

вержены черновой механической обработке для приближения к окончательного контуру заготовки и термически обработаны, участок зубьев должен проверяться на соответствующую твердость. Для поверхностно-закаленных деталей окончательная твердость зубьев должна проверяться не разрушающимися методами.

Обеспечение высоких нагрузочных способностей и долговечности высокоскоростных редукторов [1] не возможно без обеспечения выполнения жестких требований к зубчатым передачам высокой точности и долговечности с применением специальных модификаций зубьев.

В Киевском политехническом институте на кафедре инструментального производства начаты работы по конструированию металлорежущего инструмента (червячной фрезы) для изготовления зубчатых колес таких высокоскоростных редукторов, с последующим изготовлением опытных зубчатых пар и проведение необходимого комплекса испытаний.

#### Общие выводы.

1. Разработка и внедрение национального стандарта [1] позволит гармонизировать технические требования по конструированию и изготовлению высокоскоростных зубчатых редукторов специального назначения для нефтеперерабатывающей и газовой промышленности.

2. Для выполнения требований к зубчатым колесам таких редукторов необходимо проведение комплекса конструкторских работ по созданию режущего инструмента с последующим изготовлением опытных образцов зубчатых колес и проведением их испытаний в рабочем диапазоне нагрузок.

**Список литературы:** 1. *ДСТУ ISO 13691* Нафтова і газова промисловість. Редуктори зубчасті високошвидкісні спеціального призначення. 2. *ГОСТ 16162* Редукторы зубчатые. Общие технические условия. 3. *ДСТУ ISO/TR 13593* Редуктори для промислового застосування. 4. *ДСТУ ISO 81400-4* Вітрові установки. Частина 4. Конструкція і технічні умови редукторів. 5. *ISO 1328-1:1995*, Cylindrical gears – ISO system of accuracy – Part 1: Definitions and allowable values of deviations relevant to corresponding flanks of gear teeth. 6. *ISO 3448:1992*, Industrial liquid lubricants – ISO viscosity classification. 7. *ДСТУ ISO 6336-5:2005* Розрахунок навантажувальної здатності циліндричних прямозубих і косозубих передач. Частина 5. Міцність і якість матеріалів. 8. *ДСТУ ISO 8579-1:2005* Правила приймання зубчастих передач. Частина 1. Правила випробування на шум. 9. *ДСТУ ISO/TR 10064-4*: Циліндричні зубчасті передачі – Практичні правила приймання – Частина 4: Рекомендації, що стосуються структури поверхні і перевірки плями контакту зубців.

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**P. TÖKÖLY**, Ing., PhD. student, Institute of Transport Technology and Engineering Design, Faculty of Mechanical Engineering, Slovak University of Technology, Bratislava, SR

**M. GAJDOŠ**, Ing., PhD. student, Institute of Transport Technology and Engineering Design, Faculty of Mechanical Engineering, Slovak University of Technology, Bratislava, SR

**M. BOŠANSKÝ**, Assoc. Prof., PhD., Institute of Transport Technology and Engineering Design, Faculty of Mechanical Engineering, Slovak University of Technology, Bratislava, SR

### EFFECT OF TOOTH SHAPE TO SIZE OF CONTACT STRESS NONINVOLUTE GEARING

В статье описывается влияние формы зуба с размером контактной напряженности. В геометрических параметрах выпукло-вогнутой передач можно изменить форму зубьев, которая может быть достигнута за счет сокращения размеров контактных напряжений. Важную роль играет сокращение радиусов кривизны, которая существенно влияет на размер этих напряжений. Приравниваясь два места вокруг самого начала (точка B) и конца (точка D) на линии зацепления, которые находятся в непосредственной близости от этих точек представлены одно и двухпарным зацеплением.

The article describes the effect of the tooth shape to the size of the contact stress. Using the geometrical parameters convex-concave gearing can change the tooth profile, which can be achieved by reducing the size of the contact stress. Important role played by reduced radii of curvature, which significantly affect to the size of these stress. Comparative two places around the beginning (point B) and end (point D) on the line of action.

**Introduction.** Gear transmission, as the most important member of mechanical transmission, for its function must meet demanding requirements in terms of geometric, material, manufacturing and operational conditions for a given operating load. Generally require gearing the following features: long life, low noise, positive mesh and slip rates, increased carrying capacity and wear resistance. Some of these properties can be achieved by selecting non-standard type of gearing and an appropriate choice of geometrical parameters. For non-standard type of gearing can be considered such a tooth profile whose geometry is different from the standard profile ( $\alpha=20^\circ$ ,  $h_a^*=1m_n$ ,  $r_f=0,38m_n$ ), or whose shape of involute is modified.

On the Institute of Transport Technology and Engineering Design the grant 1/0189/09 deals mainly with non-standard convex-concave (K-K) gearings [1, 2, 3,4, 5, 6, 7] and partly HCR gearings (High Contact Ratio) which has a coverage factor which is contact ratio  $\varepsilon_\alpha \geq 2$ .

Convex-concave (C-C) gearing is characterized as a gearing, the tooth flank which forms a curve composed of two arcs with convex and concave part as inflex point in pitch point C. This gearing arises if path of contact is S-shaped (Fig. 1). Existing studies C-C gearing have shown that compared with involute gearing touch the lower contact pressures [8] as well as keenly process slip conditions [9] which implies

the possibility of using ecological lubricant with lower viscosity, respectively lubricant without EP (extreme pressure) ingredients.

In carrying out strength analysis noninvolute type of gearing is not well-defined procedures and standards as to for involute gearing, so it is appropriate to use in their strength design of modern numerical methods (FEM – finite element method). Size of the contact stress are deal with the software environment, enabling the program ANSYS to simulate and provide results that are not normally get established calculation methods, which proved to be a need for a solution with non-standard C-C gear.

The aim of this paper is to carry out strength analysis of two pairs of C-C gearing (KK-1, KK-3), which are different geometric parameters of the profile tooth curves. Will be considered the two areas (area where are mesh one pair teeth: 1PZ and where are mesh two pairs of teeth: 2PZ) two characteristic points on the line of action – points B and D (Fig. 2).

**Material and methodology.** Limiting factor for most types of mechanical transmission is the contact stress combined with carrying capacity and durability of functional areas. When generating carrying members of mechanical gearing transmissions occur to the mutual contact of the relatively small area. The influence of load leads to distortions in the surface layers and have a touch on the desktop. The calculation of pressure is based on the Hertz formula derived from the power relationship between two circular cylinders with parallel axes of rotation, with a radius and width gearing  $b$ . Evenly distributed force  $F$  raises in the point of contact elastic deformation, which leads to a thin rectangle contact area. Spreading pressure is parabolic and its maximum value is given by:

$$\sigma_H = p_{\max} = \sqrt{\frac{\frac{F}{b} \cdot \left( \frac{1}{\rho_1} + \frac{1}{\rho_2} \right)}{\pi \cdot \left( \frac{1 - \mu_1}{E_1} + \frac{1 - \mu_2}{E_2} \right)}}, \quad (1)$$

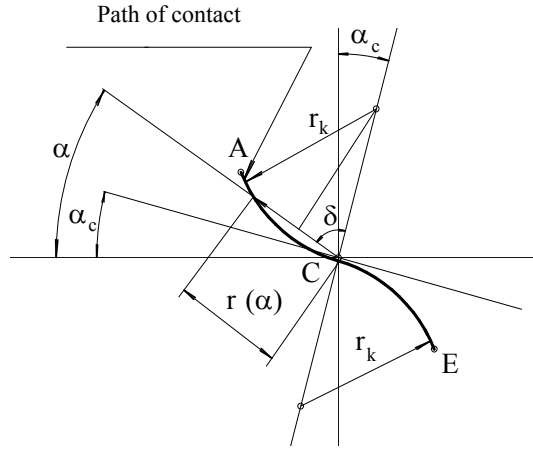


Fig. 1 – S shaped liene of action C-C gearing

where  $E_1, E_2, \mu_1, \mu_2$  are elastic constants of materials of each cylinder.

For  $\mu_1 = \mu_2 = 0,3$  is possible formula (1) modify the shape:

$$\sigma_H = p_{\max} = 0,418 \cdot \sqrt{\frac{F \cdot E_H}{b \cdot \rho_H}}, \quad (2)$$

where  $E_H$  [MPa] is modulus of elasticity reduced by Hertz,

$$E_H = \frac{2 E_1 \cdot E_2}{E_1 + E_2}. \quad (3)$$

$\rho_H$  is radius of curvature reduced by Hertz:

$$\rho_H = \frac{\rho_1 \cdot \rho_2}{\rho_1 + \rho_2}. \quad (4)$$

As seen from the formula (2), the size of contact pressure has affects to he tooth profile together meshing wheel, which is in the relationship represented by reduced radius of curvature  $\rho_H$ . On this basis, it can be concluded that reducing of the size of contact presures may be achieved by such a change in the shape of the tooth, which will increase the reduced radius of curvature of the teeth flanks in gearing. For the size of a radius of curvature at a specific point, make a formula (5) [8] – Fig. 3:

$$\rho_{1,A} = m r + \frac{2 r_1 r_k \sin \alpha \cos(\alpha - \alpha_c)}{2 r_k \cos(\alpha - \alpha_c) - r_1 \cos \alpha} \quad (5)$$

$$\rho_{2,A} = \pm r + \frac{2 r_2 r_k \sin \alpha \cos(\alpha - \alpha_c)}{2 r_k \cos(\alpha - \alpha_c) - r_2 \cos \alpha}$$

While the upper sign applies to the points over the top of path of contact (over the x axis) and the lower sign applies to the points below the axis of x. Then reduced the radius of curvature  $\rho_H$  for C-C gearings will be [8]:

$$\rho_H = \frac{\left\{ \frac{r_1 r_2 \sin^2(2\alpha - \alpha_c)}{\cos^2(\alpha - \alpha_c)} - 2 r_k \operatorname{tg}(\alpha - \alpha_c) [r_k \sin(\alpha - \alpha_c) \pm (r_1 + r_2) \sin(2\alpha - \alpha_c)] \right\}}{(r_1 + r_2) \sin \alpha}, \quad (6)$$

$r_{1,2}$  – centredes of pinion and wheel;  $r_k$  – radius of curve path of contact ( $h$ =top

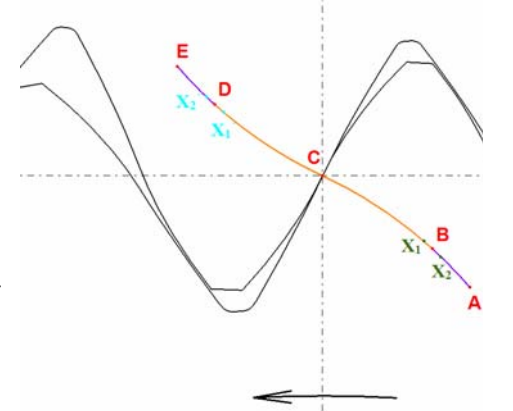


Fig. 2 – Line of action with specific points in which is realized strength analysis

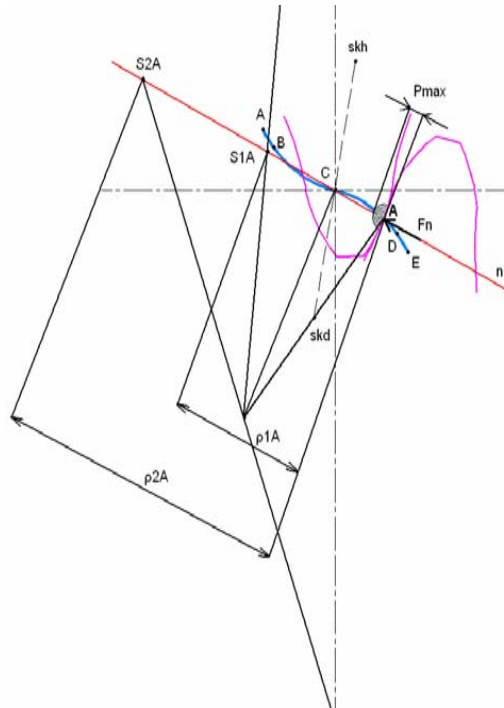
radius,  $d$ =bottom radius);  $\alpha$  – rake angle of path of contact in anywhere mesh point;  $\alpha_c$  – rake angle of path of contact in point C.  
The normal force expressed by the formula:

$$F_N = \frac{F_t}{\cos \alpha} \quad (7)$$

For the size of pressure force  $F$  is necessary get on the resulting force in the gearing  $F_n$ . In the area of meshing one pair of teeth is true equality  $F=F_n$ , but in the area of meshing two pairs of teeth force  $F_n$  is the distribution between the two pairs of teeth static uncertain, but for simplicity it is assumed that:  $F=0.5F_n$  [10].

On the basis of formulas (1) to (7) is possible at any moment to express the size of the scope of contact pressure meshing sides of the teeth as a function of position contact point on the line of action, while in role of position coordinate leaving the radii of curvature respectively. reduced radius of curvature. This procedure is possible expressed by analytical equations applied to noninvolute types of gearing only in the event of a plane model (2D), where there is no problem detect size of contact stress. It is preferable to use a three-dimensional model (3D), which has a comparison with the 2D model of the advantage that it is possible to depict and see the course of the

contact pressure in the longitudinal direction. In addition to finding the maximum pressures can also depict the resulting distortions arising from the contact forces in the direction of path of contact, which is important for estimating the life of gearing. Possible is also the course of finding the resulting relative stiffness of gearing depending on the position of the point on the path of contact, which is essential for the dynamic calculations of drives to the possible use of this type of gearing. In this 3D model is clearly demonstrated the advantages or disadvantages of the gearing modifications to the size of the contact pressures, or the influence of other inaccuracies.



**Experimental methods.** For an accurate analysis of C-C gearing were in AutoCAD using AutoLISP macro generated first plane 2D models of gears by the following parameters: top and bottom radius of path of contact –  $r_{kh}$ ,  $r_{kd}$ ; rake angle of path of contact in point C –  $\alpha_c$ ; normal module gearing –  $m_n$  and number of teeth –  $z_1$ , respectively.  $z_2$ . Thus was created by four gears with the parameters listed in Table. 1.

For each wheel there are two pairs of gearings, which is then adjusted, so that for strength calculation model was used only part of gear transmission, which is suitably replace the entire gearing. Areas in which it was executed strength calculation can be see in Fig. 2. It is the vicinity of beginning (point B) and end (point D) where is in mesh one pair teeth, while turning in the direction (in opposite the direction) loads the value by the  $0.5^\circ$  we get points  $X_1$  and  $X_2$  ( $X_2$ , and  $X_1'$ ). Points  $X_1$  and  $X_1'$  represent area where one pair of teeth mesh (1PZ) and points  $X_2$  and  $X_2'$  represent area where two pairs of teeth mesh (2PZ).

Table 1 – Characteristic of models C-C gearings

Type of gearing	$Z_1$	$Z_2$	$m_n$	$\alpha_c$	$r_k$	$\alpha$
	[-]	[-]	[mm]	[°]	[mm]	[-]
KK-1	27	40	4	25	15	1,15468
KK-3					21	1,20977

The geometric model of C-C gearing was constructed as the dextrorotatory system with two teeth segments (for the pinion) respectively. of three teeth (for wheels). Height of segment, amount the rim of pinions and wheel and from dedendum circle has a value 3.5 times the module gearing. Thus prepared a model has impact on the overall reduction in computing time performed strength analysis.

To construct a geometric model were used from the library of ANSYS-elements type of PLANE 42 and SOLID 45th With free networking was developed planar model of a network in the tooth. This model was subsequently elongate function "Extrudo", thereby creating a solid 3D model of the gearing. This will create in the contact mapped network, which in the transverse direction contained 40 nodes (Fig. 4). The size element in the lateral tooth profile, where there is active to create a contact or where the contact expect was 0.05 mm. Other side curve C-C gearing has size elements set to 0.5 mm. For the transient curve of the tooth and on the addendum circle distance elements was 0.25 mm, and the size of elements towards the active lateral area of the tooth was temper. The rest of the gearing on the rim wheel and pinions has element value set to 1 mm.

Contact areas were defined by the contact elements CONTA 174 and TARGED 170 and depending on (1PZ) respectively. (2PZ) were defined the contact area, too. For a description of the material used for the manufacture of

pinion and wheel were chosen a linear, elastic and isotropic material with modulus of elasticity value  $EX=2.1e5$  MPa and the Poisson number  $NUXY=0.3$ .

Contact surfaces were defined using the contact boundary conditions are defined as follows: rim of pinions was firmly connected via the contact elements CONTA 175 and TARGED 170 bond RIGID with established NOD in the beginning of the global coordinate system. In made NOD was allowed rotation around its axis of rotation (Z axis) and was granted torque 285 Nm. Rim of wheel had taken all the degrees of freedom, thereby was created fix body.

In the analyzed gearing during the meshing were evaluated two most commonly used hypothesis of equivalent stress: the hypothesis of maximum shearing stress (Sint) and hypothesis HMH (SEQV) and to determine the size of the contact stress (CONTPRES).

**Results and discussion.** Already on the basis of a comparison of geometric parameters (Table 1, Fig. 5) of the C-C gearing can be seen that the constant parameters ( $z_1, z_2, m_n, C$ ) the effect of the radius of curvature path of contact ( $r_k$ ) are also growth factor of the contact ratio ( $\epsilon_a$ ) [8] – Fig. 6. Together with the growth of radius of curvature decreases reduced radius of path of contact C-C gearing (Fig. 7). Specifically, the value of the results of strength analysis, which were examined reduced tension under hypothesis of maximum shearing stress and

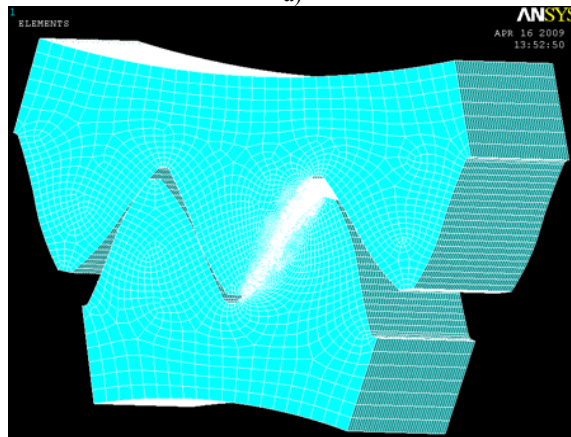
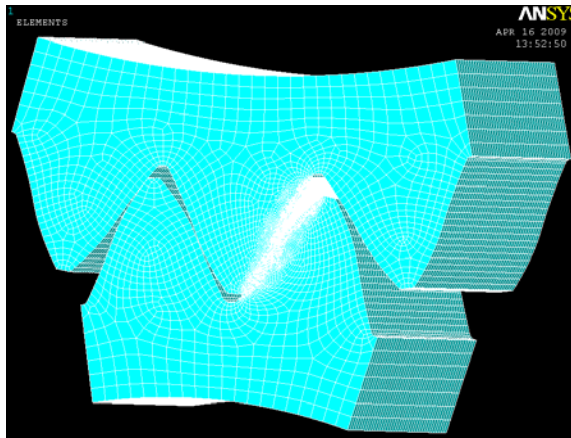


Fig.4 – Discretization of three-dimensional model C-C1 gearing:  
a) 1PZ, b) 2PZ

under the HMH hypothesis, as well as the size of the contact stress are shown in Table 2.

From the table of results it is evident that as to the size of reduced stress and the size of the contact pressure, is better gearing C-C-1. The results are more favorable than in the vicinity of point B, both in the vicinity of point D. The results confirmed by the Fig. 7, where the reduced radius of curvature of gearing C-C-1 is greater than in the C-C-2.

Graphical representation of all the results of the equivalent stress according to the HMH hypothesis for one pair teeth mesh in the vicinity of point B and point D for the gearing of C-C-1 is in Fig. 8. Fig. 9 are graphical results one pair teeth mesh gearing C-C-2. For better visibility of the course and distribution of colors was limited maximum stress in the range from 0 to 250 MPa. From Fig. 8a) and 9a), we see that the layout of stress is almost identical for both considered gearing in a given location. Pinion (bottom segment) is almost at the beginning of one pair teeth mesh and stress is spreading from contact point towards which the normal force acts in the place of contact. At this point, is the second most heavily weighted local area dedendum of pinion and the dedendum of wheel. This also applies to Fig. 12a) and 13a).

Vicinity of point D is shown in in Fig. 8b), 9b) and 12b). 13b) – is ending of one pair teeth mesh area, where the sizes of all the reduced stress as well as the contact pressures are less than it is in the vicinity of point B, almost at the beginning of one pair teeth mesh. The stress in the surrounding parts of gearing is spreading in the direction of normal forces and again second the most weighted place are the two dedendum both gearing.

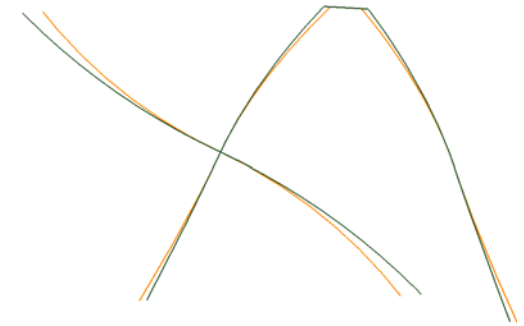


Fig. 5 – Effect of change of radius of curvature of path of contact ( $r_k$ ) to shape ones and profile curve of the tooth (orange: C-C-1, green: C-C-2)

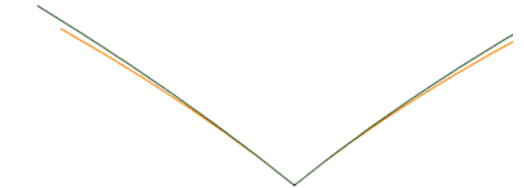


Fig. 6 – Effect of change of radius of curvature of path of contact ( $r_k$ ) to contact ratio ( $\epsilon_a$ ) (orange: C-C-1, green: C-C-2)

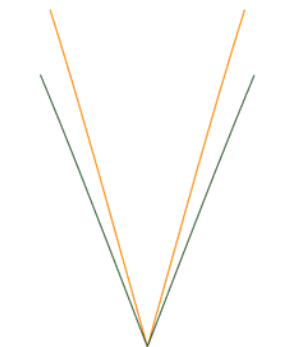


Fig. 7 – Effect of change of radius of curvature of path of contact ( $r_k$ ) to reduced radius of curvature ( $\rho_H$ ) (orange: C-C-1, green: C-C-2)

Table 2 – Result of stress analysis in designated pint of meshing

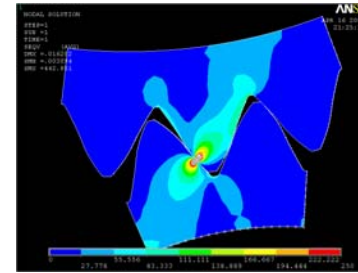
Type of gearing	results	Vicinity of point B [MPa]		Vicinity of point D [MPa]	
		X <sub>1</sub> (1PZ)	X <sub>2</sub> (2PZ)	X <sub>1</sub> (1PZ)	X <sub>2</sub> (2PZ)
KK-1	SEQV	442,851	348,813	416,48	325,618
	SINT	466,022	366,27	440,396	342,493
	CONTPRES	2429	1972	2326	1785
KK-2	SEQV	458,835	408,19	446,252	384,921
	SINT	488,404	436,599	469,452	399,792
	CONTPRES	2530	2229	2434	2108

In the two pair teeth mesh in the vicinity of point B (Fig. 10a), 11a), 14a) and 15a)) achieved all reduced and the contact stress lower value, as it is in one pair teeth mesh. It's understandable, since the normal operation of force is located on the two teeth. The greatest stress are in the contact area near the point B, where contact occurs and is the place where the approaching two pair teeth mesh in transition to one pair teeth mesh (in Fig. 2 is it between points A and B).

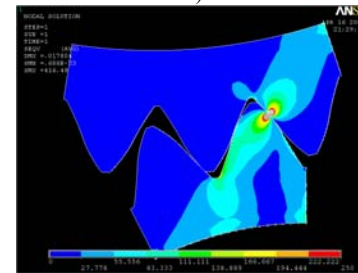
Second two pair teeth mesh area, near the place of D (Fig. 10b), 11b), 14b) and 15b)) is compared with the previous most congested in the addendum of pinion, but here all the stress has reached lower values, as in previous two pair teeth mesh area. Naturally, there is also stress distribution between the two teeth. This stress distribution is almost similar in two pair teeth mesh areas, where the important role played geometric parameters of gearing.

**Conclusion.** Article was designed to assess the size reduced and the contact stress in two pairs C-C gearings. Individual gearings to distinguish between the change in radius of curvature path of contact (see tab. 1). Judged were two nearby areas of the two characteristic points of path of contact, with both points into the one pair teeth mesh and two pair teeth mesh. Final results are displayed in the tab. 2 and show that the change in radius of curvature is growing contact ratio, but also decreases the size of the reduced radius of curvature, resulting in an increase in contact pressure in the gearing. In this example it is evident that suggest C-C gearing is not easy, because by changing the geometry can improve some parameters of gearing, but other parameters worsen. It is therefore necessary to make a compromise between the proposed geometry, depending on the functional parameters.

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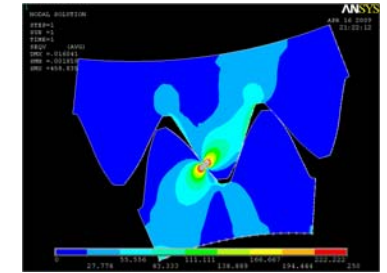


a)

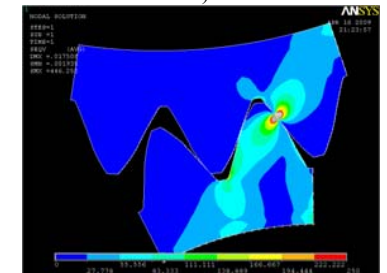


b)

Fig. 8 – Equivalent stress by the hypotesiz HMH (SEQV) for C-C-1 (1PZ): a) X<sub>1</sub>, b) X<sub>1</sub>'

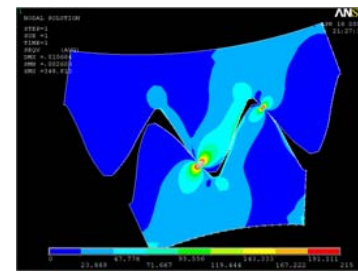


a)

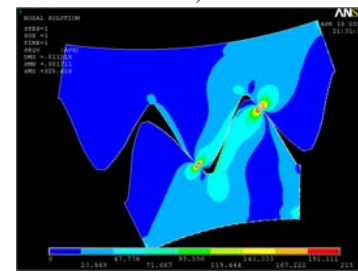


b)

Fig. 9 – Equivalent stress by the hypotesiz HMH (SEQV) for C-C-2 (1PZ): a) X<sub>1</sub>, b) X<sub>1</sub>'

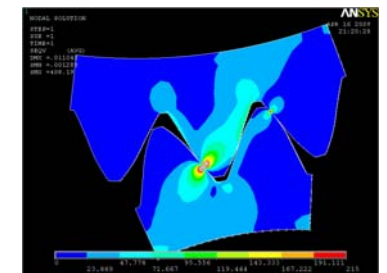


a)

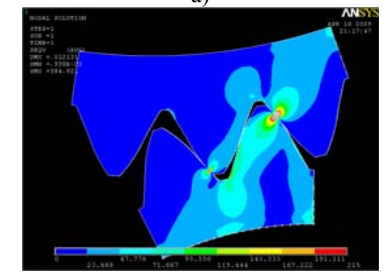


b)

Fig. 10 – Equivalent stress by the hypotesiz HMH (SEQV) for C-C-1 (2PZ): a) X<sub>2</sub>, b) X<sub>2</sub>'

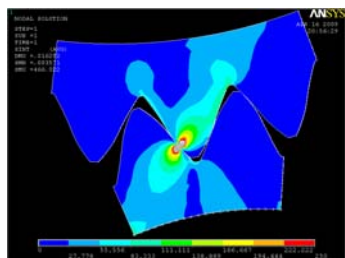


a)

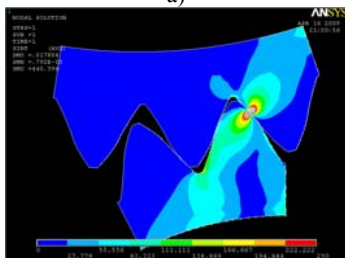


b)

Fig. 11 – Equivalent stress by the hypotesiz HMH (SEQV) for C-C-2 (2PZ): a) X<sub>2</sub>, b) X<sub>2</sub>'

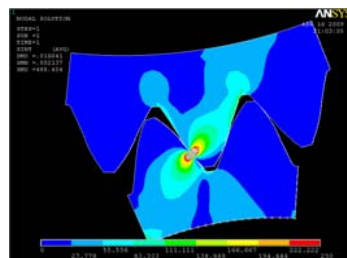


a)

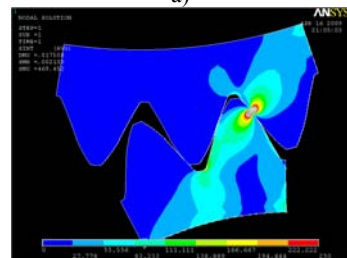


b)

Fig. 12 – The Equivalent stress by hypothesis of maximum shearing stress (Sint) (SINT) for C-C-1 (1PZ): a) X1, b) X1'

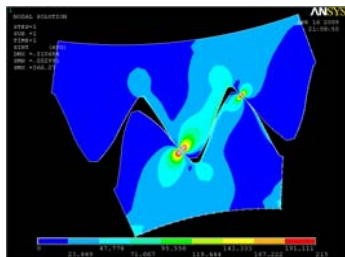


a)

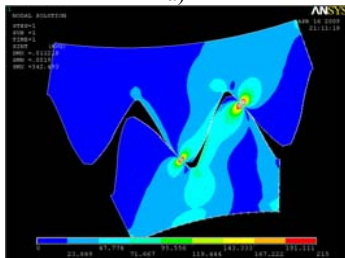


b)

Fig. 13 – The Equivalent stress by hypothesis of maximum shearing stress (Sint) (SINT) for C-C-2 (1PZ): a) X1, b) X1'

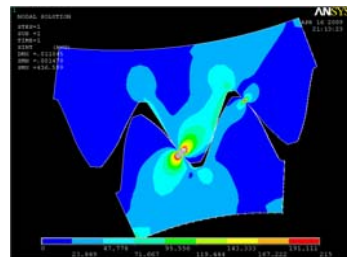


a)

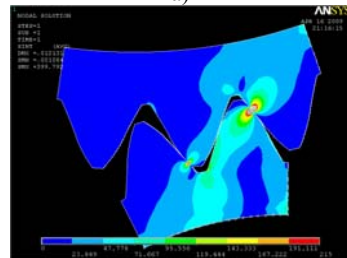


b)

Fig. 14 – The Equivalent stress by hypothesis of maximum shearing stress (SINT) for C-C-1 (2PZ): a) X2, b) X2'



a)



b)

Fig. 15 – The Equivalent stress by hypothesis of maximum shearing stress (SINT) for C-C-2 (2PZ): a) X2, b) X2'

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*Д.А. БАБИЧЕВ*, аспирант каф. МСП ТюмГНГУ (Нефтегазовый университет)  
*Д.Т. БАБИЧЕВ*, д. т.н., профессор каф. "Детали машин" ТюмГНГУ  
*А.А. СЕРЕБРЕННИКОВ*, д. т.н., зав. кафедрой МСП ТюмГНГУ  
*Е.Б. ПАНФИЛОВА*, ассистент каф. ПТ ТюмГАСУ (Архитектур. строит. ун-т)

## УНИВЕРСАЛЬНОЕ ПЛОСКОЕ ЗАЦЕПЛЕНИЕ И ТИПОВЫЕ ПЛОСКИЕ ПРОФИЛИ

Описане плоске універсальне зацеплення, у системи координат і рухів якого можна вписати вищу пару й утворюючі їхні ланки з будь-якого плоского механізму: зубчастого, кулачкового, із планетарним рухом, зубчато-кулачкового й інших. Запропонована система типових плоских профілів: від геометричних примітивів, до профілів зубчастих вінців зі зламами. Дані математичні моделі.

Plane universal gearing was describer, in system co-ordinates and moving of which can inscribed highest couple and created them link of any plane gear: caged, cammed, caged-cammed and etc. Suggested system of standard flat profiles: from geometrical primitives to profiles of gear rings with fracture. Made mathematical models.

**Введение.** Главные задачи классической теории зубчатых зацеплений (ТЗЗ) [1, 2]: 1. Найти сопряженную поверхность (задача формообразования); 2. Найти передаточную функцию и условия касания поверхностей (задача