UDC 621. 833

M. BOŠANSKÝ, assoc. prof**.** , PhD., Mechanical engineering faculty, SlovakUniversity of Technology Bratislava
M. VEREŠ, assoc. prof**.** , PhD., Mechanical engineering faculty, Slovak University of Technology Bratislava

THE CONVEX-CONCAVE GEARING AS A POSSIBILITY OF INCREASING THE LOAD CAPACITY OF GEARING TO THE CONTACT

У статті є показаний математичний опис конвексно – конкавної / опукло – увігнутої / шестірні на основі векторного вираження ружу точки захоплення. Також тут введені та показані геометричні параметри конвексно – конкавної / опукло – увігнутої / шестірні та основні умови коректності / точності / захоплення цього тиру шестірні. Тут також описується вплив основних геометричних параметрів конвексно – конкавної / опукло – увігнутої / шестірні, щодо можливості підвищення несучої здатності у місці дотику конвексно – конкавної / опукло – увігнутої / шестірні. В заключній частині показані результати та иорівняння конвексно – конкавної/ опукло – увігнутої / та еволвентної шестірні з точки зору величини коитактних /дотиових/ тисків.

The problem of research. The formation of the surface damage of the tooth flank in gearing means the putting of gearing and thereby also all equipment out of service. Therefore the questions relating to the surface damage of the tooth flank in gearings are constantly in the centre of attention of professional interest. As for the surface damage the following damages arise in particular during operation of gearings: wear, seizing, fatigue damage of the tooth flank by pitting, breaking out of the surface layer and plastic deformation. The analysis of the mentioned surface damages of tooth shows that the magnitude of the contact stresses plays an important role in a matter of the load capacity of gearings. At a choice of the geometric parameters of the convex-concave gearing it is necessary therefore to base on a concrete requirements laid on such-and-such gearing. These requirements are given in detail in [1], resulting the task to design such type of the convex-concave gearing in which the lower values of the contact stress in comparison with the standard convex-concave gearing or the involute tooth system with the comparable geometric parameters would be reached. The analysis of individual failures of the surface damage of the tooth flank (pitting), seizing and formation of plastic deformation shown the magnitude of the reduced radiuses as a criterion. With regard to the divergence of effect of the individual geometric parameters the maximum values of the reduced radius of curvature of the convex-concave gearing in the pitch point C and at the same time also in points of the solitary mesh can not be reached. It follows that it is necessary to design the some rational values of the basic geometric parameters of the convex-concave gearing (the radius of curvature of the line of contact r_k and an angle of centre of the line of contact α_C), which would be optimal with regard to required aim.

Acknowledgement
The article was written within the solution of the project APVT 20-007602 with financial grant of Slovak APVT agency and project VEGA 1/3184/06 of Slovak VEGA Agency.

The analysis of literature. The convex-concave gearing belongs to the group of plane cylindrical gearing. The main feature of this gearing is that the shape of lateral curvature of tooth is formed from two general curvatures namely the convex and the concave ones. It is knowledge from theory that profile curves of toothing are dependent on the shape of the line of contact [1,9]. The line of contact defined for the convex-concave gearing consists of two circular arcs, see Figure No. 1, from which we receive the mentioned profile with inflex point in C point by means of parametric equation of motion of point along the line of contact.
The gearing of this sort arises hence in that case if the line of contact has „S“ shape, while in case of symmetric arcs the gearing is concave-convex – see Figure No.2 and in case of non-symmetric arcs the gearing is convex-concave – see Figure No.3 [1]. The point coordinates can be expressed mathematically by two ways namely from Altman construction [7], or from vector expression of motion of the mesh point [8]. The second possibility is more simple for calculation, where the point coordinates of the rack (1) and the line of contact (2) are directly functional dependences.

$$x = \mp 2r_{kh,d}\sin(\alpha-\alpha_C) \pm \frac{2r_{kh,d}}{r_1}\left[(\alpha-\alpha_C)\cos\alpha_C + \sin\alpha_C \lg\frac{\cos\alpha_C}{\cos\alpha}\right] \quad (1)$$

$$y = \pm 2r_{kh,d}\sin(\alpha-\alpha_C)\sin\alpha$$

$$x = \mp 2r_{kh,d}\sin(\alpha-\alpha_C)\cos(\alpha+\varphi_r(\alpha)) + r_1\sin\varphi_r(\alpha)$$

$$y = \pm 2r_{kh,d}\sin(\alpha-\alpha_C)\sin(\alpha+\varphi_r(\alpha)) + r_1\cos\varphi_r(\alpha) \quad (2)$$

$$\varphi_r = \pm\frac{2r_{kh,d}}{r_1}\left[(\alpha-\alpha_C)\cos\alpha_C + \sin\alpha_C \lg\frac{\cos\alpha_C}{\cos\alpha}\right]$$

The upper sign is valid for the mesh on the upper line of contact and the lower sign is valid for the lower line of contact. Similarly at calculation the value r_{kh} will be substituted for the upper part of the line of contact and the value r_{kd} will be substituted for the lower one. The intensity of calculation of profile of rack trace and subsequently also profile of the tooth trace is obvious at the mentioned equations. The accuracy of the point coordinates of profile is depended on step which expresses a change of an angle α_C. It follows that the more fine step is done at calculation the more precise gear wheel is manufactured.

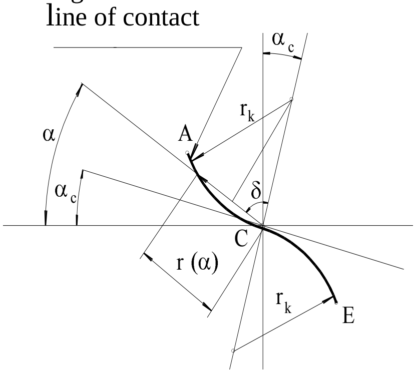

Fig.1 Line of contact convex-concave gearing

The shape of profile curve of the tooth flank of the convex-concave gearing and its basic kinematic and the geometric characteristics can be influenced by the change of the following parameters: the line of contact angle in the pitch point - α_C, the radius of curvature of upper part of line of contact - r_{kh}, the radius of curvature of lower part of line contact – r_{kd}, normal module
coefficient of height of tool head , coefficient of height of tool root and a number of teeth of pinion or wheel. A change of shape of profile curve of the tooth flank of the convex-concave gearing and thereby also a change of the mesh characteristics can be reached by the change of any of the mentioned parameters, or their combination. The change of characteristics cannot be optional but only in the limits which will ensure the creation of such tooth that in the gearing the convex part of tooth with concave part of the opposite tooth will come on the mesh and vice versa – i.e. the convex-concave mesh will be reached.

The condition of the convex-concave mesh thereafter in the gearing of which the gearing flanks will be created by two curves is evident on Figure No. 4. So that the tooth will be convex-concave in that case if the radiuses of curvature of points which create the profile of the tooth flank will be from part of **b** line of contact negative - ρ_{1b} and the radiuses of curvature of points which create the profile curve of the tooth flank will be from part of **a** line of contact positive - ρ_{1a} and vice versa at wheel the radiuses of curvature of points which create the profile of the tooth flank will be from part of **b** line of contact positive - ρ_{2b} and the radiuses of curvature of points creating the profile curve of the tooth flank will be from part of **a** line of contact negative ρ_{2a} [1,2,3].

In case that the above mentioned characteristics of the convex-concave tooth is valid thereafter the correct mesh of two teeth comes on at the convex-concave gearing that is the profile curves of the tooth flanks have the shape of the convex-concave tooth according to Figure No. 4. The basic geometric parameters of the convex-concave gearing can be chosen either individually or as a mutual combination. But at their some combination the situation can set in that the gearing is not convex-concave within the intention of the above given definition. Thereafter the tooth shape is such as is shown on Figure No. 5 from which is obvious that in this case the condition $\rho_{1b} \langle\ 0$, is not fulfilled, because the radius of curvature of pinion root $\rho_{1b} \rangle\ 0$ and C point is not inflexion point within the intention of definition of the convex-concave gearing. Thereafter the tooth of pinion is not convex-concave and therefore we cannot speak about the mesh of the convex-concave gearing.

Thereafter, it follows that the optional combination of the geometric parameters of the convex-concave gearing r_k, α_c, m_n, z, can be chosen but only such when the tooth will be convex-concave. Solution of condition of keeping the convex-concaveness of the profile curve of the tooth flank of pinion and wheel is based on correlations mentioned in [1].

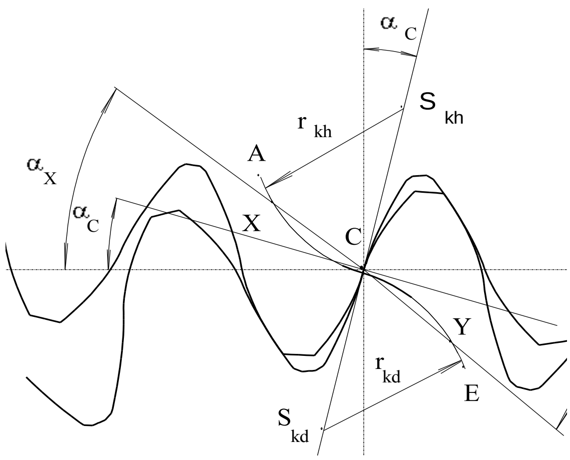

Fig. 2 Symetrical convex- concave gearing

From the point of view of the convex-concaveness for pinion the condition $\rho_{1b} \langle\ 0$ is decisive and similarly for wheel $\rho_{2a} \langle\ 0$. Thereafter the following inequation must be valid for the radius of curvature of pinion root:

$$-r+\frac{2r_1\ r_k\ \sin\alpha\cos(\alpha-\alpha_c)}{2r_k\ \cos(\alpha-\alpha_c)-r_1\ \cos\alpha}\langle\ 0 \quad (3)$$

and the following inequation must be equally valid for the radius of curvature of wheel root :

$$-r+\frac{2r_2\ r_k\ \sin\alpha\cos(\alpha-\alpha_c)}{2r_k\ \cos(\alpha-\alpha_c)-r_2\ \cos\alpha}\langle\ 0 \qquad (4)$$

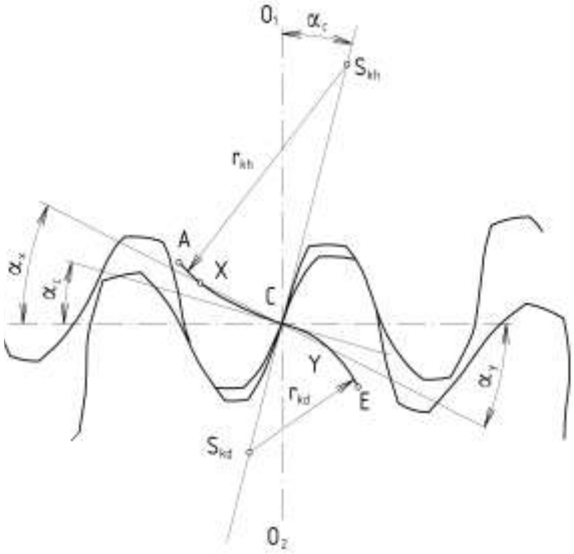

Fig.3 Nonsymetrical convex-concave gearing

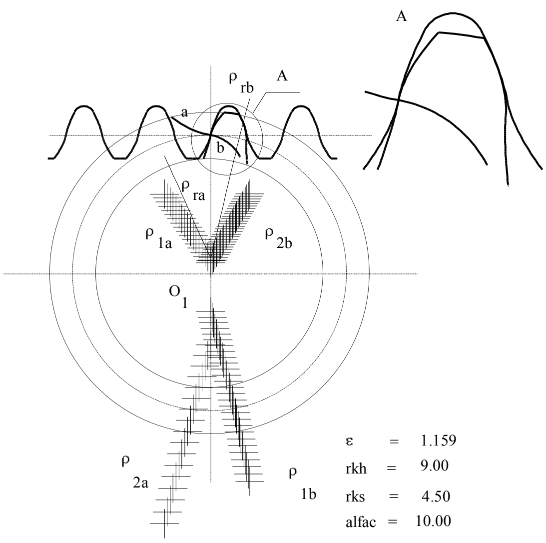

Fig. 4 correct meshingconvex-concave gearing

We came to expression of the mutual influence of the basic geometric parameters of the convex-concave gearing by solution of inequations (3) and (4) which can be expressed by the following inequality for pinion at the fulfilment of condition of correctness of mesh:

$$r_k \langle \frac{z_1\, m_n}{4} \cos\alpha_c \qquad (5)$$

and the following inequation is equally valid for convex-concaveness of the wheel tooth:

$$r_k \langle \frac{z_2\, m_n}{4} \cos\alpha_c \qquad (6)$$

The inequations (5) and (6) express the conditions of convex-concaveness but in scope of the mentioned inequations the values of the mentioned geometric parameters can be mutually changed thus the change of the mesh conditions of the convex-concave gearing can be reached. The analysis of influence of the basic geometric parameters of the convex-concave gearing to correctness of mesh which is stated in the mentioned [1] obviously shown that the mutual changes their values result in frequently opposing influence on the change of qualitative characteristics of the convex-concave gearing. At their choice it is necessary to base on a concrete requirements which are laid on such-and-such gearing.

Aim of article. Hertz was the first which engaged in problem of the power contact of two elastic bodies and therefore it is obvious from definition of Hertz stresses that their magnitude is also influenced by a tooth shape of the mating gears which is with relation to Hertz stresses specified by the reduced radius of curvature ρ_r.

Thereafter, decreasing the magnitude of the contact stresses at holding the same magnitude of other parameters can be reached by such change of a tooth shape when it come to the growth of the reduced radius of curvature of the tooth flank in gearing. However, if we want to shape the profile curve of the tooth flank so that the radius of curvature is increased it is necessary to know the influence of the geometric parameters of the convex-concave gearing on the change of a tooth shape. Figure No. 6 shows that at the constant value of a number of teeth of pinion (z_1=21) and wheel (z_2 = 36), at the constant module (m_n = 3) and the constant angle of line of contact (α_C = 10), the change of the magnitude of the reduced radius of curvature ρ_r, in point C - is constant, with the growth of the radius of curvature of line of contact, which changes from the value r_k = 4 up to r_k = 14 (the radiuses of upper and lower part are the same), does not change. But the magnitude of the reduced radius of curvature ρ_r in points of solitary mesh decreases.

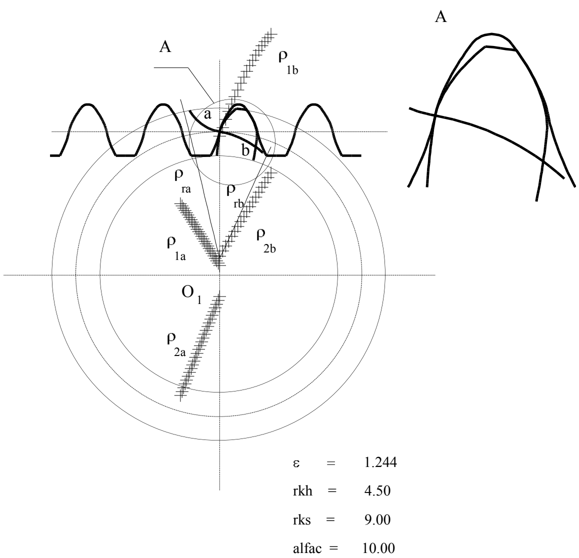

Fig. 5 Noncorect meshing convex-concave gearing

From the point of view of the shape of the convex-concave tooth, the increase of the radius of curvature of line of contact r_k in case that $r_{kh} = r_{kS}$ causes increasing the width of tooth on top and decreasing the width of tooth on root and vice versa as obvious similar on Figure No. 7. The same shows No. 8, at the constant value of a number of teeth of pinion and wheel, at constant module and the magnitude of the radius of curvature, at the convex-concave gearing with the growth of an angle of line of contact in the pitch point, which changes, for example, from the magnitude α_C = 8 degrees up to α_C = 26 degrees the reduced radius of curvature ρ_r, in point C grows. From the point of view of the shape of the convex-concave tooth, the increase of an angle of line of contact in the pitch point causes the decrease of the width of tooth on top and the increase of the width of the tooth on root and vice versa as obvious on Figure No. 9. Similarly, the width of tooth on top and the magnitudes of the radiuses of curvature ρ_r in points of solitary mesh decreases with the growth of the magnitudes of an angle of line of contact in the pitch point and the growth of the magnitudes of the radius of curvature of line of contact. On Figure No. 8 is also obvious at the same time that the growth of the magnitudes of the reduced radius of curvature ρ_r in point C and the growth of the width in root of the convex-concave tooth s_F increases. In case that the radius of curvature of upper part is different from the radius of curvature of lower part of line of contact thereafter at unchanged magnitudes of other geometric parameters the width of tooth on top s_a with the growth of the magnitudes of the radius of curvature of upper part of line of contact r_{kh} slightly increases while the width of tooth on root remains unchanged [1].

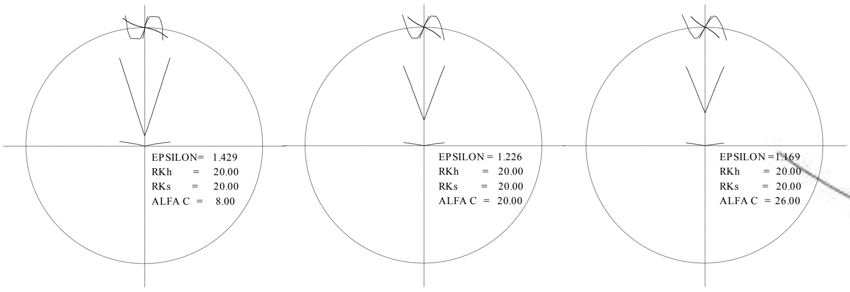

Fig.6 Influence of radius of curvatute on means radius of curvature

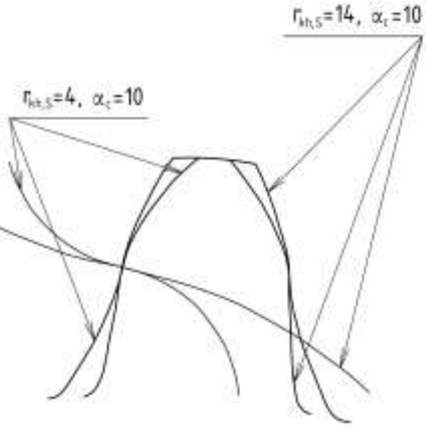

Fig.7 Influence of radius of curvature line of contact on tooth shape

Conclusion. On basis of the above defined relations it is necessary to conclude at determination of the geometric parameters of the convex-concave gearing with the defined mutual dependence of the basic geometric parameters according to relations (6.8) and (6. 9) stated in [3]. Thereafter the values processed in spatial graph [3] are valid for the mutual dependence of module m_n, a number of teeth of pinion z and the concrete angle of centre of line of contact α_C =18. They state the maximum values of the radius of curvature of line of contact r_k from point of defined aims. All smaller values of the radius of curvature of line of contact under surface in graph will fulfil the condition that the gearing will be convex-concave in case if we will take into account the constant value of an angle of centre of line of contact α_C =18. The similar graph can be constructed by the same way for the different value of an angle of centre of line of contact.

On basis of the detailed analysis of influence of the individual geometric parameters of the convex-concave gearing on qualitative factors in case that we base on requirement of the minimum value of the contact stresses, we deem the choice of the maximum possible values of the radius of curvature of line of contact r_k and the maximum possible values of an angle of centre of line of contact α_C the most suitable so as to the convex-concave gearing was functional (the condition of convex-concaveness of mesh, coefficient of continuation of mesh $\varepsilon_\alpha \geq 1.1$, coefficient of the width of tooth on top $s_a \geq 0.2\ m_n$). [3] shows that the some value of an angle of centre of line of contact α_C, which is constant also at the change of module m_n and transmission ratio u, exists for the some a number of teeth of pinion, or wheel z. Thereafter for the convex-concave gearing with the lower value of the contact stresses in gearing we suggest to choose the values of an angle of centre of line of contact α_C for a concrete number of teeth of pinion z according to graph stated in [3].

We recommend to choose the values of the radius of curvature of line of contact r_k in the values approximately 10% below the limit value following from conditions of convex-concaveness at the given concrete values of module m_n , a number of teeth z and an angle of centre of line of contact α_C [1]. detailed analysis of the convex-concave gearing shown that with the

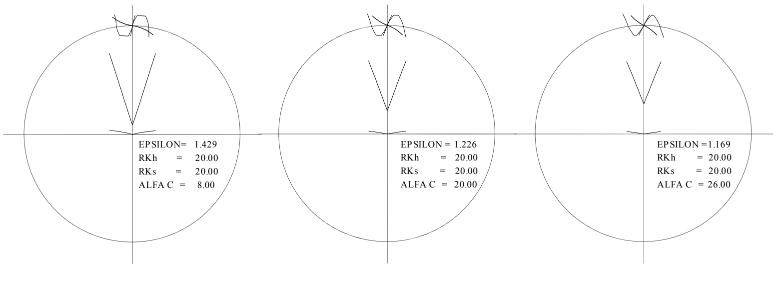

Fig.8 Influence of radius of curvature line of contact on mean radius of

growth of values of As for a influence other geometric parameters it is not such considerable. However the transmission rate, while other parameters are constant, coefficient of continuation of mesh ε_{α} grows only insignificantly. A influence of the growth of values of module m_n, when coefficient of continuation of mesh ε_{α} on the contrary decreases, is a bit significant.

However at the same time the growth of values of module m_n, at the same other parameters of gearing, will cause increasing the limit of an angle of centre of line of contact α_C, when the tooth is pointed. As for the coefficients of addendum or dedendum h_a^*, h_f^*, so the change of their values is limited by the magnitude of coefficient of continuation of mesh ε_{α}. With regard to its magnitude, the change of the mentioned coefficients can be used on rare occasions in particular in cases if the maximum possible values of an angle of centre of line of contact according to conditions of convex-concaveness [1,5,6] are chosen. The mentioned analysis of influence of individual the geometric parameters of the convex-concave gearing shown that the mutual changes of their values frequently result in divergent influence on the change of a tooth shape. With regard to the divergence of effect of the individual geometric parameters the maximum values of the reduced radius of curvature of the convex-concave gearing in the pitch point C and at the same time also in points of the solitary mesh – B, D can not be reached. It follows that it is necessary to design the some rational values of the basic geometric parameters of the convex-concave gearing (the radius of curvature of the line of contact r_k and an angle of centre of the line of contact α_C), which would be optimal with regard to required aim (decreasing of the magnitude of the contact stresses).

The mentioned analysis of influence of the individual geometric parameters of the convex-concave gearing shown that the mutual changes of their values frequently result in divergent influence on the change of a tooth shape. With regard to the divergence of effect of the individual geometric parameters the maximum values of the reduced radius of curvature of the convex-concave gearing in the pitch point C and at the same time also in points of the solitary mesh B, D can not be reached and thereafter also can not be reached such minimum values of the contact stresses, which would make it possible the use of biooils. Therefore we chose the some rational values of the basic geometric parameters with aim to reach the maximum possible values of an angle of line of contact in the pitch point α_C which can be chosen in dependence on a number of teeth of pinion from graph as stated in [1]. We recommend to choose the values of the radius of curvature of line of contact r_k in the values approximately to 10% lower than its limit values following from condition of convex-concaveness

which are expressed by inequations (1) and (2).

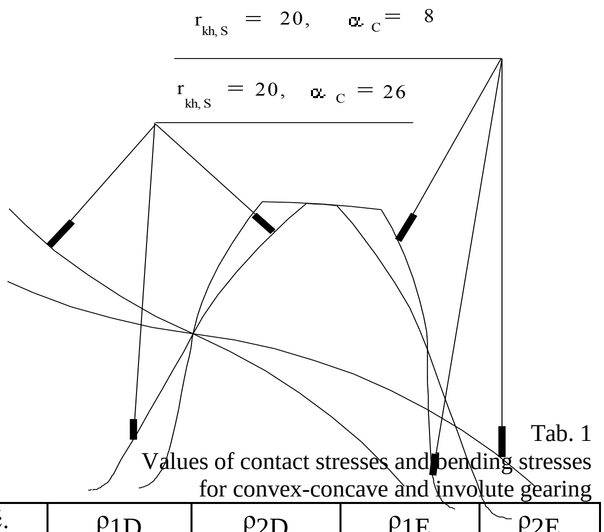

Fig. 9 Influence of radius of curvature line of contact on tooth shape

Tab. 1
Values of contact stresses and bending stresses for convex-concave and involute gearing

označ.	ρ_{1D}	ρ_{2D}	ρ_{1E}	ρ_{2E}
KK1	-404,17	28,55	779,78	150,4
E1	20,26	64,219	1138	167,1
KK2	-312,8	15,33	529,859	63
E2	9,058	29,277	778,788	61,8
KK3	-162,97	13,29	670,668	117,2
E3	9,112	26,872	902,026	124,3
KK4	-583,43	27,84	736,972	136,6
E4	26,19	37,181	1023	141,4
KK5	-126,05	9,05	883,428	141,1
E5	5,36	13,373	1289	136,9

The graphic and calculating model stated in [1,4] was used for solving of simulation of contact task of the convex-concave gearing.

Simulation of the contact task was solved by program system ANSYS. The similar mode was used also for solving of the contact task of the involute tooth system.

The result of solution show that in case when a tooth shape of the convex-concave gearing was designed from the condition of the maximum values of the reduced radius of curvature in the pitch point, the fundamentally lower values of the contact stresses in internal points of solitary mesh B,D – minimum by 25 % (see Table №1), were reached in comparison with the involute tooth system. In case of modeling a tooth shape of the convex-concave gearing with the aim to reach the maximum values of the reduced radiuses of curvature in points of solitary mesh B,D, the values of the contact stresses in comparison with the involute tooth system are still lower.

So by appropriate choice of the basic geometric parameters of the convex-concave gearing we can reach in the gearing fundamentally the lower values of the contact stresses in comparison with the involute tooth system without more fundamental worsening of other gearing parameters.

As an example we can show the gearbox with testing wheels – fig. № 10 of the little tractor. Here the original oil PP 90 (υ_{int} = 350 $\circ C$) was replaced for ecologic oil (υ_{int} = 120 $\circ C$). For the given load we found that integral temperature in the mesh for the involute tooth system was υ_{int} = 163,7 $\circ C$ and for the comparable convex-concave gearing υ_{int} = 104,4 $\circ C$, which means that the involute tooth system was not possible to lubricate with ecologic oli.

Fig. 10 Testing wheels with convex-concave gearing

On the contrary, the convex-concave gearing has satisfactory resistance to seizing also in case of using non-addititived ecologic oil with the low limit seizing temperature. At the same time the convex-concave gearing can be installed in original gearbox without any its modification.

List of literature. 1. *Bošanský, M.* : Voľba geometrických parametrov konvexno - konkáveho ozubenia z hľadiska povrchového poškodenia boku zuba. Habilitačná práca, 104 s, SjF STU, Bratislava 1997. **2.** *Bošanský, M., Vereš, M.* : Posúdenie tvaru zuba Konvexno -Konkávneho ozubenia z hľadiska jeho vplyvu na povrchové poškodenie boku zuba. Zborník referátov XXXIX. konferencie katedier častí a mechanizmov strojov, Liberec, 1998, s. 39 – 42. **3.** *Bošanský, M.,Vereš, M.* : Určenie hraničných hodnôt geometrických parametrov konvexno - konkávneho ozubenia. Zborník referátov Medzinárodnej vedeckej konferencie Strojné Inžinierstvo 98,Bratislava 1998, s. 241 – 248. **4.** *Bošanský, M.,Vereš, M.* : Pevnostná kontrola konvexno - konkávneho ozubenia pomocou MKP. Proceedings of 4th International Conference Dynamics of Machine Aggregates, Gabčíkovo 1998,s. 30 – 35. **5.** *Bošanský, M.,Orokocký, R., Vereš, M., Kožuch, I. – Nemčeková*, M. : Porovnie únosnosti K-K a evolventného ozubenia na zadieranie v interakcii s ekologickým olejom, In.: Sborník mezinárodní konference kateder části a mechanizmu stroju, Sedmihorky2005, ČR, ISBN 80-7083-951-1 s.23-26. **6.** *Bošanský, M., Kožuch, I., Vereš,* M. : Hobbing as The Possibility of C-C Gearing Production., in : The 2nd International Conference „ POWER TRANSMISSIONS 06“, 25-26 April 2006, Novi Sad, Serbia and Montenegro, p.267 – 270. **7.** *Hlebanja, J.* : Influence of the path of contact shape on sliding conditions between tooth flanks. Proceedings of MPT 91, nov. 1991 Hiroshima. **8.** *Vereš, M.* : Odolnosť ozubenia voči zadieraniu z hľadiska jeho tvaru. Kandidátska dizertačná práca, 147s, SjF SVŠT Bratislava 1987. **9.** *Vereš, M ,Bošanský, M.*: Teória čelného rovinného ozubenia, Vydavateľstvo STU Bratislava, 1999, ISBN 80 – 227 – 1226 – 4, 112 s. **10.** *Vereš, M, Bošanský, M., Gaduš, J.* : Theory of Convex-concave and plane cylindrical gearing, Slovak university of technology in Bratislava, 180 pp, ISBN 80-227-2451-3

Поступила в редакцию 01 .05.07