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FINITE-ELEMENT FREE VIBRATION AND BUCKLING ANALYSES OF SANDWICH PLATES WITH HONEYCOMB AND FOAM CORES

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Abstract
The free vibration and linear buckling analyses of the sandwich flat panels has been considered. The commercial finite element ABAQUS software was used to calculate natural frequencies and bearing capacities of sandwich plates with honeycomb and foam cores. These problems were analysed to assess the role of the core on the natural frequencies and critical loads of sandwich plates. The calculated for sandwich plates results were compared with those for solid plates having the same weight. According to the comparative results the structural performance of the sandwich plates are discussed.

1. Introduction

Because sandwich structures provide the minimum weight for given strength and stiffness, they nowadays are used in a wide range of engineering applications. In many cases these structures are subjected to various vibrations. Therefore, it is very important to study dynamic and stability responses of sandwich structures.

A review of literature shows that although extensive research has been conducted on the topic of free vibration and buckling of sandwich plates, only a few results were reported on predictions of the dynamic properties in term of face sheets and core materials as well as other structural parameters.

Structural efficiency of sandwich structures relies on the lightweight core to separate the face sheets and provide the necessary stiffness. A variety of core configurations have been employed. Most commonly used cores are foams manufactured from modern plastics and honeycomb which can be fabricated from a wide range of materials such as aluminium, titanium, impregnated paper and etc.

Thus, the objective of this work is to examine the flexural dynamic behaviour and elastic stability of sandwich plates with hexagonal honeycomb and foam cores in order to assess their structural performance depending of the different core materials. These investigations will be performed with commercially available ABAQUS software.

2. Materials

Originally, sandwiches are highly heterogeneous structures and their description requires three dimensional models, which should take into account the large number of parameters such as material and geometry of the core and face sheets. But detailed model is very difficult for developing of analytical approaches and expensive in the point of view of computational efforts. For this reason, the analysis of a heterogeneous sandwich structure is carried out by replacing the core structure with an equivalent homogeneous medium, where core thickness is much more than the face sheets thickness and its density and stiffness are much less than those of face sheets. To idealize the cellular honeycomb and foam core materials as a homogeneous materials with equivalent effective material properties, the knowledge of their mechanical properties are necessary.

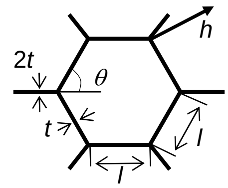
In this study, a commercially available hexagonal honeycomb material with trade mark as HexWeb™ of Hexcel Corporation was used. As a result of the fabrication process [1], material properties of honeycomb core material are highly anisotropic. Therefore they can be modelled by orthotropic homogeneous medium, which macroscopic properties depends on the cell geometry and parent material. There are three ways to estimate homogeneous elastic moduli. First, they can be taken directly from experimental investigations [1]. Second manner is to use empirical correlations. Most applicable formulae are obtained by Gibson and Ashby [2]. And finally, the homogeneous elastic moduli can be obtained by application of the unit cell homogenization technique. It should be noted that the Young's modulus E_z in the thickness direction and shear moduli G_{xz}, G_{yz} in the transverse directions are primary mechanical properties of the honeycomb material, whereas the uniaxial moduli E_x, E_y and shear modulus G_{xy} in the sandwich plane are insignificant and in practice, they can be neglected. The first-order approximation of the properties of a hexagonal honeycomb according to the formulae of Gibson and Ashby can be the following:

$$E_x = E_1 = k\frac{(1+\sin\theta)}{h\cos^3\theta}, \qquad G_{xz} = G_{13} = \frac{G_0(1+\cos\theta)}{2\sin\theta}\cdot\frac{t}{l},$$

$$E_y = E_2 = k\frac{\cos\theta}{h(1+\sin\theta)\sin^2\theta}, \qquad G_{yz} = G_{23} = \frac{G_0\sin\theta}{(1+\cos\theta)}\cdot\frac{t}{l},$$

$$G_{xy} = G_{12} = k\frac{(1+\sin\theta)}{3h\cos\theta}, \qquad E_z = E_3 = \frac{2E_0}{\sin\theta(1+\cos\theta)}\cdot\frac{t}{l}, \tag{1}$$

$$\nu_{xy} = \nu_{12} = k\frac{\sin\theta(1+\sin\theta)}{\cos^2\theta}, \qquad \rho_{eff} = \frac{2\rho_0(1+\cos\theta)}{\sin\theta(1+\cos\theta)}\cdot\frac{t}{l}, \qquad k = \frac{E_0 h t^3}{l^3}.$$



Here t is the thickness of the hexagonal cell wall, l is the dimension of the cell, θ is inclination angel, ρ_0 , E_0 and G_0 are the density, Young's modulus and shearing modulus, respectively, of foil material of which the honeycomb core is made. ρ_{eff} is the effective mass density of the honeycomb material.

The honeycomb is an extremely efficient core material, but is labour-intensive and costly compared to foams. Most commonly used foams in sandwich cores are polymer, syntactic and metal foams. The foam used in these investigations is cross-linked polyvinyl chloride (PVC) foam with trade name Divinycell™ H-grade of DIAB Corporation. Two types of foam core material referred as Divinycell H100 and H200 with densities 100 and 200 kg/m^3, respectively, were used. The data such as Young's modulus, Poisson's ratio, etc. for the used foams can be found in technical manual of DIAB [3]. The microstructure observations of PVC foams with SEM show that the foams do not possess the prevalent direction. Moreover, experimental evidences indicate that the PVC foams exhibit nearly isotropic macroscopic properties, which depend on the relative density of the foams. The mechanical response of PVC foam under uniaxial compression-tension and shear tests is linear, when strains are small and typically 3% or less. In case of higher level of strains the nonlinear behaviour of foams takes place.

3. Computational models

The modelling of sandwich plate with complex internal structure implies several simplification steps, required for reduction of this highly non-conventional structure to a standard computational model. First of them is assumption that each layer can be treated as homogeneous. In this study, the anisotropic properties of honeycomb core were represented by continuum with orthotropic properties and foam core was considered as an isotropic material. After that, to the 'standard' three-layered model of a sandwich structure can be directly applied either two-dimensional (2D) laminate theories of plate/shell or three-dimensional (3D) approach. The details of the using of such sandwich models in order to solve the free vibration and linear buckling problems can be found in many references and are commonly supposed the following sequence of actions:

- each of the displacement components is expressed as a series of products of functions of the thickness coordinate and trigonometric functions of the surface coordinates. The in-plane displacements are usually represented by functions of higher order than those for the transverse displacement;
- for each pair of harmonics, substitution of the displacement expressions into the governing differential equations, and integrating in the thickness coordinate, produces a system of homogeneous algebraic equations with coefficients being the amplitudes of the assumed thickness distribution of the displacements;
- this system of simultaneous linear algebraic equation are solved to obtain the eigenvalues (natural frequencies or critical loads) and the associated eigenfunctions (mode shapes).

Consequently, the accuracy of response quantities predicted by different computational models of sandwich structures will depends on the accuracy of the homogenization procedure of a sandwich core and the method of an analyse. The Finite Element Method (FEM) is more general and can be performed using any of the available FE-codes.

ABAQUS software is a widely used finite element code and the Standard (implicit) version [4] is here used for predictions of natural frequencies and critical loads of sandwich plates. The sandwich structures, as three-layered plates treated were investigated under the following assumptions:

– the displacements are small, meaning that a linear elastic behaviour is valid;
– the skins and core are perfectly bonded together;
– the face sheets are thin compared to the core, this means that the local flexural rigidity of skins can be ignored.

In principle, FEM models of a sandwich structure in ABAQUS can be built up in four different ways:

– shell element with math model assuming the composite cross section of a sandwich (*COMPOSITE option as denotes in ABAQUS);
– solid element with the composite cross section of a sandwich;
– both sheet faces and core are modelled of solid elements stacking through the total thickness;
– the sheet faces are modelled by shell elements and the core by solid elements staking through the total thickness.

Moreover, there are two possibilities to model 2D plate-type structures either by conventional shell element with 6 degrees of freedom (DOF) per node or continuum shell elements with 3 DOF per node. Both conventional and continuum shell elements based on the first order shear deformation theory (FSDT). The stack of continuum elements corresponds to the case of layerwise laminate theory.

4. Numerical studies

The benchmark and convergence studies of free vibration problem were performed in the example for which experimental results are available [5]. The geometry and material properties of simply supported rectangular $a \times b$=1.83×1.22 *m* sandwich plates are given as in [5]: the face sheets: E=68.9 *GPa*, ν= 0.3, ρ_f=2.77×10^3 *kg/m*^{3}, h_f=4.06×10^{-4} *m*; the core: G_{13}=0.134 *GPa*, G_{23}=0.052 *GPa*, ν= 0.32, ρ_c=1.22×10^2 *kg/m*^{3}, h_c=0.0064 *m*. The results of the calculations are tabulated in Table 1. Here, as denoted in ABAQUS, *S4* is a 4-node double curved general-purposed shell element, *S8R5* is an 8-node doubly curved thin, reduced integration shell element using 5 DOF per node, *S8R* is an 8-node doubly curved thick, reduced integration shell element, *SC8R* is an 8-node quadrilateral in-plane general-purposed, reduced integration continuum shell element with hourglass control and *C3D20R* is a 20-node quadratic brick, reduced integration solid element. The symbols '*comp*' and '*stack*' mean the composite option of an element and stacking sequence of elements. The present numerical results obtained with a 6×9 mesh in the full plate are found to be close to the experimental results and exact solution [5] as well as numerical [6, 7] ones. From Table 1 it is evident that the more exact results, especially for higher frequencies, are found with the continuum shell and solid stacking elements.

Table 1.

Natural frequencies (*Hz*) of simply supported rectangular sandwich plate.

Mode	*Expe-riment*	*3D theory*	*Khare et al.*	*Nayak et al.*	*ABAQUS*					
					S4	*S8R5*	*S8R*	*SC8R*		*C3D20R*	
								comp	*stack*	*comp*	*stack*
1	-	23	23.47	23.42	23.71	23.27	23.21	23.71	23.87	25.88	23.25
2	45	45	44.96	45.00	46.17	44.54	44.50	45.89	48.17	49.79	44.51
3	69	71	72.55	71.44	78.78	70.94	70.26	78.84	78.80	79.76	70.16
4	78	80	80.59	81.26	88.19	79.62	79.80	86.40	92.08	89.82	79.69
5	92	91	93.49	92.66	98.89	91.72	91.02	98.87	107.9	104.1	90.91
6	129	126	128.28	128.3	136.6	126.0	125.5	135.6	151.3	146.2	125.3
7	133	129	-	133.7	158.3	128.1	129.1	151.5	154.3	146.5	128.4
8	152	146	-	150.4	192.6	148.9	146.2	192.8	158.2	170.9	145.0
9	169	165	-	171.0	199.9	168.8	166.1	194.7	175.8	197.2	165.0
10	177	174	-	179.8	208.2	173.2	173.6	208.7	209.6	208.7	172.9

In order to study the effect of core thickness influence on the frequency parameter $\overline{\omega} = \omega \cdot a^2 / h \sqrt{\rho_c / E_c}$, symmetric sandwich plates made up of the 3-layered cross-ply fibre reinforced plastic (FRP) face sheets separated by heavy H200 and light H100 PVC foam core materials were investigated. In the numerical example the following data was considered: the aspect ratio a/b=1 and the length-to-thickness ratio a/h=5. The core thickness to total thickness ratio h_c/h was varied from 0.11 to 0.89. The properties of sandwich plate materials are given in the Table 2. The influence of boundary conditions on the sandwich plate response was taken into account. It can be seen in Figures 1 and 2 that there is the identical tendency for light and heavy foam cores: as the core thickness to total thickness ratio h_c/h increases the fundamental frequencies decrease up to approximately 0.75 of the thicknesses ratio for all plates and then start to slightly increase. This response is enhanced with the greater constrains and it is most significant for the full clamped sandwich plate.

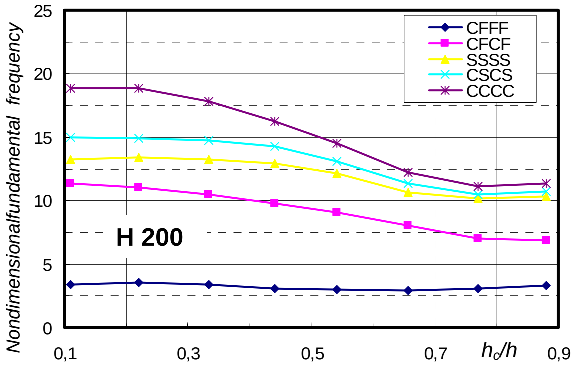


Fig. 1.

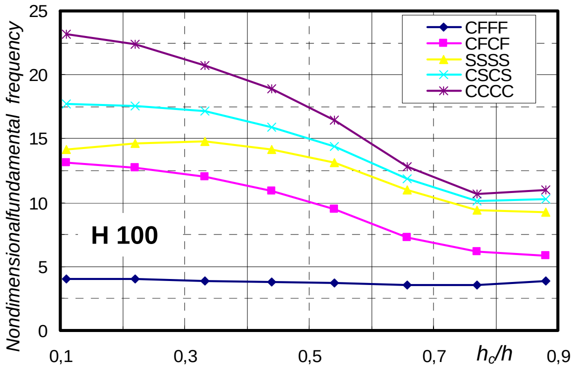


Fig. 2.

The influence of core thickness and the total thickness on the fundamental frequency parameter of the composite sandwich plates was further studied. In this purpose, the different length-to-thickness ratios corresponding to: thin, moderate thick and extremely thick plates for the simply supported edges were considered. As shown in Figures 3 and 4, the vibration behaviour is similar for both types of sandwich plates with light and heavy foam cores. Here it is obvious that there is a trade-off between plate rigidities and their mass. However, cases of thin and thick plates are quite different.

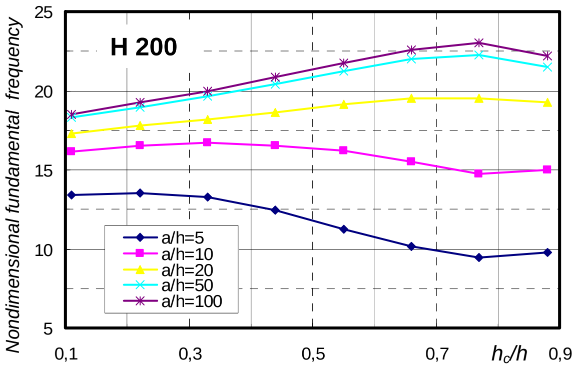


Fig. 3.

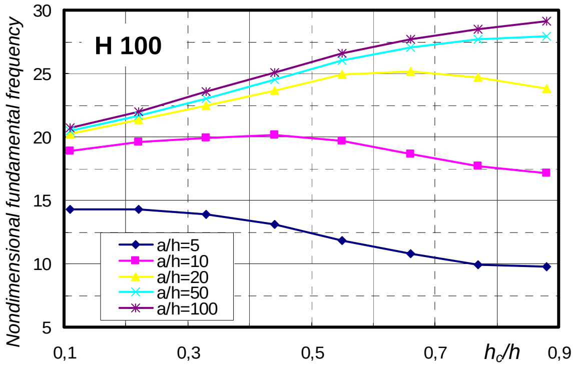


Fig. 4.

The next figures represent free vibration characteristics of square $a=b=1$ m honeycomb sandwich plates. The boundary conditions of plates corresponded to the SCSC edge supports. The structural data details of the hexagonal honeycomb material are the following: 1) the parent material: E=69 GPa, ν=0.3 and ρ=2800 kg/m^3; 2) the unit cell: t=0.0254 mm, l=1.833 mm and θ=30°. The face sheets were treated as isotropic material: E_f=69 GPa, G_f=26 GPa, ν_f=0.33 and ρ_f=2770 kg/m^3. The equivalent core properties calculated according to (1) are listed in Table 2. The values of first four natural frequencies versus cell wall thickness and cell size of hexagonal honeycomb material are shown in Figures 5 and 6. The results indicate that natural frequencies increase as the wall thickness increases and they decrease as the cell size increases.

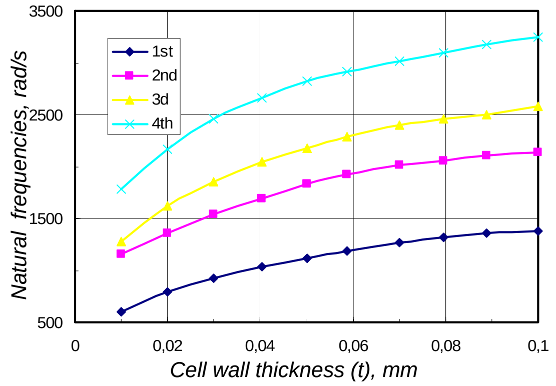


Fig. 5.

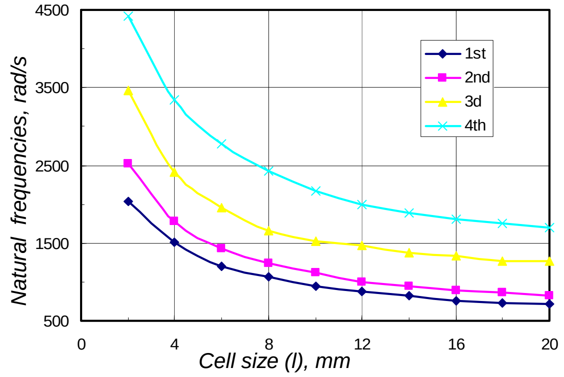


Fig. 6.

Linear stability of square simply supported symmetric sandwich plates with 5-layered cross-ply face sheets with both foam and honeycomb cores subjected to in-plane uniform load were studied. The material parameters of sandwich plate with only foam core are given as the following: 1) the face sheets: E_1=172.7 *GPa*, E_2=7.2 *GPa*, G_{12}=G_{13}=3.76 *GPa*, ν_{12}=0.30; 2) the core: E_c=0.105 *GPa*, G_c=0.045 *GPa*, ν=0.32. The material characteristics of sandwich plates with honeycomb core are the following: 1) face sheets: E_1/E_T=19, E_2/E_T=1, E_3/E_T=0.338, G_{12}/E_T=0.52, G_{13}/E_T=3.34, G_{23}/E_T=1.34, ν_{12}=0.32; 2) core: $E_1/E_T=3.2\times10^{-5}$, $E_2/E_T=2.9\times10^{-5}$, E_3/E_T=0.4, $G_{12}/E_T=2.4\times10^{-3}$, $G_{13}/E_T=7.9\times10^{-2}$, $G_{23}/E_T=6.6\times10^{-2}$, ν_{12}=0.99, $\nu_{13}=\nu_{23}=3\times10^{-5}$. Figure 7 presents values of a critical stress versus thickness face sheets ratio for thick and extremely thick honeycomb plates. The obtained results are compared with those in [8]. Figure 8 shows the value of a critical stress versus thickness-to-length ratio for sandwich plates with H100 PVC foam with various thickness core parameters.

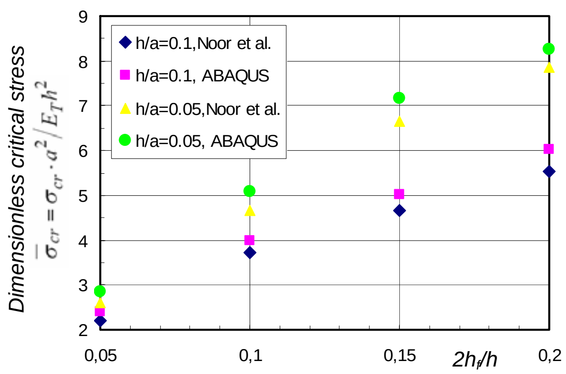


Fig. 7.

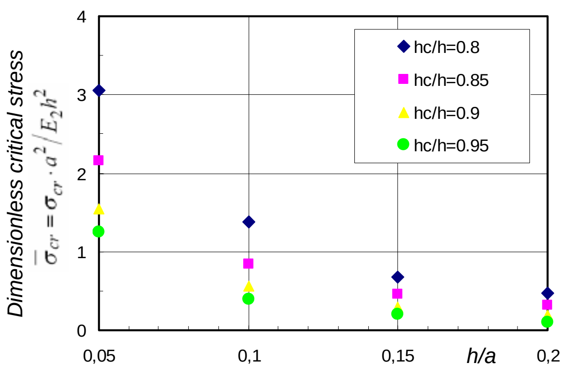


Fig. 8.

Finally, the comparative studies of free vibration and load capacity of sandwich plates *a*=*b*=1 *m* of equal mass with honeycomb and light and heavy foam cores as well as without core were carried out. Geometric data and material properties are listed in Tables 2. The magnitudes of natural frequencies are specified in Table 3. Obtained results show that the core material plays an essential role in the vibration response. It not only changes the magnitude of frequencies but also shifts the mode order. The same conclusions can be made for linear buckling behaviour of such plates. As can see from Table 3, the maximum load capacity corresponds to the sandwich plate with honeycomb core.

5. Conclusions

From present investigations the following conclusions can be drawn:

- finite element models taking into account transversal shear deformation should be used even for cases of 'thin' sandwich plates;
- most preferred FE models of free vibration and buckling analysis should be based on the stacked continuum of finite elements;

– the calculations have shown strong dependence of natural frequencies and critical loads on the effective structural parameters of cores that should be taken into account in the optimization problem for such structures.

Table 2.
Dimensions and material properties of the composite sandwich plates and solid plate.

		Sandwich: 0/90°/CORE/90°/0			*Solid*
		FRP/H100	FRP/H200	FRP/Honeycomb	FRP
m, *kg*		10.4			
h	*m*	0.02			
h_f	*m*	0.001235	0.001	0.001325	0.00145
h_c	*m*	0.0151	0.0160	0.0147	0

Material	ρ	ν	E_1	E_2	E_3	G_{12}	G_{13}	G_{23}
	kg/m^3		*GPa*					
FRP	1800	0.078	24.25	7.77	-	3.34	3.34	1.34
H100	100	0.32	0.105	0.105	0.105	0.045	0.045	0.045
H200	200	0.32	0.230	0.230	0.230	0.085	0.085	0.085
Honeycomb	57.6	0.33	0.212×10^{-3}	0.318×10^{-3}	1.427	0.018×10^{-3}	0.208	0.329

Table 3.
Natural frequencies (*rad/s*) of simply supported plates of equal mass.

Mode	*Solid*	*Sandwich*		
		Honeycomb	*H100*	*H200*
1,1	84,112	415,4	372,91	392,41
1,2	190,03	1061,4	934,35	941,97
2,1	253,41	1112,9	972,68	991,35
2,2	336,39	1616,9	1395,6	1403,0
1,3	380,13	2156,3	1747,3	1824,9
3,1	-	2261,0	1859,6	1916,5
2,3	505,12	-	-	-
Critical loads (*q, kN/m*) of simply supported plates of equal mass.				
Critical uniform load	3,74	90,92	81,84	73,85

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