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MATHEMATICAL AND COMPUTER MODELLING OF THE LOAD OF A SECTION OF THE TRANSPORT ROLLER LINE

Introduction. Due to the increased productivity of metallurgical production, the requirements for electric drive systems of mechanisms of rolling-mills are becoming stricter, including the standards for the roller lines [1]. Transport roller lines typically have a group electric drive, when a section of 3-10 rollers is driven by one or two motors. The rollers clip together by a mechanical transmission of conical gear or a gear chain [2]. Such an extensive mechanical system is characterized by a number of distributed and concentrated masses connected by spring ties and gears with gaps. It is most appropriate to conduct studies of the dynamics of the group electric drive in transport roller sections by using computer simulation in the software MatLab/Simulink. To achieve this goal we need to build with sufficient accuracy a mathematical model of the mechanical system. However, when metal is passing through the rollers, the mechanical system changes its configuration: additional variable static loads, which are distributed between the individual rollers, are increasing; the moment of the drive's momentum grows and changes over time; additional mechanical couplings between the individual rollers appear etc.

Formulating the Problem. The aim of this study is to develop a mathematical model, and a computer model based on it, of the loading (variable in time) of some rollers of a section in the transport roller line when slab is passing through them. For the sake of brevity, the basic laws of this problem will be investigated using the example of a section consisting of three rollers. Since we are determining the loading on the entire roller, the problem will be given a two-dimensional mathematical description.

Research Materials. In an arbitrary flat system of forces, the problem of finding them is statically indeterminate when the quantity of the algebraic unknowns is greater than three [3]. For roller transport lines, already in the case of the loading on three rollers the problem of finding the three forces will be statically indeterminate because we can put together only two independent equations: the balance of the projections of forces on the vertical *z*-axis, and the balance of torques relative to the axis of rotation of any roller. For all other axes of rotation, the torque equations will be a consequence of these two equations, because the right line, which connects these axes, is perpendicular to axis *z*. Based on this and the need of calculating the load on any number of rollers, we should include additional equations describing the processes of deformation in this mechanical system. These processes in the transport roller line consist in the deformations of the bending of the transported slab, which can be viewed in two dimensions (axes *x*-*z*) and described by the differential equations of the plane bending of the rod at axis *z* [4]:

$$E\,J_z(x)\frac{\mathrm{d}^2\delta(x)}{\mathrm{d}x^2} = -T_z(x), \qquad (1)$$

where *E* is the Young's modulus, which for steel is $E = 2{,}1\cdot10^5$ MPa; $J_z(x)$ is moment of inertia of the cross section of the rod at point *x*; $\delta(x)$ is a transverse bending of the rod at point *x*; $T_z(x)$ is the sum of torques of external forces at point *x*.

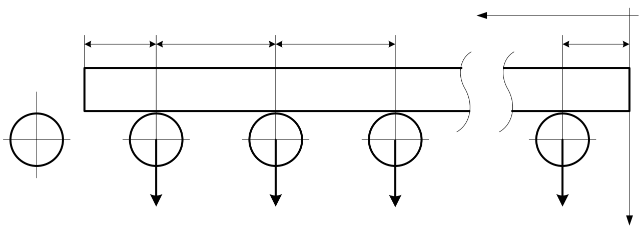

Fig. 1. The transport line with the slab, resting on *k* rollers

Let us build a mathematical model that describes the running loads, which the slab makes on the rollers, for an arbitrary number of them. Fig. 1 shows a two-dimensional image of the transport line with the slab, resting on *k* rollers. We will assume that the distance between the axes of rollers is the same:

$$m_1 = m_2 = \ldots = m_{\mathrm{k}-1} = m\,.$$

The first equation for this system is the balance of the projections of forces on axis *z*:

$$G_{\mathrm{Ri}} + G_{\mathrm{Ri+1}} + \ldots + G_{\mathrm{Ri+k-1}} = ql_{\mathrm{s}}\,,$$

where $q = \dfrac{G_{\mathrm{s}}}{m_0 + m_1 + \ldots + m_{\mathrm{k}}} = \dfrac{G_{\mathrm{s}}}{m_0 + (k-1)m + m_{\mathrm{k}}}$ is the linear weight of the slab; $G_{\mathrm{s}} = a_{\mathrm{s}}b_{\mathrm{s}}l_{\mathrm{s}}\gamma_{\mathrm{s}}g$ is the weight of slab; $a_{\mathrm{s}}, b_{\mathrm{s}}, l_{\mathrm{s}}$ are the width, the thickness and the length of the slab respectively; γ_{s} is the specific weight of the material of the slab; *g* is the gravity acceleration.

The second equation is the balance of torques relative to, for instance, the axis of the first roller:

$$mG_{\mathrm{Ri+1}} + 2mG_{\mathrm{Ri+2}} + \ldots + (k-1)mG_{\mathrm{Ri+k-1}} = [(k-1)m + m_{\mathrm{k}} - 0{,}5l_{\mathrm{s}}]ql_{\mathrm{s}}\,, \qquad (3)$$

where $(k-1)m + m_k - 0.5l_s$ is the distance from the axis of the first roller to the centre of the mass of the slab.

The remaining $k-2$ equations that are needed for determining k forces of pressure can be obtained as partial solutions of the differential equation of bending (1). Having integrated the latter twice, we will obtain a general solution as

$$E\,J_z(x)\delta(x) = C_1 + C_2 x - \frac{qx^4}{24} + H(x, m_k)G_{Ri+k-1}\frac{(x-m_k)^3}{6} + H[x,(m+m_k)]G_{Ri+k-2}\frac{[x-(m+m_k)]^3}{6} + \ldots + \\ + H\{x,[(k-1)m+m_k]\}G_{Ri}\frac{\{x-[(k-1)m+m_k]\}^3}{6}, \quad (4)$$

where C_1, C_2 are the integration constants; $H(x,y) = \begin{cases} 0 & if \;\; x \le y \\ 1 & if \;\; x > y \end{cases}$ is the explosive single function (Heaviside step function).

In points where the slab is leaning on the rollers, the size of slab transverse deformation $\delta(x)$ is equal to zero, therefore the left part of equation (4) also equals zero. By substituting in turns into equation (4) the values of x corresponding to the points of load forces on the rollers (respectively m_k, $1m + m_k$, $2m + m_k$, …, $(k-1)m + m_k$), we obtain k algebraic equations. By excluding from these equations the constants C_1, C_2, we get $k-2$ equations, which are necessary to determine the forces of pressure of the slab on the rollers.

Having completed the relevant conversions, we may write down the obtained system of algebraic equations in matrix form

$$\begin{Vmatrix} 1 & 1 & 1 & \cdots & 1 & 1 & 1 \\ 0 & m & 2m & \cdots & (k-3)m & (k-2)m & (k-1)m \\ 0 & 0 & 0 & \cdots & 0 & (1m)^3 & 2m\left[(2m)^2 - m^2\right] \\ 0 & 0 & 0 & \cdots & (1m)^3 & (2m)^3 & 3m\left[(3m)^2 - m^2\right] \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & (1m)^3 & (2m)^3 & \cdots & [(k-3)m]^3 & [(k-2)m]^3 & (k-1)m\left\{[(k-1)m]^2 - m^2\right\} \end{Vmatrix} \times \begin{Vmatrix} G_{Ri} \\ G_{Ri+1} \\ G_{Ri+2} \\ G_{Ri+3} \\ \vdots \\ G_{Ri+k-1} \end{Vmatrix} = \begin{Vmatrix} ql_s \\ qA_{k1} \\ \frac{q}{4}A_{k2} \\ \frac{q}{4}A_{k3} \\ \vdots \\ \frac{q}{4}A_{k(k-1)} \end{Vmatrix}, \quad (5)$$

where the elements of the column matrix in the right side of equation (5) depend on the position of the slab relative to the rollers on which it is leaning, which is characterized by the running coordinate m_k:

$$A_{k1} = [(k-1)m + m_k - 0.5l_s]l_s, \quad (6)$$

$$A_{k2} = \left[(2m+m_k)^4 - 2(m+m_k)^4 + m_k^4\right], \quad (7)$$

$$A_{k3} = \left[(3m+m_k)^4 - 3(m+m_k)^4 + m_k^4\right], \quad (8)$$

$$A_{k(k-1)} = \left\{[(k-1)m+m_k]^4 - (k-1)(m+m_k)^4 + m_k^4\right\}. \quad (9)$$

The resulting mathematical model (5)-(9) is the basis for building computer models of the dynamic load on the rollers of a section in the transport roller line. Since with the change of slab length the order of system of equations (5) will also change, we need to develop a computer model for a specific length of the slab. For instance, let us take the length of the slab as $l_s = 2.5m$. The subsequent numerical calculations take the following values of the parameters of the slab and the roller line: m = 0.7 m, a_s = 1.2 m, b_s = 0.2 m. For a steel slab, γ_s = 7.9·10^3 kg/m^3. Hence, $q = G_s/l_s = a_s b_s \gamma_s g = 1{,}858 \cdot 10^4$ N.

When the slab of the given length passes through the rollers, it will in turn lean on 2 or 3 rollers, as shown in Fig. 2, where numbers 1-11 mark 11 different positions of the slab against the three rollers of the selected section.

When the slab is placed on 2 rollers (k = 2), the matrix equation (5) will look as follows:

$$\begin{Vmatrix} 1 & 1 \\ 0 & m \end{Vmatrix} \begin{Vmatrix} G_{Ri} \\ G_{Ri+1} \end{Vmatrix} = \begin{Vmatrix} ql \\ qA_{21} \end{Vmatrix}, \quad (10)$$

where $A_{21} = (m + m_2 - 0.5l_s)l_s$.

The solutions of equation (10) are

$$G_{Ri+1} = \frac{q}{m}A_{21}, \quad (11)$$

$$G_{Ri} = ql_s - G_{Ri+1}. \quad (12)$$

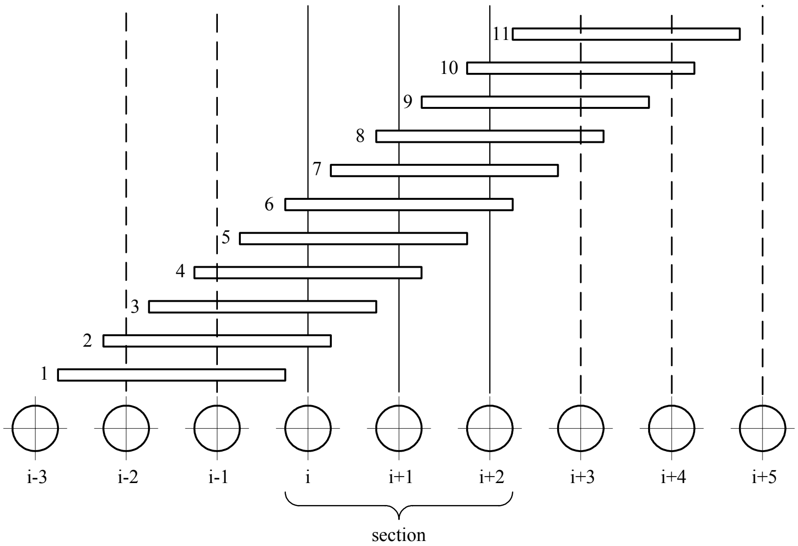

Fig. 2. Different placements of the slab (length $l_s = 2.5m$) against the rollers of the selected section

When the slab is placed on 3 rollers ($k = 3$), the matrix equation (5) will look as follows:

$$\begin{Vmatrix}1 & 1 & 1\\ 0 & m & 2m\\ 0 & m^3 & 6m^3\end{Vmatrix}\cdot\begin{Vmatrix}G_{Ri}\\ G_{Ri+1}\\ G_{Ri+2}\end{Vmatrix}=\begin{Vmatrix}ql_s\\ qA_{31}\\ \frac{q}{4}A_{32}\end{Vmatrix},\quad (13)$$

where $A_{31} = (2m + m_3 - 0.5l_s)l_s$,

$A_{32} = (2m + m_3)^4 - 2(m + m_3)^4 + m_3^4$.

The solutions of equation (13) are

$$G_{Ri+2} = \frac{q}{4m}\left(-A_{31} + \frac{1}{4m^2}A_{32}\right), \quad (14)$$

$$G_{Ri+1} = \frac{q}{m}A_{31} - 2G_{Ri+2}\,, \quad (15)$$

$$G_{Ri} = ql_s - G_{Ri+1} - G_{Ri+2}\,. \quad (16)$$

Table 1. The main dependences for modelling the load on the rollers when a slab with the length $l_s = 2.5m$ is passing through them

№ of op-tions	Terms	Amount of loaded rollers	Loaded rollers of section	Value of distance m_k	№ of loading force from Table 2		
					roller i	roller i+1	roller i+2
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	$x_c < 0$	0	-	-	1	1	1
2	$0 < x_c < 0{,}5m$	3	i	$m_3 = x_c$	6	1	1
3	$0{,}5m < x_c < m$	2	i	$m_2 = x_c$	3	1	1
4	$m < x_c < 1{,}5m$	3	i, (i+1)	$m_3 = x_c - m$	5	6	1
5	$1{,}5m < x_c < 2m$	2	i, (i+1)	$m_2 = x_c - m$	2	3	1
6	$2m < x_c < 2{,}5m$	3	i, (i+1), (i+2)	$m_3 = x_c - 2m$	4	5	6
7	$2{,}5m < x_c < 3m$	2	(i+1), (i+2)	$m_2 = x_c - 2m$	1	2	3
8	$3m < x_c < 3{,}5m$	3	(i+1), (i+2)	$m_3 = x_c - 3m$	1	4	5
9	$3{,}5m < x_c < 4m$	2	(i+2)	$m_2 = x_c - 3m$	1	1	2
10	$4m < x_c < 4{,}5m$	3	(i+2)	$m_3 = x_c - 4m$	1	1	4
11	$4{,}5m < x_c$	0	-	-	1	1	1

Table 2. Expressions for calculating the load forces on the rollers

№ of loading force	Amount of loaded rollers	Loading force	Expression to identifing of force
1	**2**	**3**	**4**
1	0	-	-
2	2	G_{Ri}	(11)
3	2	G_{Ri+1}	(12)
4	3	G_{Ri}	(14)
5	3	G_{Ri+1}	(15)
6	3	G_{Ri+2}	(16)

As a running coordinate, which determines the position of the slab, we select the distance x_c from the axis of roller i to the front edge of the slab (shown in Fig. 2 for variant 5), since the loading of the first of the three rollers of the selected section begins at $x_c > 0$. According to this coordinate, the key parameters and expressions that are necessary to calculate the loads on the rollers for 11 variants of slab position shown in Fig. 2 are given in Table 1. The forces of the load on each of the three rollers depending on the variants of slab position are marked with numbers 1-6 in the last three columns of Table 1, and the mathematical expressions corresponding to these numbers are listed in Table 2.

The data given in Table 1 and 2 serve as a basis for building computer models of the load of the roller section when the slab with the length $l_s = 2.5m$ passes through it. The computing algorithm can be divided into the following parts:

1) based on the start position of the slab and its moving speed, we determine its position regarding the selected section, which is the value of running coordinate x_c;

2) we identify the number of slab position according to the conditions given in column 2 of Table 1;

3) using column 5 of Table 1, we determine the distance (the length of the front of the slab that comes out as a console), which is the crucial coordinate in expressions for calculating the load forces and appears in formulas (6)-(9);

4) using columns 6-8 of Table 1 and Table 2, we determine the load forces on the rollers, which are passed on to the subsystem of the electric drive.

It is expedient to realize the described algorithm in such a way that according to the given coordinate m_k, the calculations of all the forces listed in Table 2 are being constantly performed, and the selection of the ones necessary is made by the number of the currently actual variant according to the columns 6-8 of Table 1.

The computer model of the system that determines the loads, which is based on this algorithm, is shown in Fig. 3. It consists of three subsystems.

In the subsystem of identification of slab position, Identification Subsystem (Fig. 4), the rollers' angular speed w, through which the slab moves, turns into a linear displacement x, which is calculated from a given starting position x0. When slab positions meet the conditions listed in column 2 of Table 1, the counter k acquires the values specified in column 1 of this table. Values of x and k are passed on to next subsystems.

In the subsystem of load calculations, Load Subsystem (Fig. 5), the distance from the front edge of the slab to the first roller on which it is based, i.e. value m_2 and m_3, is obtained on the basis of the values x and k from the previous subsystem according to the expressions given in column 5 of Table 1. On the basis of value m_2 and m_3, the parameters A_{21}, A_{31} and A_{32} are calculated first and then one calculates the forces according to the expressions listed in Table 2. The obtained values of forces in the form of a vector with 5 elements, which corresponds to column 3 of Table 2, are passed on to the next subsystem.

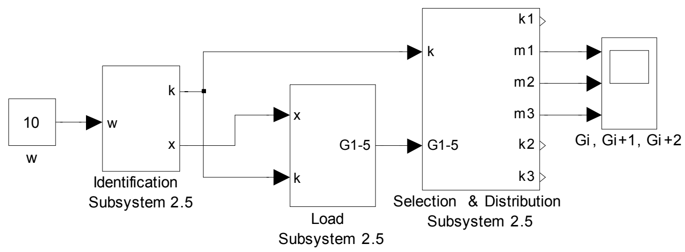

Fig. 3. The total computer model of the system of loads on the rollers when the slab is passing through them

In the subsystem of selection and distribution of loads on the rollers (Fig. 6), the formed vector of forces comes at the same time on the keys with 6 inputs Multiport Switch1 - Multiport Switch3, which according to the values of k1, k2, k3 (1 to 6) submitted to their control inputs, transfer the forces needed at the moment onto the outputs m1, m2, m3, which correspond to the load forces according to i, (i +1) and (i +2) rollers. Values k1, k2, k3, in turn, are formed by the relevant keys with 11 inputs Multiport Switch4 - Multiport Switch6, on which the forces with numbers according to columns 6-8 of Table 1 are submitted, and their selection is made according to the value of counter k, which passes onto the control inputs of all three keys.

To check the workability of the developed computer model, the constant value of the angular speed of the rollers, 10 rad/s, was given at the input of the system, which corresponds to the linear speed of the moving slab, 1.75 m/s. The obtained dependences of the load forces on each of the rollers are shown in Fig. 7. According to the developed methodology we created the analogical, but more complicate computer models for different lengths of slab. For example, Fig. 7, b shows the load forces of rollers at $l_s = 3.5m$. For each of the rollers, the nature of the load is symmetric and is repeated with a shift in time required for the passage of the distance between the rollers with the given speed of the slab.

Conclusions. As a result of solving the general problem of static loading on the rollers of a section in the roller line, we developed a methodology of computer simulation and obtained computer models that make it possible to calculate the loads on the rollers when slabs of varying lengths pass through them. These models are a subsystem of the total model of the section electric drive, which takes into account the peculiarities of the complex mechanical part.

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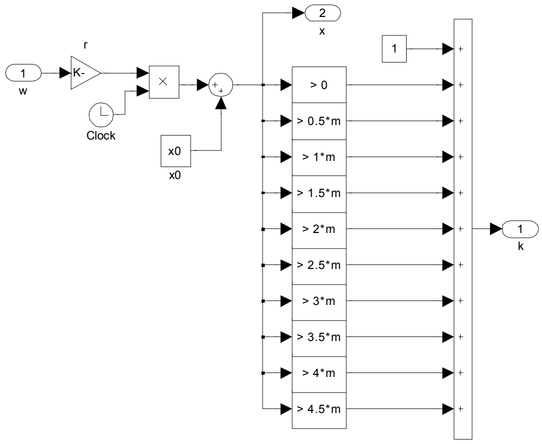


Fig. 4. The computer model of the subsystem that identifies the position of the slab (Identification Subsystem) for $l_s = 2.5m$

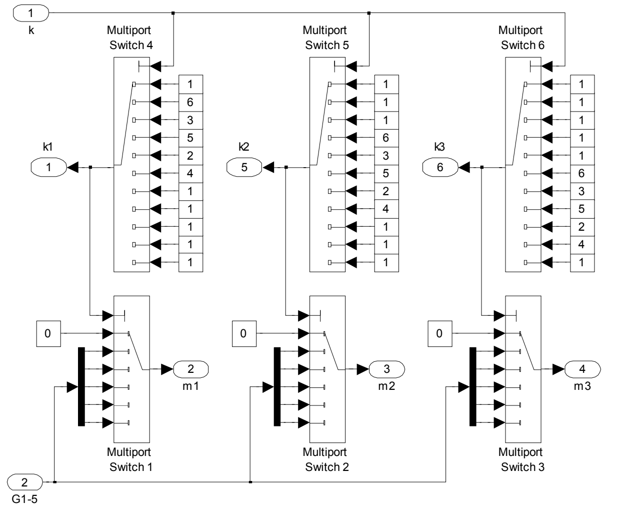


Fig. 6. Computer model of the subsystem of selecting and distributing loads on the rollers (Selection & Distribution Subsystem) for $l_s = 2.5m$

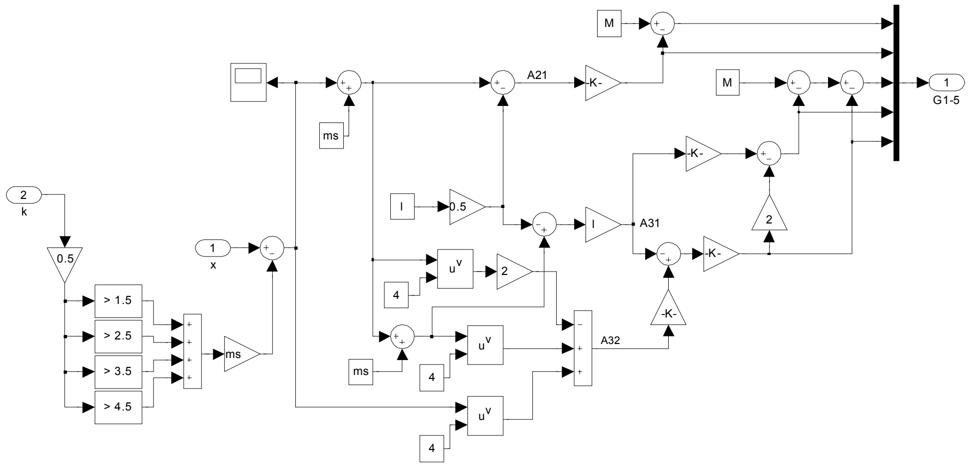
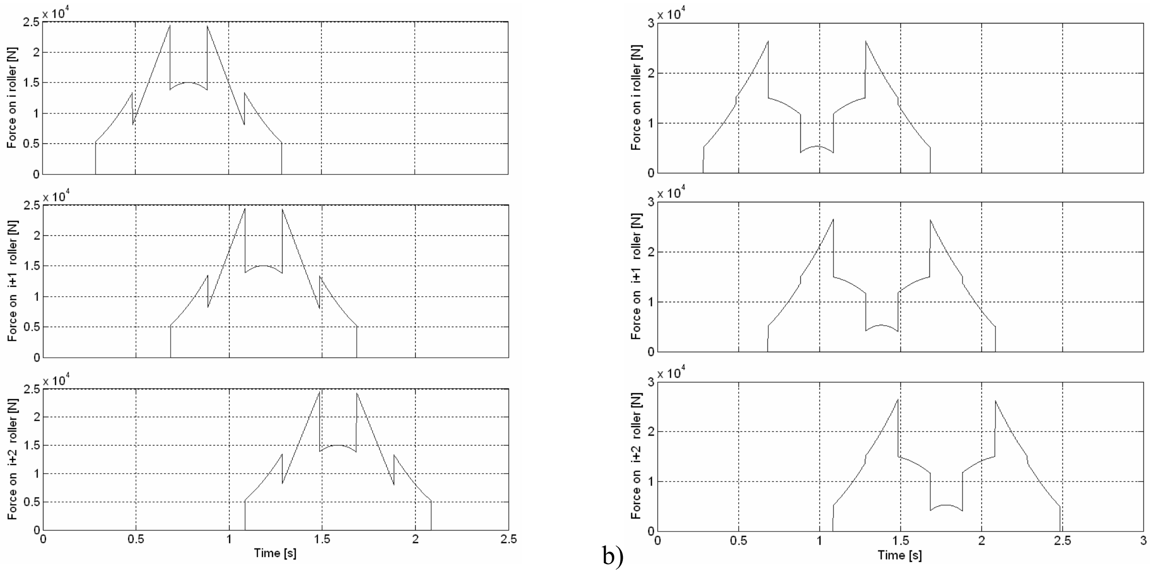


Fig. 5. Computer model of the subsystem of load calculations (Load Subsystem) for $l_s = 2.5m$



a) b)

Fig. 7. The results of computer simulations: time dependences of the load forces on the rollers G_{Ri}, G_{Ri+1} and G_{Ri+2} (kg) when the slab with length $l_s = 2.5m$ (a) and $l_s = 3.5m$ (b) passes through them at the speed 1.75 m/s