

вогнутым контактом рабочих поверхностей зубьев. Далее он успешно развивал эту тематику и в 2009 году защитил докторскую диссертацию. К этому моменту он перешел работать в Харьковский Национальный автомобильно-дорожный университет ХАДИ, но основные теоретические положения его диссертации были разработаны в НТУ "ХПИ" при научной консультации проф. А.Ф. Кириченко.

Разработкой и исследованием геометрии и контактной прочности двухпараметрических зубчатых передач на кафедре КГМ занимались *Николай Анатольевич Ткачук* и *Александр Витальевич Устиненко*. Последний под руководством проф. Н.Э. Тернюка и проф. В.Н. Загребельного защитил в 2000 году кандидатскую диссертацию по этой тематике. Также на кафедре КГМ выполнялись работы по созданию зубчатых вариаторов скорости на основе идей *Вадима Романовича Ковалюха*. Этой тематикой занимались *Роман Вадимович Ковалюх* и *Дмитрий Олегович Волонцевич*, защитивший в 1995 году кандидатскую диссертацию, посвященную синтезу вариатора скорости на основе составных зубчатых колес.

В настоящее время "центр тяжести" исследований в области зубчатых передач сместился на кафедру Теории и систем автоматизированного проектирования механизмов и машин (ТММ и САПР). Здесь ведутся работы по нескольким направлениям:

- исследования новых видов зацеплений: эволютного и двухпараметрического;
- совершенствование теоретических основ оценки усталостной изгибной прочности цилиндрических зубчатых передач;
- оптимизация зубчатых приводов по массогабаритным характеристикам.

Следует также отметить, что кафедра ТММ и САПР стала с 2008 года базовой по организации проведения международной научно-технической конференции "Проблемы качества и долговечности зубчатых передач, редукторов, их деталей и узлов" в Севастополе. Кафедра является ответственной за издание Вестника НТУ "ХПИ", тематический выпуск "Проблемы механического привода". Начата работа по открытию в НТУ "ХПИ" межотраслевой проблемной лаборатории "Разработка и исследования прогрессивных видов зубчатого зацепления и механического привода".

В перспективе планируется развивать направления компьютерного моделирования НДС новых видов зубчатых зацеплений и расчетно-экспериментального исследования их геометрии и прочности. Также планируется оснащение современным лабораторным оборудованием для экспериментальных исследований. Ведется работа по написанию монографий и учебников, посвященных зубчатым передачам.

Таким образом, исследования зубчатых передач, имеющие в нашем университете богатую историю, получают второе дыхание и выходят на качественно новый уровень, не уступающий мировым достижениям.

Поступила в редколлегию 14.06.10

УДК 621. 833

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### THE EFFECT OF GEAR GEOMETRY ON THE THICKNESS OF TOOTH FACE HARDENED LAYER

В статье описывается влияние геометрических параметров стандартных эвольвентных и нестандартных выпукло-вогнутых (С-С) зубчатых передач до толщины упрочненного слоя. Толщина упрочненного слоя имеет важное значение для износа зуба. В случае с эвольвентной передачей толщина слоя определяется более авторами, и также использованием стандарта STN 01 4686-5. В случае нестандартных выпукло-вогнутых (С-С) зубчатых передач нет стандартов, поэтому можно определить толщину закаленного слоя только с использованием современных методов моделирования.

This article describes the effect of the geometrical parameters of the standard involute and non-standard convex-concave (C-C) gearing on the thickness of the hardened layer. The thickness of hardened layer is important from the aspect of wear on gearing. In case of involute gearing is the thickness of the hardened layer defined by various authors, what is on the other hand determined also by the standard STN 01 4686-5. In case of C-C gearing there are not available any standards, and therefore it is possible to determine the thickness of the hardened layer only by the means of modern simulation methods.

**Introduction.** The simplest gearing is created with one pair of meshing gears, where the tooth faces are creating a kinematics couple. This kinematics couple serves to the transition of rotational movement and mechanical energy. The basic criterion of a gearing is to acquire a continual tooth mesh what is defined in the fundamental law of gearing Fig.1: the continual mesh of two profiles occurs, when the mutual normal line in the point of contact proceeds through the pole of relative motion C in every moment [1]. Point C then divides the axis spacing in ratio, which equals to the gear ratio [2]:

$$i = \frac{\omega_1}{\omega_2} = \frac{z_2}{z_1} = \frac{r_{w2}}{r_{w1}} = \frac{O_2C}{O_1C} = const. \quad (1)$$

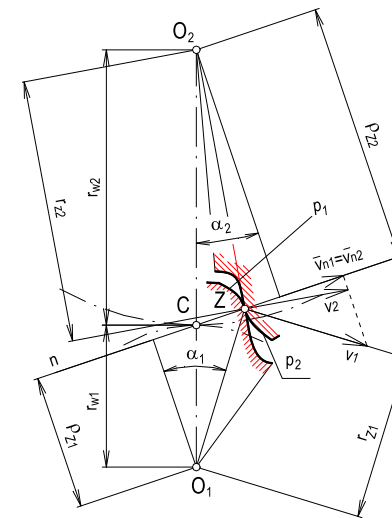


Fig.1 – Criterion of gear mesh

$\omega_{1,2}$  – angular speed (velocity),  $z_{1,2}$  – number of teeth,  $r_{w1,2}$  – rolling radius.

The elemental displacement of both profiles in the direction of the normal line has to fulfill the criteria (1), it means that the normal components of the peripheral velocities must be equal:

$$V_{n1} = V_{n2} = V_n, \quad (2)$$

$v_{n1,2}$  – normal components of peripheral velocity.

The profiles meeting the requirements (1) and (2) are called conjugated profiles and the C-C gearing must also fulfill these requirements. Except of the constant gear ratio (with exception, when it does not acts on the gearing with variable gear ratio) the gearing in general has to meet also other requirements [3]:

- Functional requirement: teeth not undercut, determined width of tooth addendum ( $s_a=0,2m_n$  or  $s_a=0,4m_n$ ), absence of interference, minimum meshing duration ( $\alpha_v > 1,2$ )
- Operation requirements: similar values of bending strength and gearing slides, high stiffness in contact pressure transmission, wear resistance.
- Economical requirements: simple manufacturing, simple control and assembly, easy operation and maintenance.
- Continuous and silent operation at the required period of service.

**Material and methodology.** It is evident from the fundamental law of gearing, that the main structural element of the gearing is the tooth face profile, characterized by basic geometrical parameters:

Involute gearing – the path of contact is a straight line (Fig.2):  $z_{1,2}$ ,  $m_n$ ,  $\alpha_w$ ,  $x_{1,2}$  and the tool parameters  $h_a^*$ ,  $h_p^*$ ,  $r_f^*$ .

C-C gearing – the path of contact is created from two arcs with inflection point C (Fig.3):  $z_{1,2}$ ,  $m_n$ ,  $\alpha_c$ ,  $r_{kb}$ ,  $r_{kd}$  and the tool parameters  $h_a^*$ ,  $h_p^*$ ,  $r_f^*$ .

Geometrical parameters and the shape of the path of contact clearly determines the type of the gear. The meshing conditions of the spur gears are defined on the face plane, where the general point of contact X is translating through the path of contact defined with point AE, while for the moment are two teeth in mesh (points AB, DE) – Fig.4,5.

For defining the reduced radius of curvature  $S_{red}$  in case of involute gearing (mesh of two convex surfaces) the following equation applies:

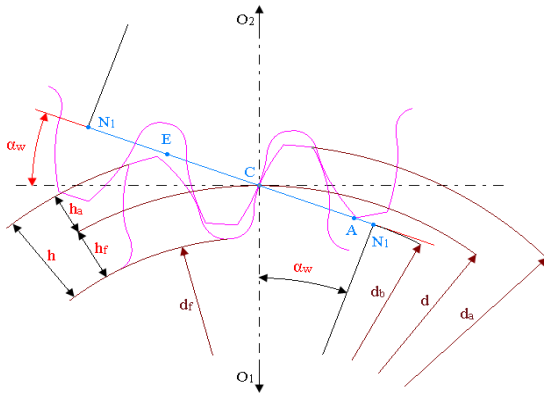


Fig.2 – Basic geometrical parameters of the involute gearing

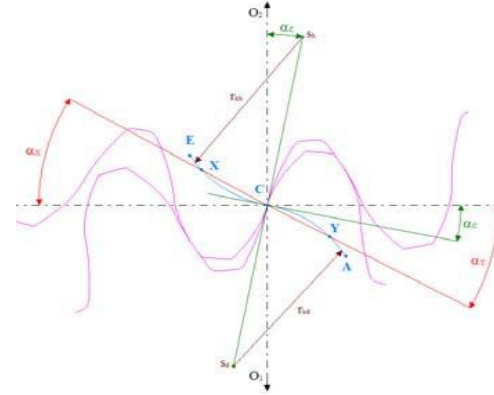


Fig.3 – Basic geometrical parameters of the C-C gearing [4]

$$\frac{1}{\rho_{red}} = \frac{1}{\rho_1} + \frac{1}{\rho_2} = \frac{\rho_1 + \rho_2}{\rho_1 \cdot \rho_2}, \quad (3)$$

$\rho_{1,2}$  – radius of curvature  
In case of C-C gearing and also the inner involute gearing (mesh of concave – and convex + surface) the following applies:

$$\frac{1}{\rho_{red}} = \frac{1}{\rho_1} - \frac{1}{\rho_2} = \frac{\rho_2 - \rho_1}{\rho_1 \cdot \rho_2}. \quad (4)$$

The magnitude of Hertz's pressure can be determined by [5]:

$$\sigma_H = 0,418 \cdot \sqrt{\frac{F_N}{b} \cdot \frac{2 \cdot E_1 \cdot E_2}{E_1 + E_2} \cdot \left( \frac{1}{\rho_1} \pm \frac{1}{\rho_2} \right)}, \quad (5)$$

$\rho_H$  – reduced radius of curvature by Hertz;  $F_N$  – normal force;  $b$  – face width;  $E_1$ ,  $E_2$ ,  $\mu_1$ ,  $\mu_2$  – are elastic constants of materials of each cylinder

The tangential component of the peripheral velocity plays a significant role, where with the right correction of the gearing it is possible to influence and reduce

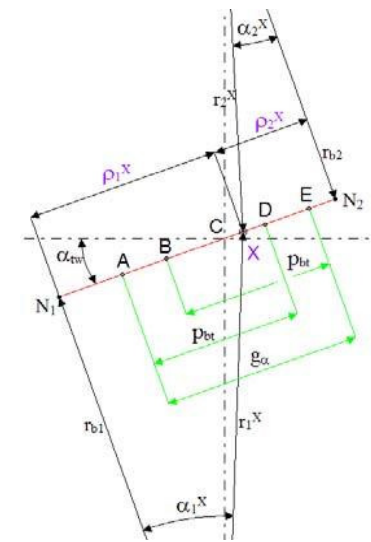


Fig.4 – Meshing parameters and radius of curvature of involute gearing

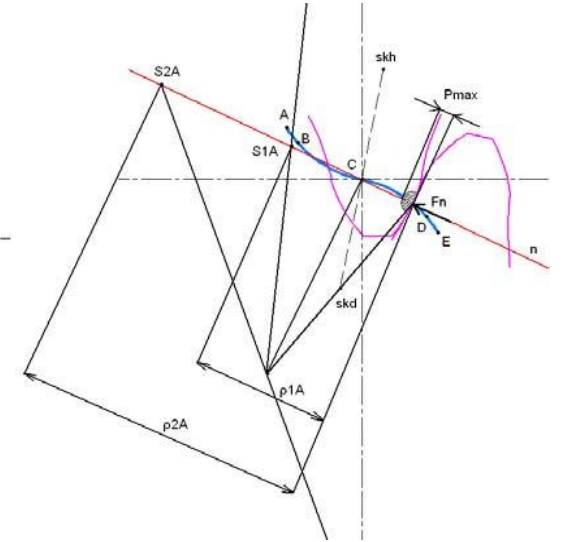


Fig.5 – Meshing parameters and radius of curvature for C-C gearing [5]

the magnitude of friction which comes with a temperature reduction in the contact area of the gearing. The calculation of the tangential components of the peripheral velocity in case of C-C gearing is more complicated [6] (the location and orientation of the components of the peripheral velocity is changing) because of the shape of the path of contact. Distribution of the normal and tangential components of the velocity in the involute gearing is evident from Fig 6, in case of C-C gearing (Fig.7) these components cannot be clearly determined due to the curved shape of the path of contact.

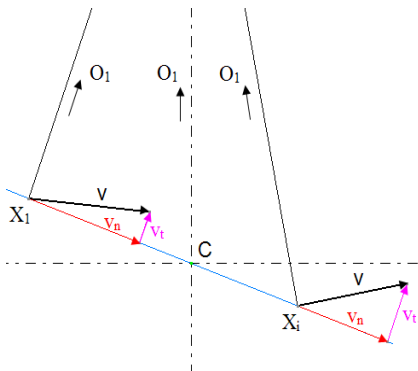


Fig.6 – Distribution of velocity components in case of involute gearing [6]

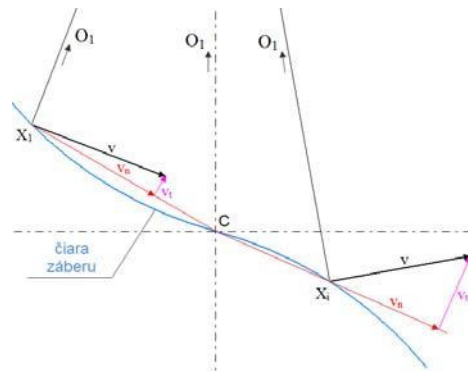


Fig.7 – Distribution of velocity components in case of C-C gearing [6]

The determination of the gearing slide could be done with the following expressions:

$$g_1 = \frac{v_{t1} - v_{t2}}{v_{t1}} ; \quad g_2 = \frac{v_{t2} - v_{t1}}{v_{t2}} , \quad (6)$$

$v_{t1,2}$  – tangential component of peripheral velocity

From Fig.8 it is evident, that the values of gearing slide in case of C-C gearing are lower than in case of involute gearing [6].

Based on the aforementioned facts it is possible to compare the advantages and disadvantages of the involute and C-C gearings.

Involute gearing – the benefits are due to the characteristics of the involute profile:

- simple profile and easy manufacturing,
- possibilities of tooth profile correction,
- correct mesh and constant gear ratio, even after the change of center distance,
- good reliability, lifetime and mechanical efficiency.

The drawbacks of the involute gearing:

- possibility of undercutting the tooth dedendum for the low number of teeth,
- risk of teeth tapering for the high angle of mesh,

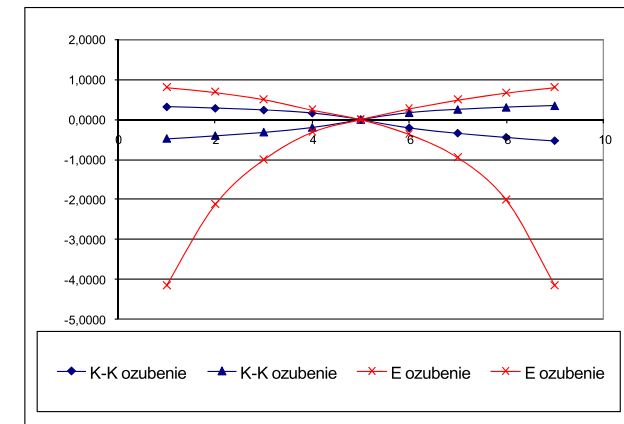


Fig.8 – Progress of gearing slides in C-C gearing [6]  
( $m=4\text{mm}$ ,  $z_1=16$ ,  $z_2=24$ ,  $r_{hi}=r_{hd}=8\text{mm}$ ,  $\alpha_c=20^\circ$ )

- development of higher surface pressure (two convex surfaces engaging on each other- external gearing),
- development of higher gearing slide and higher losses by friction,
- noise and vibration,
- the rigid constraint not allows vibration damping and dynamical loading,
- requirements on the accuracy.

C-C gearing – the benefits comes from the advantage of mesh of convex tooth faces with concave tooth faces [7]:

- lower contact pressures (high load carrying capacity in contact),
- better gearing slide ratio what can affect the lower wear, noisiness, and losses by friction with final, effect on the longer life time and durability.

C-C gearing disadvantages:

- more complicated shape of tooth face,
- high requirements on finishing operation,
- reaching the accurate distance between axes.

**Experimental methods.** In force transmissions the widely used base material is steel where at the maximum load rating (high carrying capacity) it has to preserve its stiffness with ductile core and increased hardness of tooth faces with the following characteristics [7].

- gear teeth resistant against brittle fracture at impact loading,
- high stiffness and hardness of tooth faces in contact,
- good resistance of tooth faces against wear and seizing,
- increasing of fatigue strength of surface layer in tensile.

The geometry itself with the properly selected material does not ensure increased hardness of tooth faces, therefore it is necessary to modify the material by

the means of thermochemical design: cementation, carbonitriding, nitriding, boriding, or surface hardening (Table 1).

Table 1 – Datas of thermochemical surface engineering techniques [8]

Technology	Cemented	Carbonitriding	Nitriding	Nitrocemented	Borided	Surface hardened
Difused elements	C	C+N	N	N+C, N+C+O	B	-
Temperature [°C]	850-950	600-630	500-550	820-860	800-1000	30-70 °C over Ac3, 30-70 °C over Ac1
Thickness of layer [mm]	0,15-0,2,mm (max. 2 mm)	0,05	0,2-0,6	0,4-0,8 (ammonia) 0,05-0,2 (bath)		2,5-6 or 1-2
Layer hardness [HV],[HRC]	60-62 HRC	1000 HV	60-65 HRC, (1000-1200 HV)	56-60 HRC, (700-800 HV)	1500-2000 HV	45-55 HRC

Hard layers established with the aforementioned technologies on the surface are of high resistance against abrasion, while the depth of the layers and the resulting hardness are not equal (Fig.9).

The nitride and carbonitride layers are the hardest and fatigue resistant, but they have small depth which not allow high loading with surface pressures (the core would deform and the nitrided layer would damage), therefore they are appropriate for gearings with intensive abrasive loadings and lower surface pressures. For higher surface pressures it is necessary to use alloyed steels with higher toughness and core strength.

Cemented and nitrocemented layers has lower hardness (~ 800HV ) but are essentially thicker and therefore has good wear resistance and if the core strength is high enough they tolerate higher surface pressures.

Surface hardened layers have lower hardness (max. 750 HV) but they are the thickest and therefore they tolerate high surface pressures.

Other possibilities to increase the carrying capacity is the creation of coatings by de-

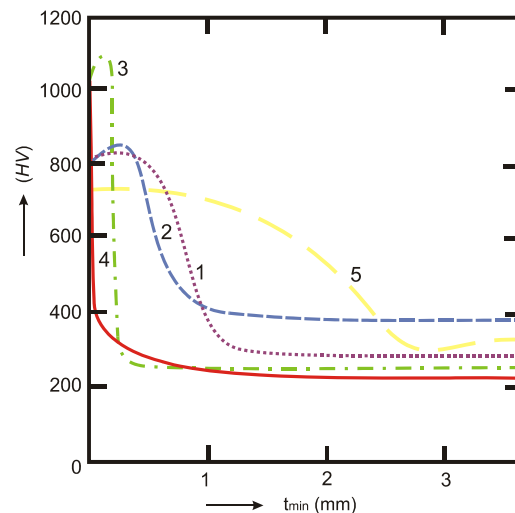


Fig.9 – Comparing the hardness of various layers [9]  
1 - cemented, 2 - nitrocemented, 3 - nitriding,  
4 - carbonitriding, 5 - surface hardened

positing thin layers on the base material. Methods of coating deposition are the following [10], [8]:

1) *CVD Method* (Chemical Vapour Deposition) – The principle of CVD is in heating up the coated substrate in vacuum to high temperature (900-1050 °C) and with response of chemical compounds, supplied to the surface of the material in vapour state, the solid state is formed.

**Benefits:** High temperature stability of the created layers, possibility to develop complicated layers not only nitrides of metals, high adhesion and wear resistance, uniform thickness of the layers on the surfaces with complex shape.

**Disadvantages:** maintaining the base substrate at high temperature to reach steady chemical structure and high power demands long operation cycle (8-10 h) due to long heating and cooling, ecological problems with disposal of exhaust gases produced during deposition, tensile strengths in the layer (different coefficient of thermal expansion).

2) *Coating with PACD method* (Plasma assisted chemical vapour deposition) – Presence of plasma allows to lower the temperature of layer forming at 470-530°C. No dimensional changes occur during the coating process. These coatings have extremely low coefficient of friction (below 0,1). The equipment for PACVD coatings enables except of coating deposition also surface nitriding and cleaning the parts by the means of ion etching. Coatings created in this way complies to high requirements of quality, mainly abrasion resistance, life and hardness.

3) *PVD Method* (Physical vapour deposition) – The principle of PVD is based on transformation of deposited material to vapour phase (ion sputtering) in vacuum and depositing on the substrate at low temperatures (150-500°C). The coating thickness is ranging from 1-to 5 µm.

**Benefits:** the most friendly method of coating deposition (no toxic materials are used), high wear resistance of the layers, low coefficient of friction, possibility to form wide range of various combinations of the layers, small thickness and easy reproduction of the layers, possibility to form accurate layer thickness, possibility to control internal tensions in the coating, high speed of coating deposition with good adhesion.

**Drawbacks:** difficulties with deposition on polymers, high costs on purchasing and operation.

4) *Thermal spraying* – The material is deposited as a powder or wire and it is brought to the equipment where it is melted and forwarded to the substrate. With impacting the surface the smelted particles are spread and the drops are coupled among each other, where the coating is formed during the cooling process. The thickness is ranging from 0,2-2 mm.

**Benefits:** good wear resistance, excellent tribological characteristics, oxidation resistance, corrosion, electro insulation and electro-conductive coatings.

From the mentioned methods of deposition of thin coatings the PVD method seems to be the most suitable to deposit thin coating on gears, which belongs to the

most advanced methods, operates with low temperatures (max. 500°C) and allows to create also several hundred layers, so called multilayers and nanolayers with thickness up to few μm and last but not least it is economically reasonable [15].

Before deposition it is necessary to remove impurities from the surface (due to good adhesion). The most important requirements in coating deposition are, that the layers must have good mechanical stability (no cracking) and must have good adhesion to the substrate (no delamination). From the aspect of increasing the carrying capacity of gear wheels it is necessary to improve the following mechanical characteristics:

high surface hardness, high stiffness, resistance against corrosion and high temperature oxidation, abrasion resistance, long lifetime [11]. Moreover the coating has to resist to temperatures around 400°C, low coefficient of friction, maximum surface roughness  $Ra=0,6 \mu\text{m}$  etc. [10].

**Results and discussion.** From the aspect of minimum hardened layer thickness determination on the geared transmissions are dangerous mainly high shear stresses, developing from the contact pressures in the surface layers, which can cause pitting and plastic deformation in the area below the coating and to fracture of the layer [12].

#### Involute gearing:

1) The magnitude of minimum depth of hardened layer is specified by standard STN 01 4686-5 in the control of fatigue in contact

a) for cemented and nitrocemented gear wheels the following applies:

$$h_{\min} = \frac{J_{HV}}{J_{HV} - 120} \cdot 4,16 \cdot 10^{-3} \cdot d_1 \cdot \frac{u}{u+1}, \quad (7)$$

$h_{\min}$  – min thickness of the hardened layer;  $J_{HV}$  – tooth core hardness;  $d_1$  – circular pitch;  $u$  – transference number.

b) for nitrided gear wheels applies:

$$h_{\min} = \frac{J_{HV}}{J_{HV} - 150} \cdot 2,38 \cdot 10^{-3} \cdot d_1 \cdot \frac{u}{u+1}. \quad (8)$$

2) According to GLAUBITZ [13] has to be the minimum thickness of the hardened layer  $t_E$  greater than the depth of maximum shear stress:

$$t_E = 2 \cdot (z)_{\tau_{\max}}, \quad (9)$$

$(z)_{\tau_{\max}}$  – max shear stress depth.

3) LINHART [14] in his work determines the requirement of minimum hardened layer in the way, that on the interface of the layer and the core must not exceed the yield stress:

$$0,5\sigma_{Kt} = p_H \left\{ 1 - \frac{t_{\min}}{a} \left[ 1 + \left( \frac{t_{\min}}{a} \right)^2 \right]^{0,5} \right\} \frac{t_{\min}}{a}, \quad (10)$$

$\sigma_{Kt}$  – yield strength;  $p_H$  – max. tensile stress according to Hertz;  $t_{\min}$  – min thickness of hardened layer;  $a$  – half width of contact area.

**C-C gearing:** In case of C-C gearing it is possible to determine the minimum thickness of hardened layer on the base of numerical simulations [11]. Maximum von MISESS stresses and maximum shear stresses were evaluated, while the minimum thickness of the hardened layer was determined from the maximum shear stress. Simulation were run on 108 models, which varied in angle of path of contact in point C ( $\alpha_c$ ), which acquired values:  $\alpha_c = 6^\circ \div 23^\circ$ ; further they varied in radius of curvature of path of contact ( $r_k$ ), which acquired values:  $r_k = 13, 17, 22 \text{ mm}$ . Other geometrical parameters of C-C gearing are shown in Table 2.

Table 2 – Gearing parameters

$\alpha_c$	$m$	$z_1/z_2$	$D_1/D_2$	$D_{a1}/D_{a2}$	$D_{f1}/D_{f2}$	$a=a_w$	$j=j_w$	$r_k$ [mm]		
								13	17	22
[°]	[mm]	[–]	[mm]	[mm]	[mm]	[mm]	[mm]	$\epsilon_a$ [–]	$\epsilon_a$ [–]	$\epsilon_a$ [–]
23	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,167	1,219	1,266
22	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,176	1,23	1,281
21	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,185	1,242	1,296
20	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,194	1,254	1,311
19	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,204	1,268	1,328
18	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,216	1,283	1,346
17	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,228	1,299	1,367
16	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,24	1,315	1,387
15	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,253	1,332	1,408
14	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,267	1,351	1,431
13	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,282	1,37	1,456
12	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,297	1,39	1,481
11	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,314	1,411	1,507
10	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,331	1,434	1,535
9	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,349	1,457	1,564
8	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,367	1,481	1,594
7	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,387	1,507	1,626
6	4	28/42	112/168	120/176	102,8/158,8	140	0,142	1,407	1,533	1,659

$\alpha_c$  – angle of point C,  $m$  – modulus,  $z$  – number of teeth,  $D$  – diameter of the pitch circle,  $D_a$  – addendum diameter,  $D_b$  – dedendum diameter,  $a_w$  – axial distance,  $j_w$  – backlash

From the results of maximum shear stresses were specified equations for the determination of minimum thickness of hardened layer in point B and D (Fig.5),

which where defined by a trend line.

Than for the point B applies:

$$t_{\min} = 0,3012 \cdot \alpha_C^{0,0056}, \quad (11)$$

and for the point D applies:

$$t_{\min} = 0,2913 \cdot \alpha_C^{0,0282}. \quad (12)$$

**Conclusion.** The presented article deals with the effect of geometrical parameters of C-C gearing on the determination of the minimum thickness of hardened layer. This was specified by numerical simulation by the means of FEM analysis, where for the point B applies the expression (11) and for the point D expression (12) [11]. In the mentioned expression the minimum thickness of the hardened layer depends on the angle of the path of contact in point C –  $\alpha_c$ , while the final value has to be round to hundredth. Starting point for the design of hardened layer then could be the most unfavorable situation from the expressions (11) and (12) to prevent its failure and damage of the base material.

Subject for further examination is to continue in research on increasing the load carrying capacity of C-C gearing by the means of thin hard layers (multi layers, nano layers,) while in [10] is evident the effect of TiN thin hard layer. Further research is carried on with the application of mono-, multi- and nano layer on the model of strength analysis.

The work was elaborated within the solution of grant projects VEGA 1/0189/09.

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Поступила в редколлегию 05.05.10

УДК 621.01; 621.833; 621.852

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## ИСПОЛЬЗОВАНИЕ ПРОИЗВОДНОЙ ОТ УСКОРЕНИЯ ВНЕДРЕНИЯ ПРИ НАХОЖДЕНИИ ТОЧЕК НА ОГИБАЮЩЕЙ, ЗНАЯ ТОЧКИ НА ОБВОЛАКИВАЮЩЕЙ

Запропонована модифікація кінематичного методу обчислення координат крапок на поверхнях, формованих методами огинання. Метод легко вписується в існуючі системи аналізу процесів формоутворення недиференціальними методами, суттєво зменшуючи загальний обсяг обчислень. Показано, що застосування похідної від прискорення впровадження на порядок підвищує точність кінематичного методу знаходження крапок на поверхні, що обгинає.

There is a modification to the undifferentiated surface points' estimation methods that are formed by envelope methods. This method: doesn't need a lot of additional calculations; easily fits the existing analysis systems of forming processes; essentially reduces the overall total of calculations at adequate accuracy.

**Постановка задачи.** В [1] изложен кинематический метод нахождения точек на огибающей поверхности, имея точки на обволакивающей [2]. Метод основан на использовании понятий скорость и ускорение внедрения и, как показано в [1], позволяет при том же числе “резов” на 1-3 порядка уменьшить величину отклонения найденных точек от теоретической огибающей. Компьютерные расчеты, часть которых представлена в [1], показали, что характер отклонения найденных точек на  $\Sigma_3$  от огибающей  $\Sigma_2$  (см. рисунок 1), подчиняется вполне определенным зависимостям: а) в направлении роста радиуса кривизны огибающей  $\Sigma_2$  точки  $\Sigma_3$  располагаются внутри тела огибающей  $\Sigma_2$ ; б) отклонения точек  $\Sigma_3$  от  $\Sigma_2$  связаны степенной зависимостью.

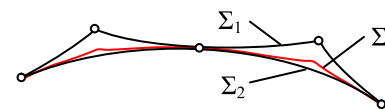


Рисунок 1 – Виды поверхностей:

$\Sigma_1$  – обволакивающая;

$\Sigma_2$  – огибающая;

$\Sigma_3$  – найдена кинематическим методом

Всё это соответствует физическому смыслу и указывает на то, что в анализируемом примере погрешности вычислений обусловлены влиянием третьих производных. Ведь скорость внедрения вычисляется через первые производные; ускорение внедрения – через вторые производные; а более высокие производные в кинематическом методе [1] не учитываются. Заметим, что высокие производные предлагалось использовать при анализе и синтезе зацеплений и ранее. Так Г.И. Шевелёва разработала метод степенных рядов [3]. Д.Т. Бабичев писал [4] о полезности понятий высоких производных от скорости внедрения, но идея эта до настоящего времени ни в методиках, ни в расчетах не была реализована.