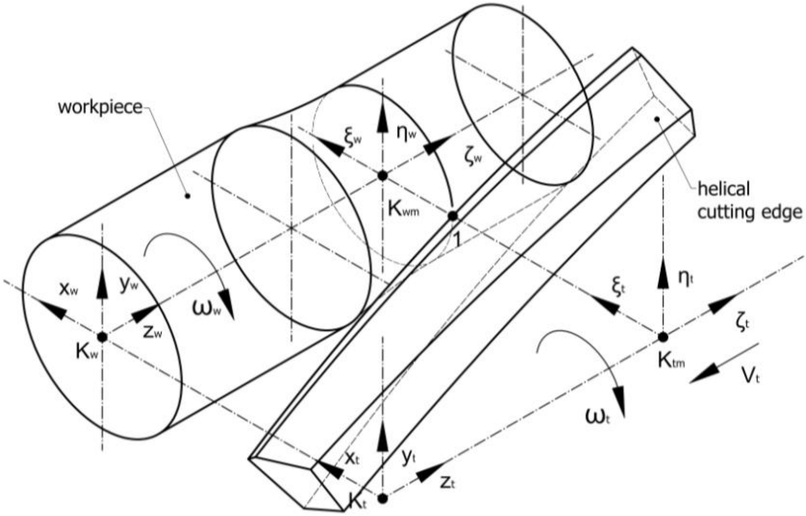

Figure 1 – Kinematical model of rotational turning

Defining the transformational equations

After the determination of the coordinate systems, the transformational equations describing the relations between them can be written down. A general attribute of these equations is that they contain a rotation matrix and a linear translation vector [17, 18]. If the vector equation is given in the initial system the form of the equation can be written down on the other system. Three transformational equations are needed because there is four coordinate systems. The equations are the followings:

$$\mathbf{r}_t = \mathbf{R}_{t,tm}\mathbf{r}_{tm} + \mathbf{t}_{t,tm} = \begin{bmatrix} \cos\omega_t t & -\sin\omega_t t & 0 \\ \sin\omega_t t & \cos\omega_t t & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{r}_{tm} + \begin{bmatrix} 0 \\ 0 \\ l_m - v_t t \end{bmatrix} \tag{1}$$

$$\mathbf{r}_w = \mathbf{R}_{w,t}\mathbf{r}_t + \mathbf{t}_{w,t} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{r}_t + \begin{bmatrix} -a_w \\ 0 \\ 0 \end{bmatrix} \tag{2}$$

$$\mathbf{r}_{wm} = \mathbf{R}_{wm,w}\mathbf{r}_w + \mathbf{t}_{wm,w} = \begin{bmatrix} \cos\omega_w t & \sin\omega_w t & 0 \\ -\sin\omega_w t & \cos\omega_w t & 0 \\ 0 & 0 & 1 \end{bmatrix}\mathbf{r}_w + \begin{bmatrix} 0 \\ 0 \\ -l_m \end{bmatrix} \tag{3}$$

The above equations can be summed up with the solution of the following equation:

$$\begin{aligned}\mathbf{r}_{wm} &= \mathbf{R}_{wm,w}\mathbf{r}_{w} + \mathbf{t}_{wm,w} = \mathbf{R}_{wm,w}\left[\mathbf{R}_{w,t}\left(\mathbf{R}_{t,tm}\mathbf{r}_{tm} + \mathbf{t}_{t,tm}\right) + \mathbf{t}_{w,t}\right] + \mathbf{t}_{wm,w} = \\ &= \mathbf{R}_{wm,w}\mathbf{R}_{w,t}\mathbf{R}_{t,tm}\mathbf{r}_{tm} + \mathbf{R}_{wm,w}\mathbf{R}_{w,t}\mathbf{t}_{t,tm} + \mathbf{R}_{wm,w}\mathbf{t}_{w,t} + \mathbf{t}_{wm,w}\end{aligned} \tag{4}$$

Accordingly the form of the vector equation (defined in the tool's moving coordinate system) is the following in the moving coordinate system of the workpiece:

$$\mathbf{r}_{wm} = \begin{bmatrix} \cos(\omega_t t - \omega_w t) & -\sin(\omega_t t - \omega_w t) & 0 \\ \sin(\omega_t t - \omega_w t) & \cos(\omega_t t - \omega_w t) & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{r}_{tm} + \begin{bmatrix} -a_w \cos\omega_w t \\ a_w \sin\omega_w t \\ -v_t t \end{bmatrix} \tag{5}$$

3. DEFINITION OF THE MACHINED SURFACE EQUATION

The proper equation of the cutting edge is necessary to define of the machined surface. The vector equation of a helical curve with $\mathbf{r_t}$ diameter and $\mathbf{90\text{-}\alpha_t}$ pitch angle is the following:

$$\mathbf{r}_{tm}(\zeta_p) = r_t \cos(\zeta_p)\mathbf{e}_\xi + r_t \sin(\zeta_p)\mathbf{e}_\eta + \frac{r_t}{\tan(\alpha_t)}\zeta_p \mathbf{e}_\zeta \tag{6}$$

Describing the two-parametric equation of the surface

The equation of the surface can be obtained if the equation written down in the moving coordinate system of the workpiece is rotated around the ζ axis by the following equation:

$$\mathbf{r}_s(\zeta, \upsilon) = \begin{bmatrix} \cos\upsilon & -\sin\upsilon & 0 \\ \sin\upsilon & \cos\upsilon & 0 \\ 0 & 0 & 1 \end{bmatrix} \mathbf{r}_{wm}(\zeta) \tag{7}$$

If we write the equation of the edge (6) into the defined transformation equation (5), then we do the previous multiplication (7), we will get the two-parametric equation of the surface:

$$\mathbf{r}_s(\zeta_p, \upsilon) = \begin{bmatrix} r_t \cos[\zeta_p + \upsilon + (\omega_t t - \omega_w t)] - a_w \cos(\upsilon + \omega_w t) \\ r_t \sin[\zeta_p + \upsilon + (\omega_t t - \omega_w t)] - a_w \sin(\upsilon - \omega_w t) \\ r_t \cot(\alpha_t)\zeta_p - v_t t \end{bmatrix} \tag{8}$$

Describing two-dimensional (one-parametric) equation of the surface

After defining the two-parametric equation, the one-parametric equation can only be defined with the determination of a side-condition. This condition is that the resultant vector must be in the **[ζ,ξ]** plane, that is the **η** component must be zero.

$$\eta_{r_s} = r_t \sin[\zeta_p + \upsilon + (\omega_t t - \omega_w t)] - a_w \sin(\upsilon - \omega_w t) = 0 \tag{9}$$

The **υ** parameter of the two-parametric equation (8) can be expressed from Equation 9. in function of **ζ**:

$$\upsilon(\zeta_p) = \arctan\left[-\frac{r_t \sin[\zeta_p + (\omega_t t - \omega_w t)] + a_w \sin \omega_w t}{r_t \cos[\zeta_p + (\omega_t t - \omega_w t)] - a_w \cos \omega_w t} \right] \tag{10}$$

The resulting equation helps to write down the two-parametric equation in the form of the one-parametric equation. We can use this equation to describe the machined surface during the cutting. In Figure 2. some of the cases (40°,45° and 50° pitch angle) can be seen. The hyperboloid shape can be easily observed.

$$\xi(\zeta) = r_t \cos\left[\frac{\zeta + v_t t}{r_t \cot(\alpha_t)} + \upsilon(\zeta) + (\omega_t t - \omega_w t) \right] - a_w \cos(\upsilon(\zeta) + \omega_w t)$$

$$\upsilon(\zeta) = \arctan\left[-\frac{r_t \sin\left[\frac{\zeta + v_t t}{r_t \cot(\alpha_t)} + (\omega_t t - \omega_w t) \right] + a_w \sin \omega_w t}{r_t \cos\left[\frac{\zeta + v_t t}{r_t \cot(\alpha_t)} + (\omega_t t - \omega_w t) \right] - a_w \cos \omega_w t} \right]$$

(11)

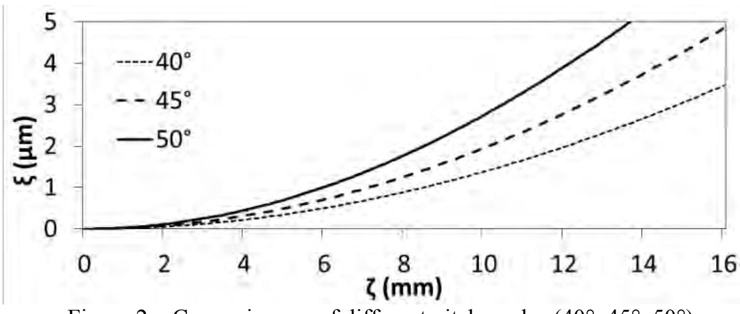

Figure 2 – Curves in case of different pitch angles (40°, 45°, 50°)

SUMMARY

Rotational turning is a method of complex kinematics and it requires complex tools for machining. The mathematical description of the cutting process is not an easy task as can be seen from the described equations. In general we can say that the exact mathematical model can be solved with the defined description. Other important geometrical attributes of the surface (2D and 3D surface roughness parameters) and the topography can be determined when other parameters are taken into consideration. There is also an opportunity to observe the complex shaped cross section of the chip and we can continue to further study the process.

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