The influence of spin-orbit interaction and Zeeman effect on the persistent current in a one-dimensional ring of correlated electrons

M.V.Moskalets
Fl.48, 93-A prospect Il'icha, 310020 Khar'kov, Ukraine
(Dated:)

The effect of both spin-orbit interaction and Zeeman interaction on a thermodynamic equilibrium (persistent) current in a one-dimensional ballistic ring of correlated electrons is considered at arbitrary temperatures. It is shown that the change of the number of spin excitations in a ring due to the Zeeman effect affects considerably the persistent current.

PACS numbers: 72.10.-d, 73.20.Dx, 71.70.Ej

I. INTRODUCTION

The Aharonov-Bohm (AB) effect [1] ensures a possibility for the influence on a quantum interference of electrons and it leads to a number of remarkable effects in mesoscopic physics [2]. So, at low temperatures thermodynamic [3, 4] as well as kinetic [5] properties of doubly connected mesoscopic samples (rings) oscillate as a function of an AB magnetic flux Φ with a period of $\Phi_0 = h/e$ [6, 7].

One of the such effects is an existence in normal (non-superconducting) rings of a thermodynamic equilibrium (persistent) current

$$I = -\partial F/\partial\Phi \tag{1}$$

(where F is the free energy for an isolated ring or the thermodynamic potential for a ring coupled to a reservoir), which was theoretically predicted in Refs.[8, 9] and was observed experimentally in Ref.[10]. The theoretical consideration [11] has shown that the properties of persistent currents considerably depend on the properties of an electron system in a ring as well as on the interaction with an environment (with a reservoir) that allows to study fundamental problems of quantum physics.

The parity effect [12]-[24] (i.e., the dependence of persistent current properties on the parity of the number N_e of spinless electrons or on $N_e = N_{e\uparrow} + N_{e\downarrow}$ modulo 4 for spinfull electrons in a ring) is one of the most important properties of persistent currents. This effect takes place as in isolated rings as in rings coupled to an electron reservoir [18, 25, 26]. In particular, for the even number N_0 of electrons with spin in the ground state the period of AB oscillations equals to Φ_0 whereas if N_0 is odd then the period is $\Phi_0/2$ [17].

Though the AB magnetic flux affects only the charge degrees of freedom of an electron system the spin subsystem influences on AB oscillations also, that is due to the parity effect and it especially displays for interacting electrons (e.g.,see Ref.[4]). For instance, such an effect leads to an existence of fractional AB oscillations with a period of Φ_0/N_0 in isolated rings with a small number of electrons [20, 23, 27]. In systems with a large number of particles in the common case the fractional oscillations do not appear [23], however the effect of a spin subsystem may reduce the period of AB oscillations by a factor of two or four [26].

Thus, if we affect the spin subsystem we may change the persistent current. For example, the spin-orbit (SO) interaction results in an effective spin-dependent magnetic flux [28], that may change both the current amplitude and the period of oscillations. As was pointed out in Ref.[29], the effect of SO interaction is the manifestation of the Aharonov-Casher effect [30] which can produce persistent currents like the Aharonov-Bohm effect [31, 32]. Besides, in an inhomogeneous magnetic field the electron wave function acquires an additional phase (over a pure orbital phase) [33]. The existence of such a geometric phase in adiabatic cyclic evolution of a quantum particle with a spin, which was firstly predicted by Berry [34], affects the quantum interference also and it changes the persistent current.

Another effect due to an electron spin is the Zeeman splitting of electron levels in a magnetic field. This effect influences on persistent currents in rings without a spin polarization (namely such a case is considered below). In the present paper we consider the effect of both the SO interaction and the Zeeman interaction on the persistent current. The SO interaction affects the current in an isolated ring as well as in the ring coupled to a reservoir. In the former case if the number of electrons is odd then the SO interactions results in a persistent current even without an AB magnetic flux. If the number of electrons is even such a current (i.e., at $\Phi = 0$) is absent. The Zeeman splitting in a transversal homogeneous magnetic field does not affect the current in an isolated ring. However, in the case of a ring coupled to a reservoir such a splitting may lead to a change of the number of spin excitations in a ring that results in a change of a current magnitude and/or of a current period. Moreover, the persistent current depends on the number of spin excitations modulo 4.

II. THE MODEL AND MAIN EQUATIONS

Let us consider a one-dimensional impurity-free ring of

length L in an homogeneous transversal magnetic field H which produces the magnetic flux $\Phi = HL^2/(4\pi)$ through the ring opening. We assume, that the inelastic mean free path $L_\varphi(T)$ and the spin-flip length $L_s(T)$ are large compared to the ring circumference L. In such a case the motion of electrons in the ring is ballistic. The part of the one-electron Hamiltonian $\Delta\hat{H}_s$ including the Zeeman interaction in the magnetic field (which is applied along the axis z of a ring) and the spin-orbit interaction (in the particular case when it is proportional to the spin projection on z) is

$$\Delta\hat{H}_s = \sigma_z \left(\hbar\beta(\hat{p} - eA)_\varphi - g\mu_B H\right), \qquad (2)$$

where σ_z is the Pauli matrix; β is the spin-orbit splitting coefficient; $\hat{p}_\varphi$ is the angular momentum operator; $A_\varphi = \Phi/L$ is the tangential component of the vector potential; $\mu_B = e\hbar/(2m_0)$ is the Bohr magneton. Note, that such a form of SO interaction may be caused by the Aharonov-Casher effect in a cylindrically symmetric electric field lying in the plain of a ring [35]. In particular, such an electric field may be due to the potential which confines the electrons to the ring (e.g., for rings formed in a semiconductor heterostructure) if the crystallographic orientation is appropriately chosen [36].

However, the Eq.(2) may be used as an effective (phenomenological) Hamiltonian for calculating a charge current when the spectrum of free electrons in the ring of a radius $R = L/(2\pi)$ has a form

$$\epsilon_{n\sigma} = \frac{\hbar^2}{2m^* R^2}(n + \varphi_{AB} + \sigma\varphi_{SO})^2 - \sigma g\mu_B H, \qquad (3)$$

where m^* is an electron effective mass; n is an integer; $\varphi_{AB} = \Phi/\Phi_0$ mod1; $\varphi_{SO} = m^*\beta R$ mod1; $\sigma = \pm 1$. In particular, the spectrum (3) takes place for the adiabatic motion of electrons in an inhomogeneous magnetic field (a texture) [33].

Let us estimate the quantity φ_{SO}. For example, for the mesoscopic InAs ring with $R \sim 1\mu m$ and with a width of $15nm$ ($m^* = 0.02m_0; \hbar^2\beta = 3.5\times10^{-10} eVcm$) [37] we get $\varphi_{SO} \sim 0.9$. Thus, the SO interaction can considerably change the effective magnetic flux $\Phi_{eff} = \Phi + \sigma\varphi_{SO}\Phi_0$ piercing the ring.

We will consider the ring with a large number N_e of electrons. In this case we can linearize the electron spectrum near the Fermi points and will describe the correlated electrons as a Luttinger liquid [38]. For electrons with spin the Lagrangian L_{LL} in a bosonic form is [39, 40]

$$L_{LL} = \frac{\hbar v_\rho}{2g_\rho}\left\{\frac{1}{v_\rho^2}\left(\frac{\partial\theta_\rho}{\partial t}\right)^2 - \left(\frac{\partial\theta_\rho}{\partial x}\right)^2\right\} + \frac{\hbar v_\sigma}{2g_\sigma}\left\{\frac{1}{v_\sigma^2}\left(\frac{\partial\theta_\sigma}{\partial t}\right)^2 - \left(\frac{\partial\theta_\sigma}{\partial x}\right)^2\right\}, \qquad (4)$$

where x is the coordinate along the ring; v_i, g_i $(i = \rho, \sigma)$ are Haldane's parameters. The subscripts ρ, σ denote quantities describing the charge and spin degrees of freedom, respectively. For noninteracting electrons we have $g_{\rho(\sigma)} = 2$, $v_{\rho(\sigma)} = v_F$ (where v_F is the Fermi velocity) [39, 40].

The Lagrangian corresponding to the Hamiltonian (2) is

$$L_s = \frac{2\hbar}{L}\pi^{1/2}\varphi_{SO}\frac{\partial\theta_\sigma}{\partial t} + \frac{g\mu_B H}{L}N_\sigma, \qquad (5)$$

where $N_\sigma = \frac{1}{\pi^{1/2}}\int_0^L dx\nabla\theta_\sigma + N_{0\sigma}$ is the number of spin excitations in a ring; $N_{0\sigma} = N_{0\uparrow} - N_{0\downarrow}$ is the ground state number of spin excitations.

The Aharonov-Bohm interaction of electrons with the magnetic flux Φ with respect to the parity effect is described as follows [18, 21]

$$L_{AB} = \frac{2\hbar}{L}\pi^{1/2}\left\{\left[\frac{k_{j\rho}}{4} + \varphi_{AB}\right]\frac{\partial\theta_\rho}{\partial t} + \frac{k_{j\sigma}}{4}\frac{\partial\theta_\sigma}{\partial t}\right\}, \qquad (6)$$

where $k_{j\rho} = k_{j\uparrow} + k_{j\downarrow}$; $k_{j\sigma} = k_{j\uparrow} - k_{j\downarrow}$; $k_{j\uparrow}, k_{j\downarrow}$ are the topological numbers which subject to the parity dependent constraints [18, 38]: $k_{js} = 0$ (1), if N_{es} is odd (even), where s=$\uparrow,\downarrow$.

We consider both the case of an isolated ring ($N_0 = const$) and the case of a ring weakly coupled to an electron reservoir (which fixes the chemical potential $\mu = const$ of electrons in the ring). In the latter case the transfer of charge (and of spin) between a ring and a reservoir is allowed. Therefore, we must take into account a charging energy $E_c = e^2/(2C)$ (associated with the transfer of an elementary charge between a ring and a reservoir) which is due to a geometrical capacitance C of a ring (see Fig. 1). At low temperatures ($T \leq 1K$ at $C \leq 10^{-15}F$) the charging energy

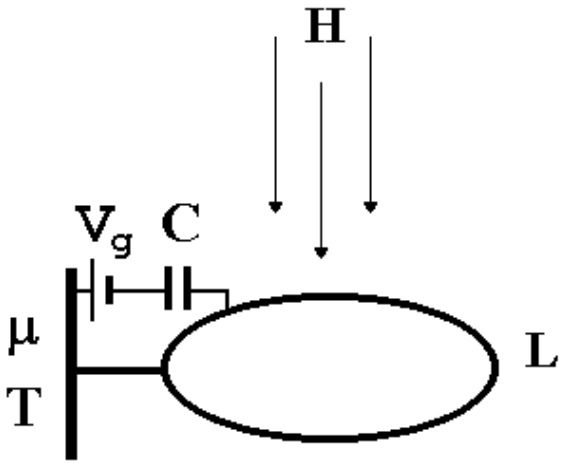

FIG. 1: One-dimensional ring of length L in a homogeneous transversal magnetic field H weakly coupled to an electron reservoir with the chemical potential μ and the temperature T. C and V_g are the geometrical capacitance and the potential difference between a ring and a reservoir, respectively.

E_c strongly suppresses the charge transfer (the Coulomb blockade regime [41]) that considerably affects the persistent current [25, 26, 42, 43]. We assume that the chemical potential μ does not depend on spin. In such a case the Lagrangian describing the particle exchange with a reservoir is

$$L_{ex} = \frac{\mu}{\pi^{1/2}}\frac{\partial\theta_\rho}{\partial x} - \frac{E_c}{L}\left(N_e - N(V_g)\right)^2. \quad (7)$$

Here $N_e = \frac{1}{\pi^{1/2}}\int_0^L dx\nabla\theta_\rho + N_0$ is the number of electrons in a ring; $N(V_g) = C(V_g - V_{g0})/e$, where V_g is the potential difference between a ring and a reservoir (see Fig.1). We assume that at $T = 0$ and $V_g = 0$ the ring is electrically neutral.

The partition function Z may be presented in the form of a path integral over the fields θ_ρ and θ_σ

$$Z = \int D\theta_\rho D\theta_\sigma \exp(-S_E/\hbar) \quad (8)$$

with the Euclidean action

$$S_E = -\int_0^L dx \int_0^{\hbar/T} d\tau L(x,\tau), \quad (9)$$

where $\tau = it$ is an imaginary time. The Lagrangian $L(x,\tau)$ is a sum of parts (4)-(6) for the isolated ring or a sum of parts (4)-(7) for the ring coupled to a reservoir.

The fields θ_ρ and θ_σ obey twisted boundary conditions [18]

$$\begin{aligned}\theta_\rho(x + k_1 L, \tau + k_2\frac{\hbar}{T}) = \theta_\rho(x,\tau)\\ +k_1\pi^{1/2}(2m_\rho + k_{M_\rho}) + k_2\pi^{1/2}n_\rho,\end{aligned} \quad (10)$$

$$\begin{aligned}\theta_\sigma(x+k_1 L, \tau+k_2\frac{\hbar}{T}) = \theta_\sigma(x,\tau)\\ +k_1\pi^{1/2}(2m_\sigma+k_{M_\sigma})+k_2\pi^{1/2}n_\sigma,\end{aligned}$$

where k_1,k_2,$n_{\rho(\sigma)}$,$m_{\rho(\sigma)}$ are integers; $k_{M_\rho} = k_{M_\uparrow} + k_{M_\downarrow}$, $k_{M_\sigma} = k_{M_\uparrow} - k_{M_\downarrow}$, where $k_{M_s}(s =\uparrow,\downarrow)$ are topological numbers characterizing the parity of additional numbers (over the ground state numbers) of electrons with spin s. Note, that the numbers k_{js} and k_{Ms} depend on the parity of the number of electrons with spin s N_{0s} in the ground state [18, 21]: $k_{js} = k_{Ms}$ if N_{0s} is odd; $k_{js} = (k_{Ms} + 1)mod1$ if N_{0s} is even. For the case of isolated ring we have $m_{\rho(\sigma)}$=$k_{M_{\rho(\sigma)}}$=0.

The considered Lagrangian is quadratic in fields $\theta_{\rho(\sigma)}$, therefore the extremal trajectories obeying the boundary conditions (10) and determining the flux-dependent part of the free energy are linear functions of both x and τ

$$\theta_{\rho(\sigma)}(x,\tau) = \pi^{1/2}\left((2m_{\rho(\sigma)} + k_{M_{\rho(\sigma)}})\frac{x}{L} + n_{\rho(\sigma)}\frac{\tau}{\hbar/T}\right). \quad (11)$$

Calculating the Euclidean action S_E for the trajectories (11) and performing the summation over $n_{\rho(\sigma)}$, $m_{\rho(\sigma)}$ and the topological numbers $k_{j\rho\sigma}$, $k_{M_{\rho(\sigma)}}$ with respect to the above parity-dependent constraints we can calculate the partition function and the free energy $F = -T\ln(Z)$.

III. THE FIXED NUMBER OF ELECTRONS

In this case the free energy is

$$\begin{aligned}\Delta F(\Phi) = -g\mu_B H N_{0\sigma}\\ -T\ln\left\{\theta_3\left(2\varphi_{AB} + k_{j\rho}/2, q_\rho^4\right)\times\theta_3\left(2\varphi_{SO} + k_{j\sigma}/2, q_\sigma^4\right)\right.\\ \left.+\theta_2\left(2\varphi_{AB} + k_{j\rho}/2, q_\rho^4\right)\times\theta_2\left(2\varphi_{SO} + k_{j\sigma}/2, q_\sigma^4\right)\right\}\end{aligned} \quad (12)$$

Here $\theta_{2(3)}(v,q)$ are the Jacobi theta functions [44]; $q_{\rho(\sigma)}$=$\exp(-T/2T^*_{\rho(\sigma)})$, where $T^*_{\rho(\sigma)} = \hbar v_{\rho(\sigma)}g_{\rho(\sigma)}/(\pi L)$. For noninteracting electrons we get $T^*_{\rho(\sigma)} = T^*$, where $T^* = \Delta_F/(\pi^2)$ is the crossover temperature for the persistent current in an isolated ring [18], and $\Delta_F = hv_F/L$ is the level spacing near the Fermi points at $\Phi = 0$.

It is seen from Eq.(12) that in the common case the Zeeman splitting (the first term) breaks down the exact periodic dependence of the free energy on the magnetic flux. However, calculating the persistent current in a ring with a large number of electrons ($N_0 \gg 1$) and without spin polarization ($N_{0\sigma} \sim 1$) we can neglect this term, because, the current $I_=$ due to this term is less than the current $I_0 \sim ev_F/L$ due to the AB effect: $I_=/I_0 \sim \frac{m^*}{m_0}gN_{0\sigma}\frac{\lambda_F}{L} \ll 1$, where λ_F is the Fermi wavelength of an electron. Note, that we neglect the spin-flip processes in the ring therefore the number $N_{0\sigma}$ does not depend on the magnetic flux.

Neglecting the first term and accounting the periodicity of the theta functions we conclude that in the common case the free energy (12) is periodic in φ_{SO} with a period of 1 and in the AB magnetic flux with a period of Φ_0. However, at $\varphi_{SO} = 0, \pm\frac{1}{2}$ the period of AB oscillations depends on the parity of the number of electrons even for noninteracting electrons [17]. At the same time, at $\varphi_{SO} = \pm\frac{1}{4}$ the period of AB oscillations is $\Phi_0/2$ [21, 28] for the even number of electrons ($N_0 = 4n$, $k_{j\rho} = 2$,$k_{j\sigma} = 0$; $N_0 = 4n + 2$, $k_{j\rho(\sigma)} = 0$; where n is an integer) and it is Φ_0 for the odd number of electrons ($N_{0\uparrow} = 2n + 1$,$N_{0\downarrow} = 2m$, $k_{j\rho} = 1$,$k_{j\sigma} = -1$; $N_{0\uparrow} = 2n$,$N_{0\downarrow} = 2m + 1$, $k_{j\rho(\sigma)} = 1$). By analogy, at $\varphi_{AB} = 0, \pm\frac{1}{2}$ the period of the dependence of the free energy on φ_{SO} is 1 for the even number of electrons and it is 1/2 for the odd number of particles. Besides, at $\varphi_{AB} = \pm\frac{1}{4}$ the considered period is 1/2 for the even number of electrons and it is 1 for the odd number of electrons.

In rings with an odd number of electrons the spin-orbit interaction produces the persistent current I even without an AB magnetic flux. The direct calculation gives

$$I(\Phi = 0) = \mp\frac{T}{\Phi_0}F(1/4, T/(2T^*_\rho))\frac{\theta_2(2\varphi_{SO} + \frac{1}{2}, q_\sigma^4)}{\theta_3(2\varphi_{SO} + \frac{1}{2}, q_\sigma^4)}, \quad (13)$$

where $F(v,p)$=$2\pi\sum_{m=1}^{\infty}(-1)^m\sin(2\pi mv)/\sinh(mp)$; the upper (lower) sign is applied to the case when $N_{0\uparrow}$ is odd (even) and $N_{0\downarrow}$ is even (odd). At $T \to 0$ the

current amplitude is $I_0 = \pi^2 T_\rho^*/\Phi_0$ and in the ballistic case ($g_\rho v_\rho = 2v_F$) it does not depend on the strength of an interelectron interaction [4]. At $T \gg T^*_{\rho(\sigma)}$ we have $I = \mp I_0 \frac{8T}{\pi T_\rho^*} \exp\left(-\frac{T}{2T_\rho^*} - \frac{T}{2T_\sigma^*}\right) \sin(2\pi\varphi_{SO})$. The dependence $I(\varphi_{SO})$ is depicted in Fig. 2.

IV. THE FIXED CHEMICAL POTENTIAL

We assume that the chemical potential μ of a reservoir does not depend on an electron spin. In this case at $H = 0$, $\delta_c = 0$ and $\varphi_{SO} = 0$ the number of electrons in a ring is even ($N_{0\uparrow} = N_{0\downarrow}$) and the number of spin excitation $N_{0\sigma} = N_{0\uparrow} - N_{0\downarrow}$ is equal to zero. Further we consider the case $N_0 = 4n+2$. Note, that the results for $N_0 = 4n$ can be deduced from that for $N_0 = 4n+2$ by changing $\varphi_{AB} \to \varphi_{AB} + \frac{1}{2}$. In this case the flux-dependent part of the thermodynamic potential is $\Delta\Omega(\Phi) = -T\ln(Z_\mu)$ with

$$Z_\mu = \sum_{i=1}^{4} \theta_i(2\varphi_{AB}, q_\rho^4)\theta_i(2\varphi_{SO}, q_\sigma^4)\theta_i(2\delta_c, q_c^4)\theta_i(2\delta_z, q_{0\sigma}^4), \tag{14}$$

where $\theta_i(v,q)$ are the Jacobi theta functions [44]; $q_{c(0\sigma)}$=$\exp\left(-\frac{\pi^2 T}{8T_{c(0\sigma)}}\right)$; T_c=$T_{0\rho}$+$2E_c$; $T_{0\rho(\sigma)} = \frac{\pi\hbar v_{\rho(\sigma)}}{g_{\rho(\sigma)} L}$; $\delta_c = \frac{eV_g+\mu}{4T_c} mod 1$; $\delta_z = \frac{g\mu_B H}{4T_{0\sigma}} mod 1$.

The magnetic field $H = 4\pi\Phi/L^2$ incorporates in $\Delta\Omega$ in a twofold way. Firstly, it is through the parameter φ_{AB} that causes conventional AB oscillations (which are periodic in Φ with a period of Φ_0). Secondly, it is through the parameter δ_z that also causes oscillations of the thermodynamic potential. The latter oscillations are physically due to as follows. With increasing magnetic field the Zeeman splitting leads to a monotonous shift of electron levels in a ring relatively to the chemical potential μ of an electron reservoir. This leads to an increase of a number of electrons with spin "up" in a ring and to a decrease of a number of electrons with spin "down". Thus, the spin excitations appear in the ring. So, if the magnetic field is distinct from zero the effect of a reservoir leads to an effective spin-flip process in a ring (it is worth emphasizing that, we neglect direct spin-flip processes within the ring $L_s(T) \gg L$) when the electron with spin "up" passes from the reservoir to the ring and the electron with spin "down" passes back. This process results in new oscillations of the thermodynamic quantities as a function of the magnetic flux with a period which is different from the period of AB oscillations. Because the number $N_0 \equiv N_{0\rho}$ of electrons in a ring does not change these oscillations is not affected by the charging energy (i.e., by the small capacitance C) in contrast with the oscillations of the thermodynamic potential as a function of the chemical potential μ (or of the gate voltage V_g).

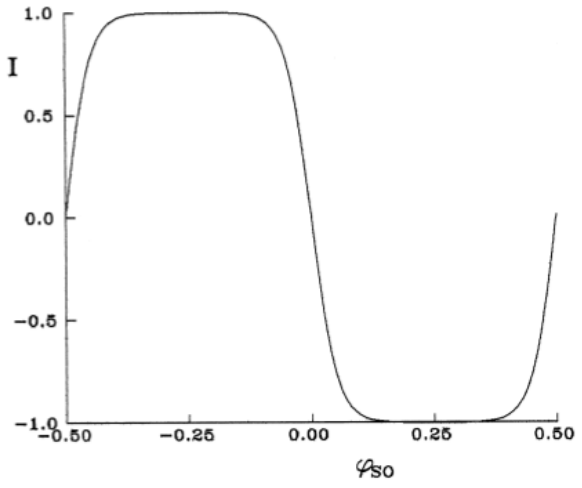

FIG. 2: The dependence of a persistent current I in units of ev_F/L on the strength of a spin-orbit interaction (the parameter φ_{SO}) for an isolated ring with an odd number of noninteracting electrons. The parameters are: $T/\Delta_F = 0.05$; $\varphi_{AB} = 0$; $N_{0\uparrow}$ is odd; $N_{0\downarrow}$ is even.

Now we calculate the period of such oscillations. From the properties of elliptic θ-functions it follows that the considered oscillations have a period (in a magnetic field) as follows

$$\Delta H = \frac{4T_{0\sigma}}{g\mu_B}. \tag{15}$$

The quantity $T_{0\sigma}$ defines the energy necessary for the increase of the number of spin excitations by 1. For noninteracting electrons ($g_{\rho(\sigma)} = 2$; $v_{\rho(\sigma)} = v_F$) we have $T_{0\sigma}$=$\Delta_F/4$. Thus, with respect to chiral and spin degeneration the period of oscillations (15) corresponds to a change of the number of spin excitations by 4. Let us estimate ΔH for the one-dimensional mesoscopic ring formed in a semiconductor heterostructure. Taking $L \sim 10\mu m$, $v_F \sim 2\times 10^5 ms^{-1}$, and $g \sim 20$ we get $\Delta H \sim 700G$ for noninteracting electrons. Note, that the period of AB oscillations is $\Delta H_{AB} \sim 5G$ at these parameters.

Further we calculate the persistent current $I = -\partial\Omega/\partial\Phi$. When we differentiate we consider the parameter δ_z as a constant, because the corresponding period ΔH (see Eq.(15)) is large in comparison with the period of AB oscillations ΔH_{AB}. Thus, the Zeeman effect does not cancel out the AB oscillations, but it leads to a periodic change of an amplitude of such oscillations with increasing magnetic field.

In the common case the dependence of a persistent current amplitude on the magnetic field is complicated and depends on both the spin-orbit interaction and V_g. Making the effect of Zeeman splitting on the persistent current more transparent we consider the case of $\varphi_{SO} = 0$. In this case we write down the expression for the quantity $I_1 \equiv I(\varphi_{AB} = \frac{1}{4})$ which is a sum of amplitudes of all the odd harmonics (note, that at $T \gg T_\rho^*$ I_1 is an

amplitude of the first harmonic of a current)

$$I_1 = \frac{T}{\Phi_0}\frac{F\left(\frac{1}{4},\frac{T}{2T_\rho^*}\right)}{Z_\mu(\varphi_{AB}=\frac{1}{4})}\theta_3\left(\frac{1}{2},q_\rho^4\right)\theta_2\left(0,q_\sigma^4\right) \times\theta_2\left(2\delta_c,q_c^4\right)\theta_2\left(2\delta_z,q_{0\sigma}^4\right), \quad (16)$$

where the function $F(v,p)$ was defined after Eq.(13), and Z_μ was defined in Eq.(14).

As it follows from above expression, at $\delta_z = \pm 1/4$ the quantity I_1 vanishes and the period of AB oscillations halves. This is due to a change of the number of spin excitations by 1. In this case the numbers of electrons with spin "up" and spin "down" have a different parity, that results in a period halving [4, 17, 26]. At the same time, at $\delta_z = 1/2$ the sign of I_1 changes by an opposite one, that is due to a change of the number of spin excitations in a ring by 2. The dependence of I_1 on the magnetic field (the parameter δ_z) is depicted in Fig. 3 (the curve 1). The curve 2 is a dependence of a sum of amplitudes of all the even harmonics $I_2 = \frac{1}{2}\left\{I(\varphi_{AB}=\frac{1}{8}) - I(\varphi_{AB}=\frac{3}{8})\right\}$ on the magnetic field. Note, that with increasing temperature $T \gg T_{0\sigma}$, when the discreteness of the spectrum of a spin subsystem is irrelevant, the considered oscillations vanish.

Now we shortly consider the case of $\varphi_{SO} \neq 0$. As it follows from Eq.(14), if any two of the three parameters $p_i \in \{\varphi_{SO}, \delta_c, \delta_z\}$ (where $i = 1,2,3$) are equal to $p_1 = \frac{n}{2}$; $p_2 = \frac{1}{4} + \frac{n}{2}$ (where n is an integer) then the period of AB oscillations halves (and it is equal to $\Phi_0/2$). In addition, at some values of the third parameter p_3 the period of oscillations reduces to four times (and it is equal to $\Phi_0/4$).

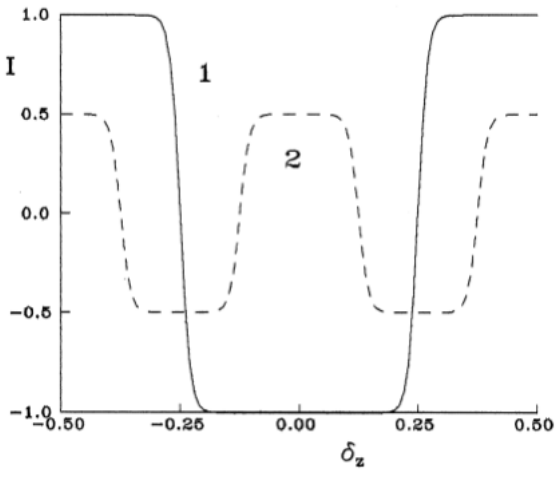

FIG. 3: The dependences of both the first harmonic amplitude I_1 (1) and the second harmonic amplitude I_2 (2) of a persistent current in units of ev_F/L on the magnetic field (the parameter δ_z) for the ring with noninteracting electrons coupled to a reservoir. The parameters are: $T/\Delta_F = 0.01$; $E_c = 0$; $\varphi_{SO} = 0$; $\delta_c = 0$.

For noninteracting electrons ($g_{\rho(\sigma)} = 2$; $v_{\rho(\sigma)} = v_F$; $E_c = 0$) this occurs at $p_3 = \frac{1}{8} + \frac{n}{4}$. The persistent current in this case is

$$I(\Phi) = \frac{4T}{\Phi_0}F\left(4\varphi_{AB}, 8T/T_\rho^*\right) \quad (17)$$

Note, that for correlated electrons the necessary value of p_3 depends on both the temperature and the strength of an interelectron interaction. For example, at $T \ll T_{0\sigma}$ and $p_1 \equiv \varphi_{SO} = 0$, $p_2 \equiv \delta_c = \frac{1}{4}$ we have $p_3 \equiv \delta_z = \pm\left(\frac{1}{8} + \frac{T}{4T_{0\sigma}}\ln\left(\frac{\theta_3(0,q_\sigma^4)\theta_3(\frac{1}{2},q_c^4)}{\theta_3(\frac{1}{2},q_\sigma^4)\theta_3(0,q_c^4)}\right)\right) + \frac{n}{2}$, where n is an integer.

V. CONCLUSION

In the present paper the effect of both the spin-orbit interaction and the Zeeman interaction on the persistent current in a one-dimensional ballistic ring with correlated electrons has considered at nonzero temperatures. An isolated ring with a fixed number of electrons (the canonical case) as well as a ring coupled to an electron reservoir (the grand canonical case) are considered. In the last case the effect of a charging energy due to a small geometrical capacitance between a ring and a reservoir is taken into account. Interelectron interactions in a ring are described within the framework of the Luttinger liquid model.

It is shown that in the canonical case the Zeeman effect in a homogeneous transversal magnetic field does not affect the persistent current. At the same time, the spin-orbit interaction is important. In particular, such an interaction produces the persistent current without an Aharonov-Bohm magnetic flux in rings with an odd number of particles. This current, in fact, is a quantum mechanical current of an additional electron ($N_{0\uparrow} \neq N_{0\downarrow}$) in an effective magnetic flux [28]. In rings with an even number of spinfull electrons ($N_{0\uparrow} = N_{0\downarrow}$) the persistent current without an AB magnetic flux vanishes, because, the spin-orbit interaction does not remove the degeneracy between states which correspond to a clockwise movement of an electron with spin "up" and a counterclockwise movement of an electron with spin "down" which carry opposite currents. At $\Phi \neq 0$ such a degeneracy is removed and the persistent current appears. Note, that in an inhomogeneous magnetic field the Zeeman interaction causes an effective spin-orbit coupling [33] and it affects the persistent current in the canonical case.

For the grand canonical case (the ring coupled to a reservoir) the Zeeman splitting may lead to a change of the number of spin excitations in a ring that affects considerably the persistent current. This effect does not cancel out the Aharonov-Bohm oscillations but it leads to a periodic change of a current amplitude with the period which is large compared to Φ_0. As a result the phase of AB oscillations may change by π or the period of oscillations may halve. Moreover,

at definite values of parameters (i.e., the spin-orbit coupling; the difference of potential between a ring and a reservoir; and the magnetic field) the period of AB oscillations may be reduced to four times. For noninteracting electrons such a set of parameters does not depend on the temperature. Whereas for correlated electrons it is not true and the change of a period of AB oscillations with the temperature may be observed.

[1] Y.Aharonov, D.Bohm, Phys.Rev. **115** (1959) 484. 1

[2] Y.Imry, In: Directions in Condensed Matter Physics, G.Grinstein and G.Mazenco (Eds.), World Scientific, Singapore, 1986. 1

[3] B.Kramer (Ed.), Quantum Coherence in Mesoscopic System, NATO Advanced Study Institute, Series B: Physics, vol.254, Plenum, New York, 1991. 1

[4] ..Zvyagin, I.V.Krive, Fiz.Nizk.Temp.**21** (1995) 687. 1, 4, 5

[5] S.Washburn, R.A.Webb, Adv.Phys. **35** (1986) 375. 1

[6] N.Byers, and C.N.Yang, Phys. Rev. Lett.**7** (1961) 46. 1

[7] F.Bloch, Phys.Rev.B **2** (1970) 109. 1

[8] I.O.Kulik, JETP Lett.**11** (1970) 275. 1

[9] M.Büttiker, Y.Imry, R.Landauer, Phys. Lett.**96A** (1983) 365. 1

[10] L.P.Levy, G.Dolan, J.Dunsmuir, and H.Bouchiat, Phys. Rev. Lett.**64** (1990) 2074; V.Chandrasekhar, R.A.Webb, M.J.Brady, M.B.Ketchen, W.J.Gallagher and A.Kleinsasser, Phys. Rev. Lett. **67** (1991) 3578; D.Mailly,C.Chapelier,A.Benoit,Phys.Rev.Lett.**70** (1993) 2020; B.Reulet, M.Ramin, H.Bouchiat, D.Mailly, Phys. Rev. Lett. **75** (1995) 124. 1

[11] R.Landauer, M.Büttiker, Phys.Rev.Lett. **54** (1985) 2049; M.Büttiker, Phys.Rev.B **32** (1985) 1846; N.Trivedi, D.A.Browne, Phys.Rev.B **38** (1988) 9581; H.F.Cheung, E.K.Riedel, Y.Gefen, Phys.Rev.Lett. **62** (1989) 587; O.Entin-Wohlman, Y.Gefen, Europhys.Lett. **8** (1989) 477; E.K.Riedel,H.F.Cheung,Y.Gefen,Physica Scripta T **25** (1989) 357; V.Ambegaokar, U.Eckern, Phys.Rev.Lett. **65** (1990) 381; G.Montambaux, H.Bouchiat, D.Sigeti, R.Friesner, Phys.Rev.B **42** (1990) 7647; E.Akkermans, Europhys.Lett. **15** (1991) 709; B.L.Altshuler, Y.Gefen, Y.Imry, Phys.Rev.Lett. **66** (1991) 88; F.von Oppen, E.K.Riedel, Phys.Rev.Lett. **66** (1991) 84; A.Schmid, Phys.Rev.Lett. **66** (1991) 80; S.Oh, A.Yu.Zyuzin, R.A.Serota, Phys.Rev.B **44** (1991) 8858; T.Chakraborty, Comments Condens. Matter Phys. **16** (1992) 35; M.Abraham, and R.Berkovits, Phys.Rev.Lett. **70** (1993) 1509; N.Argaman and Y.Imry, Physica Scripta T **49** (1993) 333; A.Kamenev, Y.Gefen, Phys.Rev.Lett. **70** (1993) 1976; P.Kopietz, Phys.Rev.Lett. **70** (1993) 3123; D.Loss and Th.Martin, Phys.Rev.B **47** (1993) 4619; P.A.Mello, Phys.Rev.B **47** (1993) 16358; A.Müller-Groeling,H.A.Weidenmüller,C.H.Lewenkopf, Europhys. Lett. **22** (1993) 193; G.Bouzerar,D.Poilblanc,G.Montambaux,Phys.Rev.B **49**, 8258 (1994); H.Kato, D.Yoshioka, Phys.Rev.B **50** (1994) 4943; A.Müller-Groeling,H.A.Weidenmüller,Phys.Rev.B **49** (1994) 4752; P.Singha Deo, A.M.Jayannavar, Phys.Rev.B **49** (1994) 13685; G.Vignale, Phys.Rev.Lett. **72** (1994) 433; R.Berkovits, Y.Avishai, Europhys. Lett. **29** (1995) 475; T.Giamarchi, B.S.Shastry, Phys. Rev. B **51** (1995) 10915; T.Chakraborty, P.Pietilainen, Phys.Rev.B **52** (1995) 1932; M.Kamal, Z.H.Musslimani, A.Auerbach, J.Physique **15** (1995) 1487; I.V.Krive, P.Sandström, R.I.Shekhter, S.M.Girvin, M.Jonson, Phys.Rev.B **52** (1995) 16451; T.P.Pareek, P.S.Deo, A.M.Jayannavar, Phys.Rev.B **52** (1995) 14657; P.Singha Deo, Phys.Rev.B **51** (1995) 5441; K.Niemela, P.Pietiläinen, P.Hyvönen, T.Chakraborty, Europhys.Lett. **36** (1996) 533; A.S. Rozhavsky, J.Phys.:Condens.Matter **9** (1997) 1521. 1

[12] H.F.Cheung, Y.Gefen, E.K.Riedel, and W.H.Shih, Phys.Rev.B **37** (1988) 6050. 1

[13] H.Bouchiat, G.Montambaux, J.Phys.(Paris) **50** (1989) 2695.

[14] D.Haldane, Phys.Rev.Lett. **67** (1991) 937.

[15] A.J.Leggett, in: D.K.Ferry, J.R.Barker, C.Jacoboni (Eds.), Granular Nanoelectronics, NATO ASI Ser.B, vol.251, Plenum, New York, 1991, p.297.

[16] F.V.Kusmartsev, JETP Lett. **53** (1991) 28; Phys.Lett.A **161** (1992) 433.

[17] D.Loss, P.Goldbart, Phys.Rev.B **43** (1991) 13762. 1, 3, 5

[18] D.Loss, Phys.Rev.Lett.**69** (1992) 343. 1, 2, 3

[19] R.A.Smith, V.Ambegaokar, Europhys.Lett. **20** (1992) 161.

[20] N.Yu, M.Fowler, Phys.Rev.B **45** (1992) 11795. 1

[21] S.Fujimoto, N.Kawakami, Phys.Rev.B **48** (1993) 17406. 2, 3

[22] J.Weisz, R.Kishore, F.V.Kusmartsev, Phys.Rev.B **49** (1994) 8126.

[23] F.V.Kusmartsev, J.F.Weisz, R.Kishore, M.Takahashi, Phys.Rev.B **49** (1994) 16234. 1

[24] A.O.Gogolin, N.V.Prokofiev, Phys.Rev.B **50** (1994) 4921. 1

[25] M.V.Moskalets, Eur.Phys.J.B **7** (1999) 645. 1, 3

[26] M.V.Moskalets, Physica E **4** (1999) 17. 1, 3, 5

[27] F.V.Kusmartsev, J.Phys.Condens.Matter **3** (1991) 3199. 1

[28] Y.Meir, Y.Gefen, O.Entin-Wohlman, Phys.Rev.Lett. **63** (1989) 798; O.Entin-Wohlman, Y.Gefen, Y.Meir, Y.Oreg, Phys.Rev.B **45** (1992) 11890. 1, 3, 5

[29] H.Mathur, A.D.Stone, Phys.Rev.Lett. **68** (1992) 2964. 1

[30] Y.Aharonov, A.Casher, Phys.Rev.Lett. **53** (1984) 319. 1

[31] A.V.Balatsky, B.L.Altshuler, Phys.Rev.Lett. **70** (1993) 1678. 1

[32] M.Y.Choi, Phys.Rev.Lett. **71** (1993) 2987. 1

[33] D.Loss, P.Goldbart, A.V.Balatsky, Phys.Rev.Lett. **65** (1990) 1655; D.Loss, P.M.Goldbart, Phys.Rev.B **45** (1992) 13544. 1, 2, 5

[34] M.V.Berry, Proc.R.Soc.London, Ser.A **392** (1984) 45; Geometric Phases in Physics, ed. by A.Shapere and F.Wilczek (World Scientific, Singapore, 1989). 1

[35] Ya-Sha Yi, A.R.Bishop, Phys.Rev.B **58** (1998) 4077. 2

[36] E.L.Ivchenko, Y.B.Lyanda-Geller, G.E.Pikus, Sov.Phys.JETP **71** (1990) 550; A.G.Aronov, Y.B.Lyanda-Geller, G.E.Pikus, Sov.Phys.JETP

73 (1991) 537; A.G.Aronov, Y.B.Lyanda-Geller, Phys.Rev.Lett. **70** (1993) 343; Yuli Lyanda-Geller, Phys.Rev.Lett. **71** (1993) 657. 2

[37] G.E.Pikus, V.A.Marushak, A.N.Titkov, Sov.Phys.Semicond. **22** (1988) 185. 2

[38] F.D.M. Haldane, J.Phys.C **14** (1981) 2585. 2

[39] H.J.Schulz, Int.J.Mod.Phys.B **5** (1991) 57. 2

[40] C.L.Kane, M.P.A.Fisher, Phys.Rev.B **46** (1992) 15233. 2

[41] D.V.Averin, K.K. Likharev, in Mesoscopic Phenomena in Solids, B.Altshuler, P.A.Lee, and R.A.Webb (Eds.), Elsevier, Amsterdam, 1991; Single Charge Tunneling, H.Grabert and M.H.Devoret (Eds.), Plenum, New York, 1992; M.A.Kastner, Rev.Mod.Phys.**64** (1992) 849. 3

[42] M.Büttiker, Physica Scripta T **54** (1994) 104; M.Büttiker, C.A.Stafford, Phys.Rev.Lett. **76** (1996) 495; M.Büttiker, C.A.Stafford, in Correlated Fermions and Transport in Mesoscopic systems, T.Martin, G.Montambaux, J.Tran Thanh Van (Eds.), Editions Frontieres, Gif-sur-Yvette (1996) 491; P.Cedraschi, M.Büttiker, J.Phys.:Condens.Matter **10** (1998) 3985. 3

[43] M.V.Moskalets, Physica B **252** (1998) 244. 3

[44] H.Bateman, Higher transcendental functions. Ed. by A.Erdélyi, New York Toronto London, Mc Graw-Hill Book Company, inc, 1953). 3, 4