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Effective elastic properties of foam-filled honeycomb cores of sandwich panels

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ABSTRACT

Filling with foams of honeycomb structures has been proposed as some enhancement of honeycomb-cored sandwich material systems. The present study considers aluminum honeycomb cores filled with polyvinyl chloride foams with the aim to predict their material elastic properties. The displacement-based homogeneous technique using 3D finite element analysis is applied to evaluate the effective elastic properties of foam-filled honeycomb cores. The special attention is paid to stress predictions at the skin/core interface and the stress distributions within the honeycomb cell walls. The influence of the foam filler on distribution of local stresses within the cell is examined. The FE modelling is performed with the commercial available software ABAQUS. The structural benefits of the foam-filled honeycomb cores are also discussed.

1. Introduction

Nowadays, sandwich constructions with the honeycomb core are being widely used in different engineering applications. It is due to their high bending stiffness and strength together with the extra-light weight. At the same time such structural components can be easily damaged by foreign object impacts. Filling of honeycomb structures with foams has been proposed as some enhancement of honeycomb-cored sandwich material systems. As a result, the honeycomb core of the same configuration is more effective for withstanding crash, impact or fatigue loading conditions [1–3]. This method is inexpensive and does not add a significant amount of weight to a sandwich structure. However, the filler existence within honeycomb cells improves not only damage tolerance of the structure, but changes its behaviors. Hence, there is a need to gain insight into the responses of foam-filled sandwich configurations.

The finite element method (FEM) provides a powerful analysis tool for solution of various engineering problems. However, a detailed representation of honeycomb cells is unsuitable and even in some cases impossible for large scale models due to computational cost. In order to attain efficiency of numerical simulations in engineering practice, the actual cellular structure of the honeycomb core is usually replaced by an equivalent homogeneous, in general, anisotropic medium using effective elastic properties. This gives the opportunity to use shell or solid elements for modelling the honeycomb core in the three-layered arrangement of the sandwich constructions. On the other hand, the reliability of the continuum model strongly depends on the accuracy of the effective core properties. Thus, the prediction of the effective properties of the honeycomb core materials must be performed as exactly as possible.

In literature, there can be found many papers devoted to the prediction of effective properties of hollow honeycomb cores. Kelsey et al. [4] have theoretically evaluated of the out-of-plane shear moduli of the hexagonal honeycomb core. They for the first time applied energy method and showed that using the theorem of minimum potential energy and a kinematically compatible uniform strain field gives an upper bound of the shear modulus, and using the theorem of minimum complementary energy and a statically compatible uniform stress field gives its lower bound. Gibson and Ashby [6] presented the analytical expressions for the in-plane and the out-of-plane effective properties as functions of the honeycomb core geometry, but the influence of the skins on the core was not taken into account. A two-scale asymptotic approach for calculation of the out-of-plane effective shear moduli accounting for the warping of cell walls was proposed by Shi and Tong [9]. Later, Masters and Evans [7] predicted the in-plane elastic constants by considering flexure, stretching and hinging effects of the honeycomb wall cells. Penzien and Didriksson [5] formulated a displacement field accounting for the skins effect to calculate the transverse shear moduli, and Becker [8] obtained the closed-form solution for the in-plane stiffnesses depending on the core thickness. Xu and Qiao [10] studied the thickness effect for evaluation of all effective elastic constants and used the multi-pass homogenization technique. Recently, Chen and Davalos [11] proposed analytical model considering the skin effect to compute both effective

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stiffnesses and interfacial stresses. The advantages of the finite element approach for the homogenization analysis were firstly used by Grediac [12], who analyzed the out-of-plane behavior of the honeycomb structure. This FE approach was applied and extended for calculation of all out-of-plane elastic moduli for the honeycomb and tubular cores by Meraghni et al. [13]. Höhe and Becker [14] derived a closed-form solution to predict components of the effective elasticity tensor and compared them with FE results. An improved analytical equation based on the FEM analysis for calculation of the equivalent in-plane Young's modulus accounting for the core height is recently proposed by Chen and Ozaki [15]. Lira et al. [16] extended the analytical method and FEM approach, developed for evaluation of transverse shear elastic properties of hexagonal honeycombs, onto novel multi re-entrant honeycombs.

Thereby, a number of analytic and numerical techniques have been proposed for predicting the effective properties of hollow honeycomb cores. However, to the authors' knowledge no investigations has been made to calculate material constants of foam-filled honeycomb cores. Therefore, the aim of this paper is to evaluate effective elastic properties of the aluminum hexagonal honeycomb filled with polyvinyl chloride (PVC) foams using the FE approach previously developed for hollow honeycomb cores. On the basis of the FE model developed, it is possible to calculate both elastic coefficients and local interfacial stresses of the foam-filled hexagonal honeycomb core. The influence of the foam on places of stress concentrations within cell walls of the honeycomb core is estimated.

2. General remarks

2.1. Materials

Prior to modelling of the mechanical properties of the foam-filled honeycomb structure, the properties of the each constituent material will be briefly considered below.

Traditionally the honeycomb structure can be manufactured either by expansion process or by corrugated method [17]. The latter is commonly used to produce high density honeycomb materials. Due to the manufacturing processes the pair of walls in the honeycomb cells is a double thickness $2t$, where t is the thickness of the aluminum foil used to produce the honeycomb core (Fig. 1a). As commercial products the hexagonal honeycomb cores are available in a large variety of specifications in terms of diameter of the cell D (or the cell wall length l) and thickness t (Fig. 1c) as well as of foil materials (aluminum alloys, titanium alloys, fiber reinforced plastic, resin impregnated paper). Although the foil materials are isotropic, honeycomb structures made of them exhibit highly orthotropic mechanical properties with respect to the L dimension (ribbon direction), the W dimension (transverse to the ribbon) and through-the-thickness T dimension, Fig. 1a, with a much higher out-of-plane than in-plane strength.

Divinycell H grade PVC closed-cell foams [18] are typically used as core materials for marine sandwich structures. These foams are available as plate-like panels in a wide range of thicknesses and densities. As using of foam materials rapidly increases during the last decades, the PVC foams recently have become a subject of extensive experimental investigations. The experimental data have shown that the linear behavior of the foams is limited to small strains, typically less than 5%. Moreover, the PVC foams with lower density up to $100\,kg\,m^{-3}$ (H100) exhibited almost isotropic mechanical properties, while denser foams such as H200 and H250 with densities $200\,kg\,m^{-3}$ and $250\,kg\,m^{-3}$, respectively, revealed a certain degree of anisotropy connected with the higher through-the-thickness tensile and compressive strength than the in-plane strength. Additional uniaxial tension and compression tests indicate that the PVC foams have also differences in the response with respect to compression and tension responses [19–21]. However, material constants related to the foams treated as an anisotropic material do not exist in literature up to now. Moreover, one can assume that the reason of the different responses for in-plane and out-of-plane directions can be associated with the foam densification through the thickness due to the roll-forming process used for the fabrication of the foam panels, i.e. in the view of the foam materials supplied by the manufacturer [21]. Besides, vacuum assisted resin transfer molding (VARTM) co-injection process [22] that is usually utilized to manufacture foam-filled honeycomb sandwich composites [23], the PVC foams are not exposed to such technology process and can be considered as bulk foam materials. Thereby, the foams may not show any anisotropy of their mechanical properties, though it is demanded for further experimental investigations. As a confirmation of this assumption, the micro-structure observations of the PVC foams with SEM did not

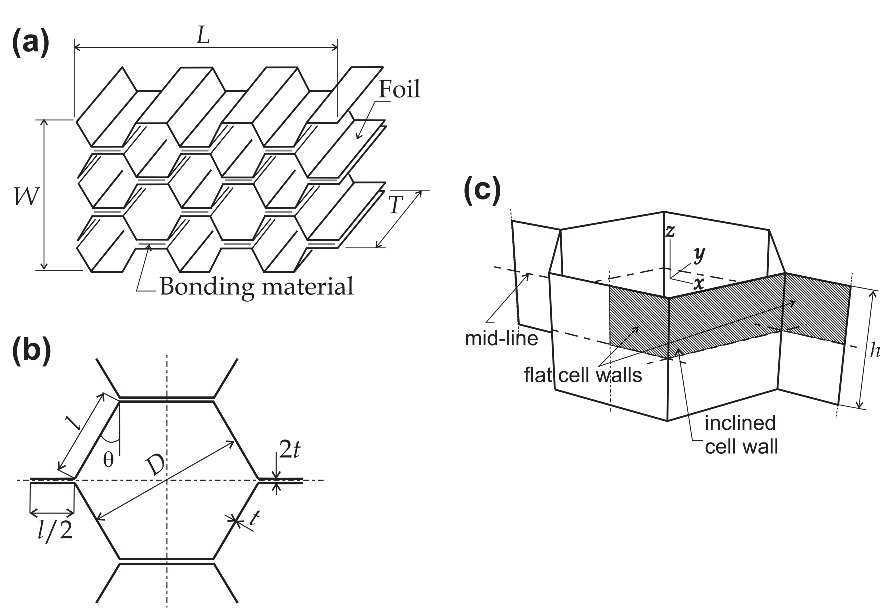

Fig. 1. Honeycomb core structure: (a) the general view; (b) the unit cell and (c) the unit cell parameters.

Table 1
Elastic properties of polyvinyl chloride (PVC) foams.

	E (GPa)	ρ (kg m^{-3})	ν
PVC H60	0.056	60	0.27
PVC H100	0.105	100	0.32
PVC H200	0.230	130	0.32

reveal any prevalent directions [20]. According to the manufacturer's data and the given reasoning, the H grade PVC foam materials placed into honeycomb cells are assumed as isotropic materials with average values of Young's moduli and Poisson's ratios taken from data [19]. Furthermore, according to Gibson and Ashby's investigations [6] the foams in their initial linear elastic responses can be considered as isotropic with the same elastic modulus in tension and compression. For the PVC foams used in this work the elastic limit has been recognized within 2–3% [19,20]. The isotropic properties of the PVC foams accepted in the paper are shown in Table 1.

2.2. Homogenization concept

The replacement of the heterogeneous honeycomb structure by an equivalent orthotropic homogeneous layer with effective material properties can be carried out on the basis of the strain energy homogenization technique of periodic media [24]. The smallest representative part of the original periodic structure, referred as the unit cell, is used to approximate macroscopic effective properties, corresponding to a real cellular structure. According to the homogenization procedure both the representative unit cell of the real honeycomb structure and the corresponding volume element of the effective homogeneous medium are subjected to the same deformation, and the strain energy stored in them is being assumed the same for both. Thereby, equivalent material properties will be related to the discrete parameters characterizing geometry and properties of the unit cell walls.

In this case the basic unit cell can be considered as a heterogeneous material assuming the elastic behavior, the strain and stress fields of which are related by a generalized Hooke's law

$$\sigma_{ij} = C^{eff}_{ijkl}\varepsilon_{kl}, \quad i,j,k,l = \overline{1,3}, \tag{1}$$

where σ_{ij} is the stress tensor averaged over the unit cell, ε_{ij} is strain tensor, corresponding prescribed strain states within the unit cell, and C^{eff}_{ijkl} are components of the effective elastic tensor. The latter can be derived as the second partial derivation of the strain density function U with respect to ε_{ij}

$$C^{eff}_{ijkl} = \frac{\partial^2 U}{\partial\varepsilon_{ij}\partial\varepsilon_{kl}}. \tag{2}$$

For an orthotropic material the elastic constitutive law (1) can be rewritten in a contracted notation through the compliance matrix $[S]$

$$\begin{pmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{yz} \\ 2\varepsilon_{xz} \\ 2\varepsilon_{xy} \end{pmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_{xy}}{E_x} & -\frac{\nu_{xz}}{E_x} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{yz}}{E_y} & 0 & 0 & 0 \\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{G_{yz}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{G_{xz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{G_{xy}} \end{bmatrix} \begin{pmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{pmatrix}. \tag{3}$$

The components of the compliance matrix highlight the effective engineering constants such as Young's moduli E_x, E_y, E_z, the shear moduli G_{yz}, G_{xz}, G_{xy} and Poisson's ratios ν_{xy}, ν_{yz} and ν_{xz} that are to be determined.

In order to obtain the effective macroscopic elastic properties of the foam-filled hexagonal honeycomb core, the 3D finite element model for the unit cell was created. The different finite element approaches [8,12,14] were proposed to predict the equivalent in-plane and out-of-plane constants of the hollow honeycomb core. In this paper the effective properties of the foam-filled hexagonal honeycomb core are determined using the strain energy-based prediction. The homogenization analyses are performed with the commercial available software ABAQUS [25].

The response of the suitable unit cell is examined under the following assumptions:

– the hexagonal cell itself is assumed to be perfectly regular with edge of the length l (Fig. 1c);
– the cell volume is filled by the foam material entirely and uniformly;
– the material behavior is considered as linear-elastically up to strain level 2%, moreover, the PVC foam materials are assumed isotropic and the difference between compressive and tensile responses are not taken into account;
– the perfect bonding between both core-to-skins and wall-to-wall contacts as well as the perfect interface between cell walls and the cell filler (foil-to-foam contact);
– buckling of the cell walls under the transverse compression loading is disregarded;
– the skins are stiff enough so that their in-plane deformation is not disturbed by distortion of the core cell structure.

3. Numerical modelling of foam-filled honeycomb cores

3.1. Out-of-plane core properties

The out-of-plane properties are the most important with the viewpoint of functionality of the core, as they resist the shear stresses and out-of-plane loading. The cellular properties were converted to structural continuum ones by using the FE model developed with ABAQUS.

In this model the unit cell is modelled with a combination of shell and solid elements. An eight-node second-order shell element `S8R5` with reduced integration is selected to model the cell walls, Fig. 2a. Here, (x,y,z) is global coordinate system, and (ξ,η) is local one situated at the cell walls. This element offers five degrees of freedom (DOF) per node (both rotation and displacement), making its ideally suited for modelling three dimensional (3D) thin-walled structures. The isotropic material model for the element, corresponding to the properties of the honeycomb parent foil material, was assigned.

A 20-node brick element with a parabolic basis function `C3D20R` was used to model the core foam material. A reduced integration element formulation was used to avoid volumetric locking, Fig. 2b. Isotropic material models were used for the foams. The size of each solid element should satisfy the compatibility conditions with the shell elements, so that their form was close to cubic. The coupling between different shell and solid elements in the mesh was achieved by the constraint of the redundant DOFs in coincident nodes, where these elements have been contacted.

To minimize the computational cost, the FE model is developed with consideration of periodicity conditions. Due to symmetric nature of the unit cell only one-quarter of its full size and half its full height were meshed (Fig. 2) with boundary conditions accounting for the symmetry. Single wall thickness were used for the model, as the double thickness walls belonging to the ribbon direction are on the split line of the repeating unit cells. The sensitivity of the mesh refinement to the value of material parameters was analyzed. The convergence studies showed that the moderately refined mesh such as 4×4 elements for the flat cell walls

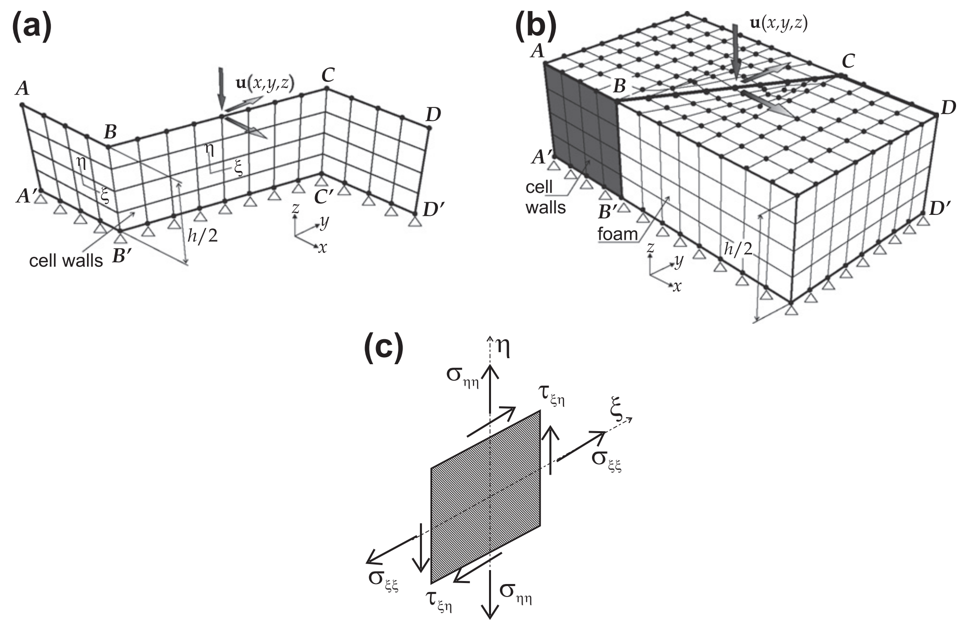

Fig. 2. FE model of one-quarter of the unit cell: (a) the hollow unit cell; (b) the foam-filled unit cell and (c) the stress state in the cell wall.

and 8×4 elements for the inclined cell wall are optimum for the homogenization analyses. The whole model of the unit cell, meshed in this manner, consisted of 2786 nodes and 544 elements.

Three independent loading cases corresponding to the conditions of mechanical tests such as shearing in two mutually perpendicular x- and y-directions and compression in the transverse direction to find appropriate engineering constants were featured by three types of boundary conditions. These displacement constraints imposed are collected in Table 2. The uniform displacement u is applied to all nodes at the top surface of the unit cell FE model along x, y or z directions, while nodes against displacement on its opposite bottom surface in the same direction are fixed for the each load case, Fig. 2. Moreover, to calculate the transverse shear moduli the bending DOFs of shell elements were restrained to obtain the pure shear state whereas each node was forced to have the same vertical displacement for calculation of the transverse Young's modulus. Symmetry constraints were imposed on the flat walls resulting in a section of the unit cell that could only displace sideways in pure shear or pure vertically.

The determination of the effective moduli G_{xz}, G_{yz} and E_z is based on computation of the stored strain energy inside the unit cell model for each deformed state mentioned above. This was done by means of the option in ABAQUS. Then, the effective out-of-plane constants, C^{eff} can be eventually obtained using the numerical differentiation of U in (2). It yields the following relationship

$$C^{eff} = \frac{2U}{\varepsilon_{eq}^2 V}, \tag{4}$$

where ε_{eq} is the equivalent homogeneous strain, i.e. ε_{zz}, ε_{xz} and ε_{yz} corresponding to the imposed displacement fields, V denotes the volume of the volume block of the homogeneous material equivalent to the honeycomb unit cell and was found as $V = 2l^2\cos\theta(1 + \sin\theta)h$. Notice, that the relative displacement u between top and bottom faces of the unit cell with height h induces the equivalent strain equal to $\varepsilon_{eq} = u/h$, according to assumption of the small displacements and strains.

In order to verify the proposed FE model, the comparative numerical studies were performed with the same material properties of the honeycomb foil material as was described in [12], i.e. $E = 72$ GPa, $\rho = 2770\ \mathrm{kg\ m^{-3}}$, $\nu = 0.31$. The properties of polyvinyl chloride foam fillers of different densities are specified in Table 1. The unit cell dimensions were applied as $l = 1$ mm, $t = 0.08$ mm, $\theta = 30°$, and h varied from 1 to 10 mm. Fig. 3 presents the comparison between magnitudes of the transverse shear modulus G_{xz} for the various unit cell heights obtained by using the FEM approach

Table 2
Boundary conditions in FE model for calculation effective out-of-plane moduli.

Edges	Load case 1			Load case 2			Load case 3		
	u_x	u_y	u_z	u_x	u_y	u_z	u_x	u_y	u_z
AA′/DD′	Free	0	0	Free	Free	0	0	0	Free
BB′/CC′	Free	0	Free	0	Free	Free	0	0	Free
AB/BC/CD	u	0	0	0	u	0	0	0	$-u$
A′B′/B′ C′/C′ D′	0	0	Free	0	0	Free	0	0	0

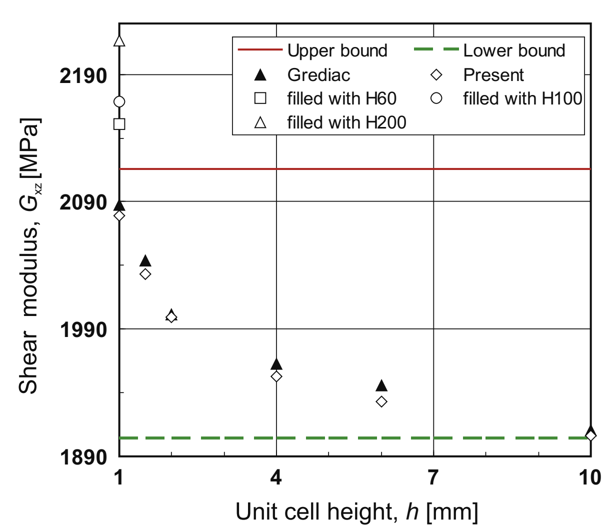

Fig. 3. Shear modulus G_{xz} predictions for the hollow and the filled honeycomb cores.

in [12] and calculated with ABAQUS in the present work. One can see a good correlation between these results. The lower and upper bounds of the transverse shear modulus G_{xz} corresponding to cases of zero and infinitely large skin effect, respectively, were found by using the Gibson and Ashby's analytical expressions in [6]. The bounds are presented as dash and straight lines, respectively, in Fig. 3. These limits values based on the assumption of an uniform stress state in unit cell walls do not account for the real stress state variation within the unit cell walls caused by skins. However, due to the quite localized character of the skin influence and in the case of high enough honeycomb cores the longitudinal shear modulus G_{xz} of sandwich constructions with the honeycomb core, in fact, coincides with the corresponding modulus of a honeycomb structure itself [10]. Thus, the correspondence of the transverse shear modulus predicted with ABAQUS to the analytical limits ensures the correctness of the proposed FE model. Besides, the proposed FE model allows comparing the effective moduli for both the hollow and the filled with the foams honeycomb cores. The Fig. 3 demonstrates such predictions regarding the shear modulus G_{xz} for the core with defined thickness $h = 1$ mm. From Fig. 3 one

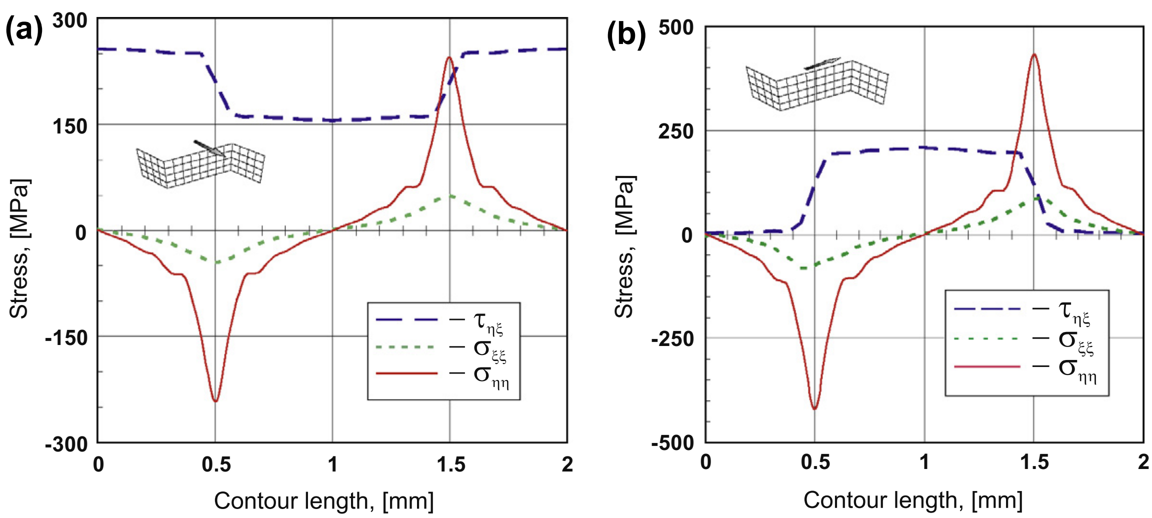

Fig. 4. Interfacial stress distributions $\sigma_{\xi\xi}$, $\sigma_{\eta\eta}$ and $\tau_{\eta\xi}$: (a) for the strain state $\gamma_{xz} = 0.01$ and (b) for the strain state $\gamma_{yz} = 0.01$.

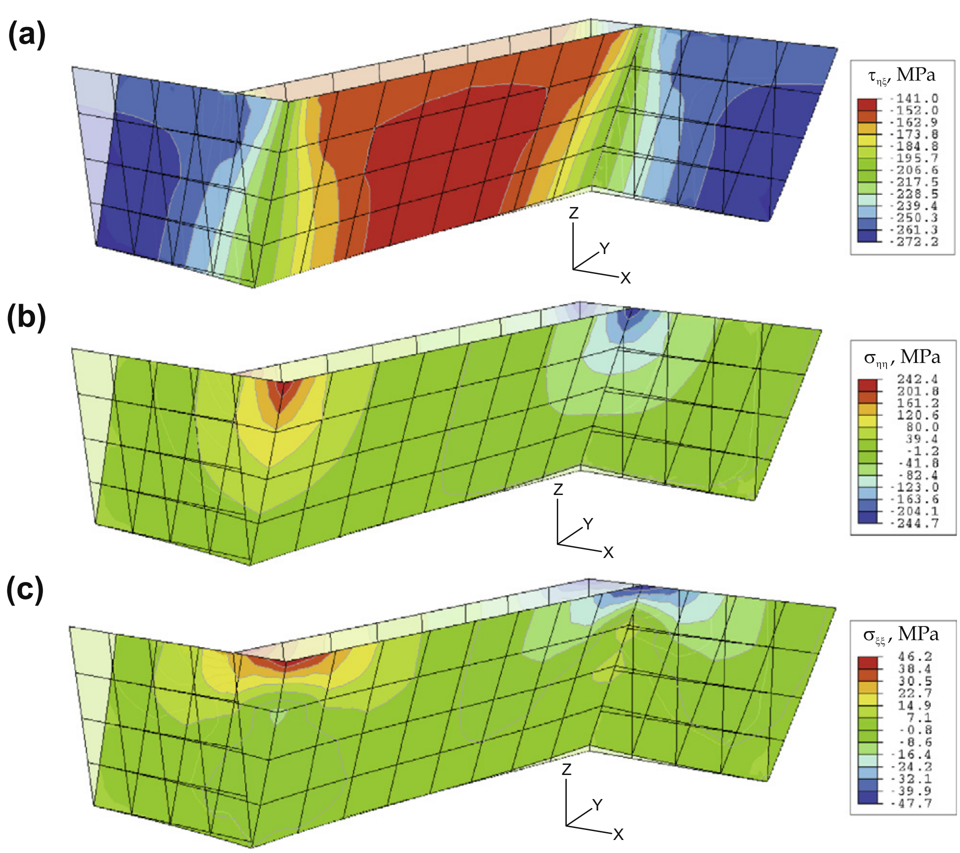

Fig. 5. Stress distributions in the hollow honeycomb cell walls under strain state $\gamma_{xz} = 0.01$: (a) the shear stress $\tau_{\eta\xi}$; (b) the normal stress $\sigma_{\eta\eta}$ and (c) the normal stress $\sigma_{\xi\xi}$.

may conclude that the foam-filling increases transverse properties of the honeycomb core and the magnitude of the transverse shear modulus are greater when the foam density increases, e.g. about 7% in case of H200 foam.

Fig. 4 shows the distributions of the interfacial stresses along top faces of the inclined and two flat cell walls (denoted as a contour length) of the hollow unit cell, the same as for the previous test study, which was subjected to the pure shear strains equal to $\gamma_{xz} = 0.01$ and $\gamma_{yz} = 0.01$. It can be observed that at the intersection of the inclined and flat cell walls not only shear stress $\tau_{\eta\xi}$ occurs under the pure shear strain state. The normal stress at this small region drastically achieves its maximal value of the same or even larger level comparing with the shear stress $\tau_{\eta\xi}$. The latter undergoes rapid change to its maximum level in this region. It should be noted that the distributions of the shear $\tau_{\eta\xi}$ and normal $\sigma_{\eta\eta}$ stresses are qualitatively similar to those obtained in [11] for the hollow honeycomb cell with the cell angle of $\theta = 10°$.

The deformed form of the unit cell and the contour plots for the shear stress $\tau_{\eta\xi}$ and normal stresses $\sigma_{\eta\eta}$ and $\sigma_{\xi\xi}$ within the unit cell walls for the case of the shear strain state $\gamma_{xz} = 0.01$ are presented in Fig. 5a–c. It is evident that the stress distributions can not be considered as uniform and it is a result of skin influence or so-called skin effect. The shear stress $\tau_{\eta\xi}$ in the flat walls are much higher than in the inclined wall, whereas the normal stresses $\sigma_{\eta\eta}$ and $\sigma_{\xi\xi}$ have high value only around the core/skins interface. This confirms that the skin effect is a localized phenomenon. Thus, the existing complex stress state concentration at the intersection of the core walls can be considered as critical for designing of the honeycomb structure. Such conclusion for the irregular honeycomb cell was earlier drawn by Chen and Davalos in [11] as well.

The influence of the PVC foam on the distributions of the interfacial stresses and the stress concentration zone is investigated in the second example. The geometry and material of the unit cell were taken from the HexWeb honeycomb specification [17], i.e. the cell size was equal to 3.175 mm and the foil material was made of 5056 aluminum alloy with the thickness equal to 0.0508 mm. The height h of the honeycomb core was equal to 5 mm. The calculated results are shown in Fig. 6 for the pure shear strain state $\gamma_{xz} = 0.01$. As it can be seen from Fig. 6a, the interfacial shear stress $\tau_{\eta\xi}$ slightly increases on the flat cell walls, while it insignificantly decreases on the inclined cell wall with increasing of the foam density. The maximum difference between the stress values on the flat and inclined cell walls for the cases of the hollow and filled with the densest foam H200 cells is about 1%. Such shear stress redistribution along the top face of the flat and inclined cell walls with increasing the foam density reflects the minute reinforcement (or stabilization) of the cell walls by the foam filler. It is obviously that the honeycomb cells will be reinforced by the foam filler placing and, then, the core stiffness will increase. Therefore, for the same level of the shear strain imposed to both the hollow and foam-filled honeycomb cells the corresponding shear stress on the most loaded flat cell walls has to increase with the foam density increasing. On the other hand, this strengthening stops the inclined cell walls encapsulated by the foam from the shearing that leads to decreasing the shear stress in them. Furthermore, it is well-known that when a transverse shear loading is applied to a sandwich structure then one of the potential failure mode is peeling-off of the face sheet due to the normal interfacial stress $\sigma_{\eta\eta}$ arising by the local bend of the cell walls near the face sheets or so-called warping effect. Fig. 6b shows that the foam-filling decreases the

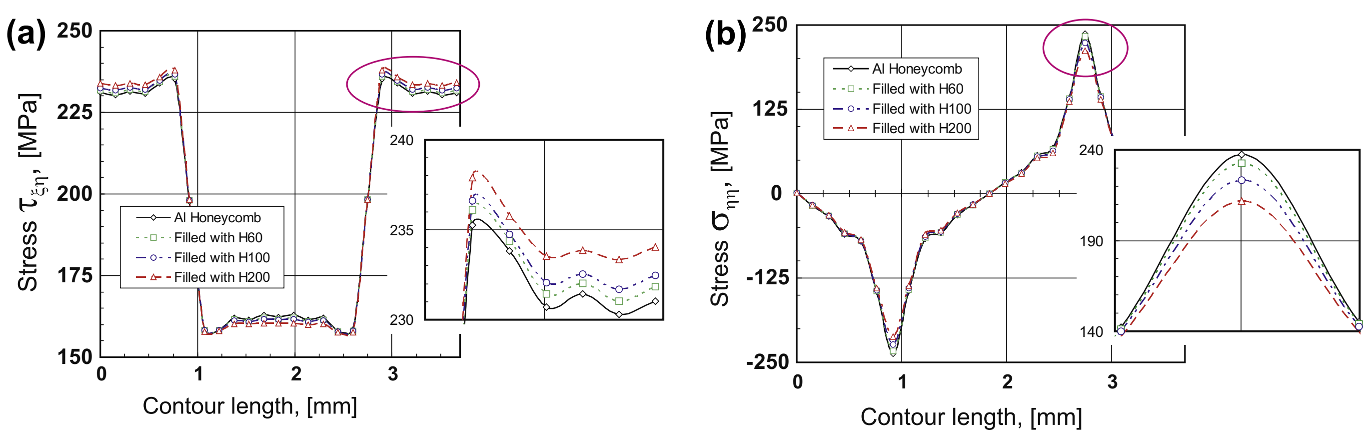

Fig. 6. Interfacial stress distributions along the top face of the cell walls for the hollow and filled with the different foams honeycomb structure subjected to $\gamma_{xz} = 0.01$: (a) the shear stress $\tau_{\eta\xi}$ and (b) the normal stress $\sigma_{\eta\eta}$.

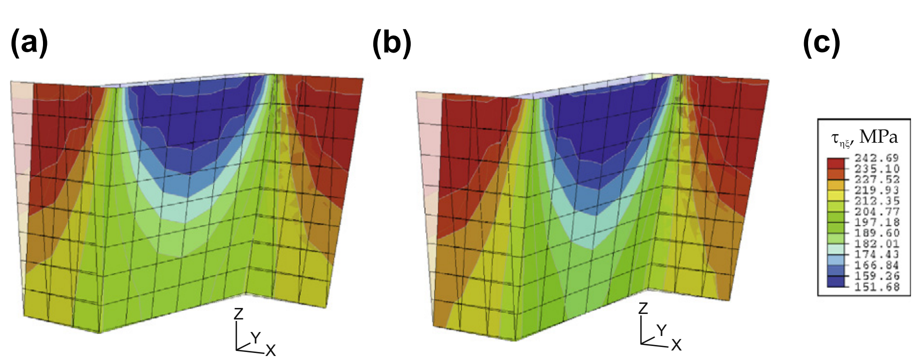

Fig. 7. Contour plots for the shear stress $\tau_{\eta\xi}$ under $\gamma_{xz} = 0.01$: (a) the hollow unit cell; (b) the unit cell filled with foam H200 and (c) scale levels.

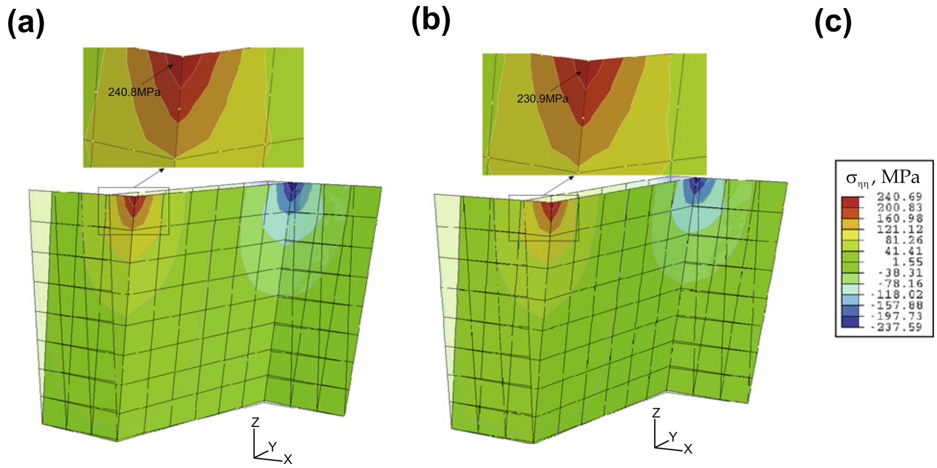

Fig. 8. Contour plots for the normal stress $\sigma_{\eta\eta}$ under $\gamma_{xz} = 0.01$: (a) the hollow unit cell; (b) the unit cell filled with foam H200 and (c) scale levels.

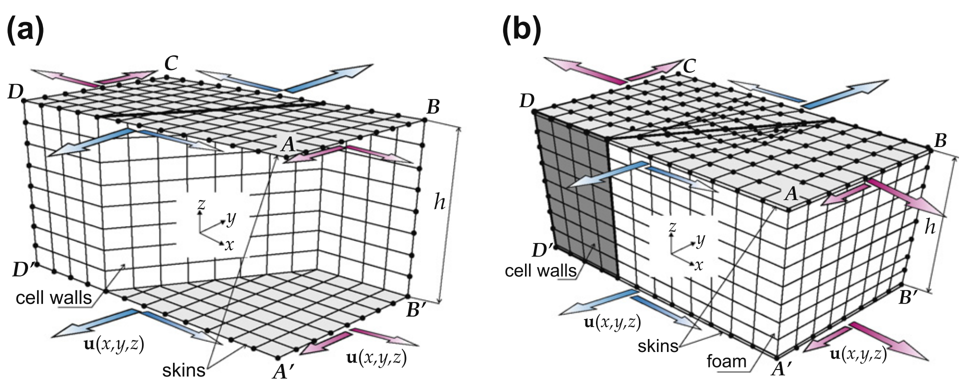

Fig. 9. One-quarter of the unit cell including the top and bottom skins: (a) the hollow unit cell and (b) the foam-filled unit cell.

magnitude of this peel interfacial stress $\sigma_{\eta\eta}$ at the critical section. The effect of the foam-filling on the normal stress $\sigma_{\eta\eta}$ is greater with increasing foam density and in the case of the foam H200 the stress reduction is approximately 11%. Thereby, one can conclude that the warping effect has trends to decreasing with increasing the foam density.

In order to illustrate more detailed stresses distributions within the honeycomb cell the contour plots of the shear $\tau_{\eta\xi}$ and normal $\sigma_{\eta\eta}$ stresses for both the hollow and filled with the PVC H200 foam honeycomb unit cells are shown in Figs. 7 and 8. By comparing Fig. 7a and b one can observe that, while the face sheet retains distortion near the interface for both the hollow and foam-filled cells, but the shear stress distribution with the height of the core differs between them. The walls of the honeycomb cell filled with the foam are loaded under lower stress level than for the hollow cell and this stress level region from the skin/core interface toward the core center is greater. It clearly demonstrates sharing of the load between the honeycomb cell walls and placed foam filler or the foam stabilization effect of the cell walls, as a result the stress distribution through the core thickness is more uniform than for the case without the foam.

It follows from the comparison Fig. 8a and b that the level of the normal stress at the stress concentration zones decreased appreciably. Obviously, it can be explained by sharing of the total shear load between the honeycomb cell walls and foam filler. Thus, filling of the honeycomb cells with the foam affects the critical peel stress and, consequently, increases the resistance of a sandwich structure to the debonding appearance. Moreover, an interfacial crack, in fact, will propagate along the skin/core interface when the crack energy release rate exceeds a critical level defined by the interface toughness of the sandwich structure. Then, because of increasing the contact area between the core and face sheet one can be expected that the foam-filled core will provide a higher level of the critical energy release rate. However, this assumption requires additional investigations and are outside the goals of this study.

3.2. In-plane core properties

The honeycomb in-plane properties are relatively low in comparison to the out-of-plane ones and they are usually ignored

Table 3
Boundary conditions in FE model for calculation effective in-plane moduli.

Load cases	Edges	u_x	u_y	u_z	φ_x	φ_y	φ_z
1	ABB′A′	u	Free	Free	Free	0	0
	CDD′C′	$-u$	Free	Free	Free	0	0
	ADD′A′/CBB′ C′	Free	0	Free	0	Free	0
2	CBB′C′	Free	u	Free	0	Free	0
	ADD′A′	Free	$-u$	Free	0	Free	0
	ABB′A′/CDD′C′	0	Free	Free	Free	0	0
3	ABB′A′	Free	u	Free	Free	0	Free
	CDD′C′	Free	$-u$	Free	Free	0	Free
	ADD′A′	$-u$	Free	Free	0	Free	Free
	CBB′C′	u	Free	Free	0	Free	Free

because all of the in-plane compression and tensile forces are carried out by the skins. Thus, there traditionally are two statements for calculation of the in-plane effective moduli. The first one ignores the skins and their interaction with core ('free modulus model'), and the second one accounts for of their influence on the core ('constrained modulus model'). Without skins a honeycomb core behaves like a spring in extension and has no significant stiffness. Consequently, the honeycomb walls will be only subjected to the pure extension. Unlike, when the skins are attached to the core, the honeycomb walls are enforced to have the same deformation as the skins. This will limit to act the honeycomb core in the spring manner and a combination of extension and bending appears.

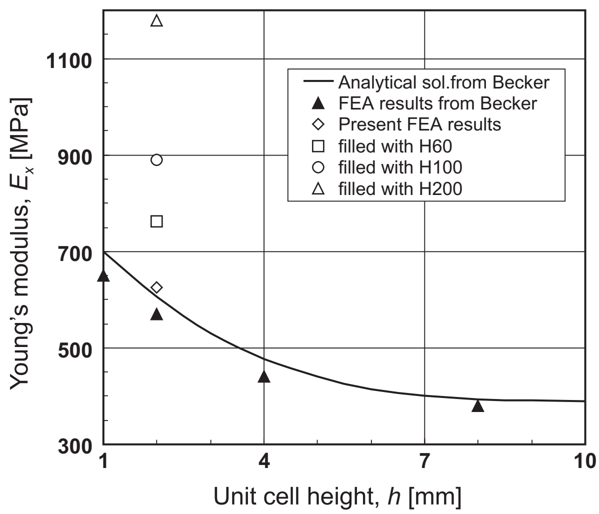

Fig. 10. Effective longitudinal Young's modulus E_x of the hollow and the filled honeycomb cores.

The effective in-plane Young's moduli E_x, E_y and the shear modulus G_{xy} were found within the developed FE approach outlined above. For calculation of the in-plane constants the honeycomb unit cell attached to the skins was considered as a representative part of the honeycomb core. Due to symmetry, one-quarter of the full unit cell can be taken into account as earlier but its full height should be used. FE models of the unit cell corresponding to the both cases with and without the foam filler are shown in Fig. 9a and b, respectively.

The mesh was comprised of the same elements that were used for prediction of the out-of-plane constants, and the skins were discretized by the similar shell finite elements those as the cell walls. The skins were attached to the cell walls by coincident nodes. The total number of finite elements in the unit cell model was 4596. Uniaxial tension loadings and in-plane shearing were simulated by imposing the uniform displacement on the skins of the unit cell along x or y directions as is shown in Fig. 9. The corresponding displacement constrains imposed are listed in Table 3. Similarly for the calculation of the out-of-plane constants, the in-plane ones were found by using the stored strain energy U of the unit cell. However, in this case the total strain energy consisted of two parts corresponding to the skins and the core, separately. In order to asses the in-plane elastic constants appropriate to the 'free modulus model' the elastic strain energy stored only in the core elements of the unit cell should be only included, otherwise the 'constrained modulus model' is considered. The effective in-plane constants were obtained using numerical differentiation of U in (4). In doing so, C^{eff} is one of the effective in-plane moduli and ε_{eq} is the equivalent homogeneous strain that relates to the strains ε_{xx}, ε_{yy} and ε_{xy}.

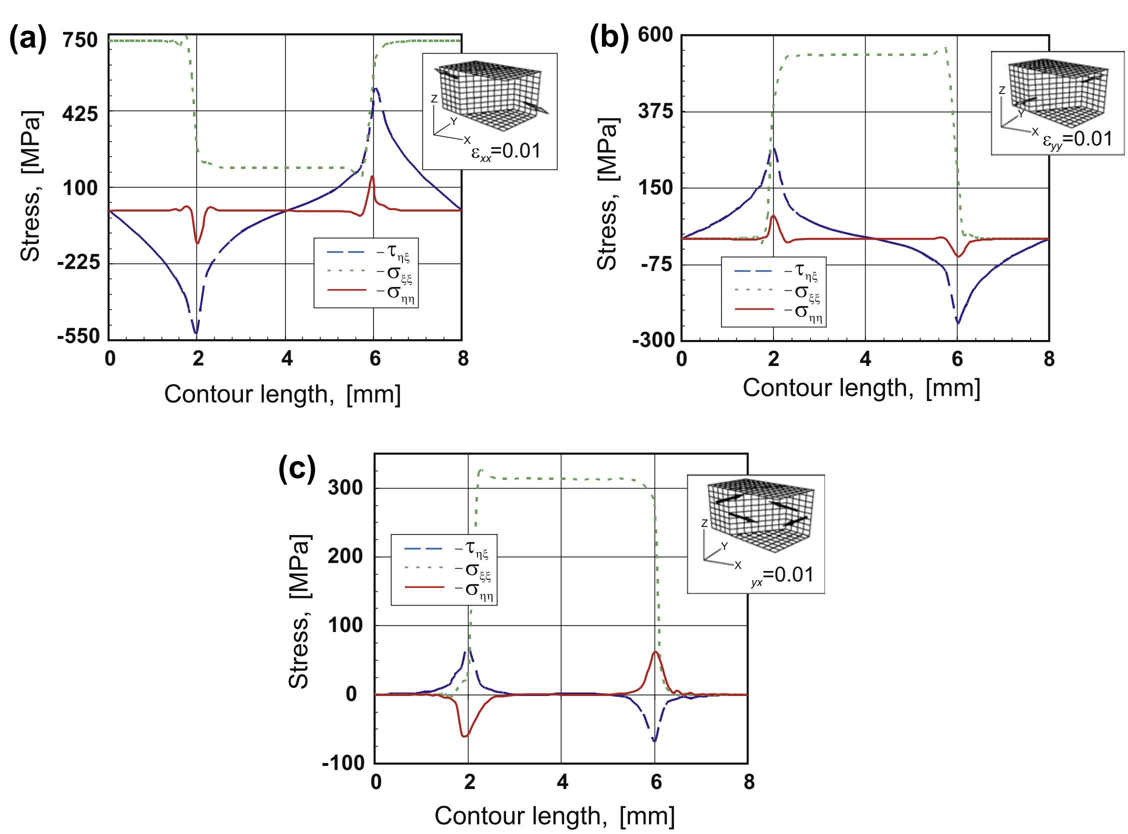

Fig. 11. Distributions of the interfacial stresses $\tau_{\eta\xi}$, $\sigma_{\xi\xi}$, $\sigma_{\eta\eta}$ for the strain states: (a) $\varepsilon_{xx} = 0.01$; (b) $\varepsilon_{yy} = 0.01$ and (c) $\gamma_{yx} = 0.01$.

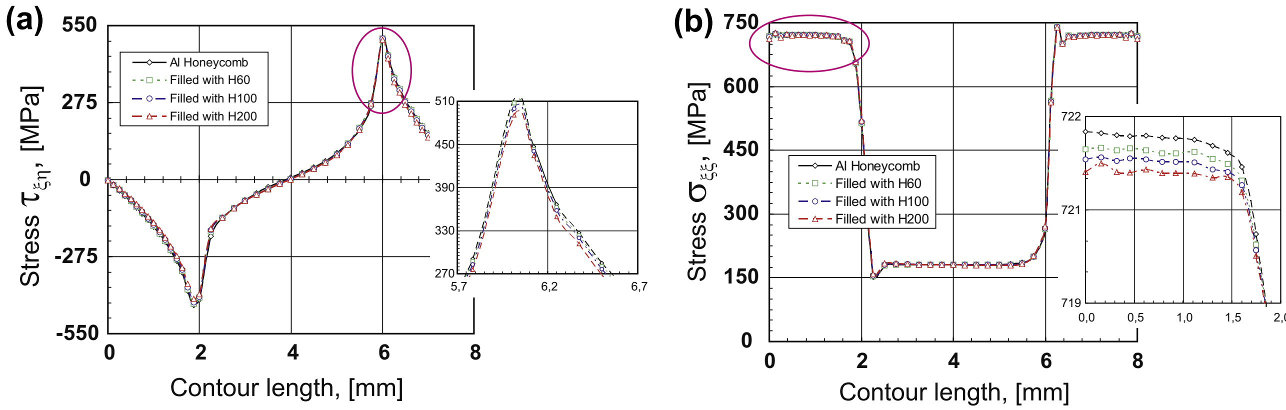

Fig. 12. Interfacial stress distributions along the top face of the cell walls for the hollow and filled with the different foams honeycomb structure subjected to $\varepsilon_{xx} = 0.01$: (a) the shear stress $\tau_{\eta\xi}$ and (b) the normal stress $\sigma_{\xi\xi}$.

For verification of the proposed model, the FE predictions of a hollow honeycomb core with dimensions and properties the same as described in [8] were carried out, i.e. the following material properties: $E = 72.2$ GPa, $\nu = 0.34$ and the next characteristic dimensions: $l = 4$ mm, $t = 0.05$ mm and $\theta = 30^\circ$ were applied (Fig. 1c). Due to the cumbersome modelling work required by FE analysis only one case of height, $h = 2$ mm was considered. Thus, the influence of the thickness core on material properties was not examined in this paper.

Fig. 10 shows values of the effective 'constrained' Young's modulus E_x calculated for both the hollow and the foam-filled cores and presents a comparison with data from the paper [8]. It can be seen that the filling with foam of the honeycomb core increases its initial longitudinal in-plane stiffness almost twice for the case of the PVC H200 foam.

Fig. 11a–c demonstrate the distributions of the interfacial shear stress $\tau_{\eta\xi}$ and the interfacial normal stresses $\sigma_{\eta\eta}$ and $\sigma_{\xi\xi}$ due to imposing on the unit cell the uniform strain states such as $\varepsilon_{xx} = 0.01$, $\varepsilon_{yy} = 0.01$ and $\gamma_{yx} = 0.01$, respectively. It is worth to notice that the same distribution of the interfacial stresses $\tau_{\eta\xi}$ and $\sigma_{\eta\eta}$ along the cell contour length in the case of the regular honeycomb cell when the unit cell was subjected to the strains ε_x and ε_y are for the first time obtained in [11], where the core thickness was equal to 4 mm. Thereby, the interfacial shear and normal stresses calculated in this study and given in Chen-Davalos05 were compared. It seems that two results were similar qualitatively and differed insignificantly quantitatively due to the different core heights accepted. By looking at Fig. 11a–c one can conclude that the interfacial shear stress $\tau_{\eta\xi}$ reaches its maximum at the cell walls intersection and vanishes at the mid-spans of cell walls for the all cases of strain states imposed. The highest magnitude of the interfacial tension–compression stress $\sigma_{\xi\xi}$ occurs along the flat walls under stretch in x-direction, Fig. 11a. While the maximal value of this stress appears along inclined wall in the cases of the strain states caused by $\varepsilon_{yy} = 0.01$ or $\gamma_{yx} = 0.01$, Fig. 11b and c, respectively. The interfacial transverse normal stress $\sigma_{\eta\eta}$ that defines tension–compression in z-direction due to Poisson's effect, is small in comparison to the two previous ones. Thereby, one can conclude that because almost no peel force occurs at the skin/core interface, the in-plane strain states create less critical stress states at the skin/core interface than the transverse one. Nevertheless, a complex stress state taking place at the interface due to skin effect under in-plane stretching makes a stress concentration zone at the intersection of the cell walls critical in design of such honeycomb structures.

The positive, from a strength viewpoint, foam-filling influence on the distribution of the interfacial shear $\tau_{\eta\xi}$ and normal $\sigma_{\xi\xi}$ stresses under the imposed tension strain $\varepsilon_{xx} = 0.01$ is shown in Fig. 12. One can see that the presence of the foam filler within the honeycomb cell slightly decreases the peak values of the interfacial in-plane stresses about 1% for the shear stress and up to 3% for the normal stress. Fig. 13 demonstrates the von Mises stress variation through the core thickness at the critical section for both the hollow and filled with the foam H200 unit cell subjected to the tension strain $\varepsilon_{xx} = 0.01$. The stress state level decreasing and skin effect reducing in the honeycomb cell with the foam filler presence are clearly observed. This demonstrates the reinforcement of core cell walls against a local buckling. The latter can be considered as a possible way for improvement of the conventional honeycomb structure to the impact resistance under in-plane loadings [26].

The deformed state under the tension strain $\varepsilon_{xx} = 0.01$ and contour plots of the shear $\tau_{\eta\xi}$ and normal $\sigma_{\xi\xi}$ stresses distributed within unit cell walls for both cases of hollow and filled with foam PVC H200 are presented in Figs. 14 and 15. As one can see, the slight stresses redistribution within the whole unit cell takes place. Thereby, the foam filler stabilizes the cell walls making the stresses distributions more uniform.

Finally, the influence of foam fillers on the effective elastic constants of a honeycomb core was studied. The hexagonal honeycomb core with thickness equal to 5 mm and made of foil of 5052 aluminum alloy with thickness $t = 0.0254$ mm and cell wall size of $D = 3$ mm ($l = 1.833$ mm) as is supplied by HexCel Corp. [17] was considered. The material data corresponded to the values of $E = 70$ GPa, $\nu = 0.3$, $\rho = 2680$ kg m^{-3}. The various PVC foam fillers

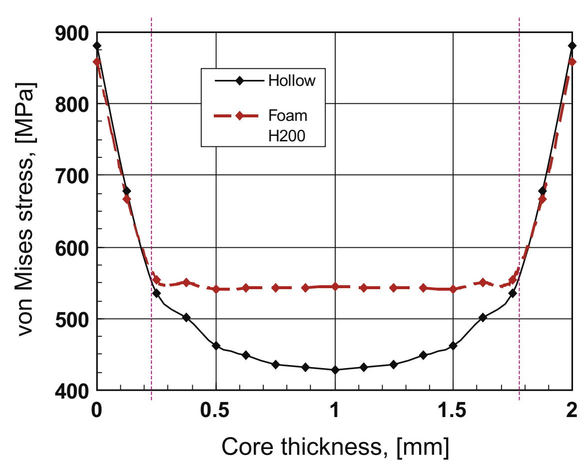

Fig. 13. Von Mises stress distribution vs. core thickness.

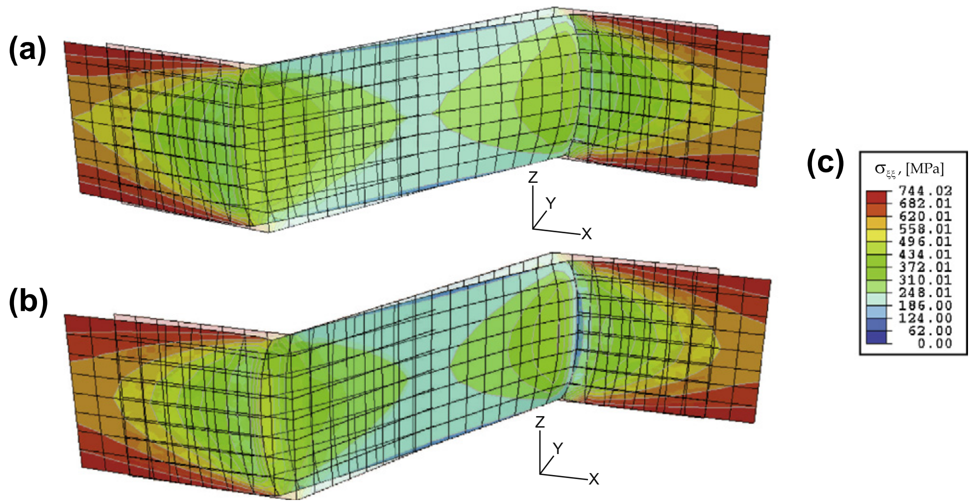

Fig. 14. Contour plots ($\varepsilon_{xx} = 0.01$) for the normal stress $\sigma_{\xi\xi}$: (a) the hollow cell walls; (b) the filled with H200 foam cell walls and (c) scale levels.

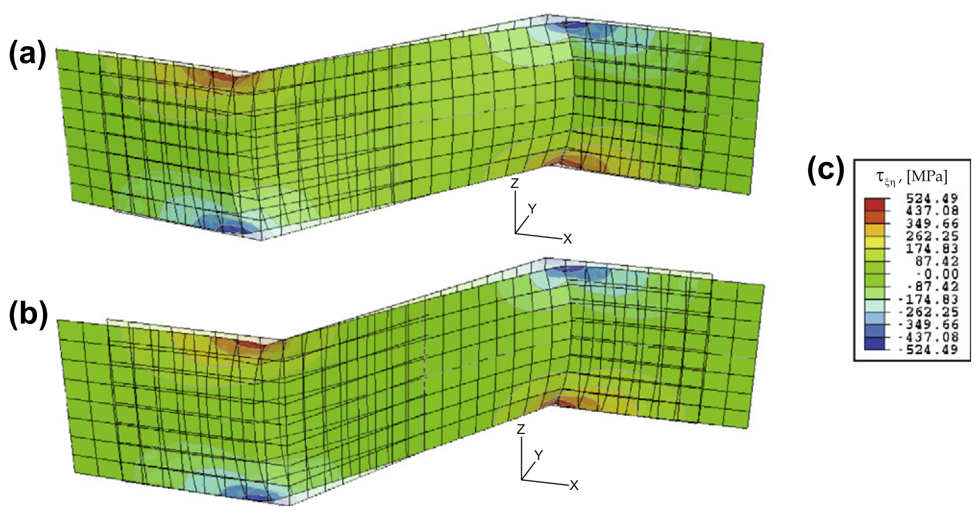

Fig. 15. Contour plots ($\varepsilon_{xx} = 0.01$) for the shear stress $\tau_{\eta\xi}$: (a) the hollow cell walls; (b) the filled with H200 foam cell walls and (c) scale levels.

Table 4
Effective elastic properties of the core materials.

	E_x (MPa)	E_y (MPa)	E_z (MPa)	G_{xy} (MPa)	G_{xz} (MPa)	G_{yz} (MPa)	ρ (kg m^{-3})
AlH [6]	0.430	0.430	1493	0.108	341.1	205.4	57.17
Present	0.461	0.461	1494	0.119	341.7	192.1	57.17
FfH60	0.672	0.672	1549	0.238	356.2	212.1	101.95
FfH100	0.788	0.788	1598	0.282	374.3	224.4	141.31
FfH200	1.061	1.061	1722	0.386	422.1	238.9	170.83

were taken into account. The effective elastic out-of-plane and 'free' in-plane constants of such core materials were calculated based on the described FE model and are listed in Table 4. It is worth to notice that the densities of the foam-filled honeycomb cores were obtained on the basis of Rule of Mixtures.

By comparing the mechanical properties of the hollow and filled with densest foam H200 honeycomb cells given in Table one can conclude that in-plane 'free' Young's moduli E_x and E_y of the foam-filled cell increased in 130%, whereas its 'free' in-plane shear modulus G_{xy} rose in 100%. The out-of-plane properties of the honeycomb cell with the foam H200 adding changed in the different way. The transverse Young's modulus E_z increased in 15%, whereas the transverse shear moduli G_{yz} and G_{xz} were higher in comparison to hollow structure about 24%. It is visible that application of the PVC filling of the honeycomb structure causes more significant strengthening in-plane properties of the composite panel than its out-of-plane ones.

4. Conclusions

The aim of the paper was investigation of the foam-filling influence on the elastic behavior of the honeycomb structure. The strengthening effect in the honeycomb sandwich panels by application of the filling PVC foams is important. The foam increases stiffness of the sandwich panels, enhances the resistance to the damage caused by debonding. The major conclusions resulting from numerical investigations can be summarized for the global effects as following:

- the global elastic behavior of sandwich plates with aluminum hexagonal honeycomb core filled with PVC foam are qualitatively similar to hollow one;
- filling with PVC foam H200 of the honeycomb structure causes substantial increasing the out-of-plane transverse Young's modulus and effective shear moduli in 15% and 24%, respectively;
- the in-plane effective material properties of the honeycomb structure are more strongly affected by the foam filler presence so that its filling with PVC foam H200 causes rising of in-plane shear modulus and Young's modulus in 100% and 130%, respectively;
- the local shear stress $\tau_{\eta\xi}$ reaches its maximum value in the flat cell walls, whereas the local normal stress $\sigma_{\eta\eta}$ has the highest value at the intersections of the unit cell walls;
- filling with PVC foam causes small reduction of the maximum magnitudes of the stresses at the critical section such as intersection of cell walls and, as consequence, leads to more uniform stress state within cell walls. This effect is magnified by higher density of the filling foam.

Acknowledgements

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