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BUILDING A SINGLE-NODE CONVEYOR LINE MODEL WITH A CONSTANT SPEED

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Аннотация. *This paper focuses on the model of the production line of the conveyor type. The system of equations to model of the conveyor line at a constant speed of the moving of the subjects of labor was written in the one-moment description and was receive its solution in analytic form that allowed to calculate the parameters of the system at any given time. It is shown that the solution is determined of the kind of initial and boundary conditions. Here was received an expression for calculation of the production cycle of the conveyor line*.

Ключевые слова: production line, PDE model, the subject of labor, control system, technological process, semiconductor production, conveyor model.

1. General statement of the problem and its relevance

Behavior of the line parameters largely determined by the fact of interaction between the subjects of labor during processing [1,2]. Technological cooperation appears at the presence of technological constraints, which define the sequence of process steps and the order of movement of the subjects of labor on technological route (pic.1) [3].

The system of equations that determines the behavior of the production line parameters in the single-step description has the following form [1]

$$\frac{\partial[\chi]_0(t,S)}{\partial t}+\frac{\partial[\chi]_1(t,S)}{\partial S}=0, \qquad (1)$$

$$[\chi]_1(t,S)=[\chi]_{1\psi}(t,S), \qquad (2)$$

at initial

$$[\chi]_0(t_0,S)=\Psi(S), \qquad (3)$$

and the boundary condition

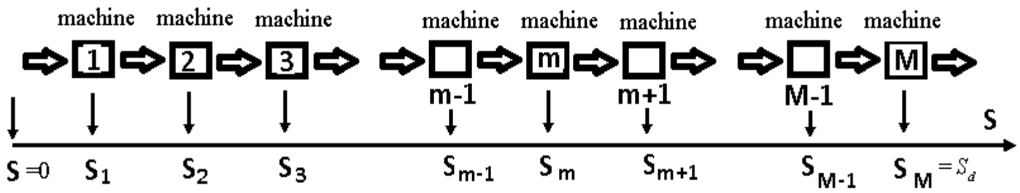
$$[\chi]_0(t,S_0)=\Phi(t), \qquad (4)$$

Where $[\chi]_0(t,S)$ – the density distribution of the subjects of labor in work in progress on the technological positions; $[\chi]_1(t,S)$ – temp of processing of the subjects of labor on technological positions at the time t. The position of subject of labor on the technological route is characterized by the coordinate $S\in[0;S_d]$.

When using the coordinates of the costs space constant S_d corresponds to the cost of production.

A number of studies used dimensionless quantity $\xi\in[0;1]$ [2], as the coordinate that defines the position of the subject, the value of which gives an idea of the degree of processing of the subject of labor: $\xi=0$ corresponds to the initial stage of processing subject of labor; $\xi=1$ It corresponds to a final processing stage of the subject of labor, when the subject of labor turns into a finished product. Selection of variable $\xi\in[0;1]$ is quite diverse. One of the ways to represent this variable can be the ratio of the transferred costs of the resources $S\in[0;S_d]$ for labor to its variable cost S_d : $\xi=\frac{S}{S_d}$. Initial condition (3) determines the number of subjects of labor at the time t_0 at each processing step. The boundary condition (4) determines the number of the subjects of labor for the technological position S_0 at arbitrary time t.

The boundary condition (4) can be set from the assumption that the size of interoperable storage is limited or determined by the terms of supply of raw materials and materials to the appropriate technological operation The boundary condition (4) can also be claimed in the case, if the model of flow line takes into account the shipment of the subjects of labor in the form of semi-finished products from a technological position, characterized by a coordinate S_0, or the presence of factors, which determine the count of the number of defective products. The presence of the initial (3) and the boundary (4) conditions leads to discontinuous nature of the solutions of the system of equations (1),(2), which is typical for the behavior of stream parameters of modern production conveyor lines [4].



Pic.1. Scheme of a single-node flow conveyor line

Solution of the system (1)–(3) for constant speed of subjects of labor on the production line or on it's separate part is presented in the works [1,p.235], [2,p.910], [5,p.97]. However, a detailed analysis of the resulting solution is not given. At the same time, the calculation of state parameters for conveyor lines is an important production task, which requires the use of new methods of solving. Special place occupies the problem of calculating the state parameters of the production system, which is composed of several interconnected conveyor lines, that are arranged as a series or parallel [6]. In the work [2] is presented a numerical calculation of the production line, that consists of three sections $\xi \in [0,0.2]$, $\xi \in [0.2,0.8]$, $\xi \in [0.8,1]$, on each of them the velocity of the subjects of labor g is constant for a particular section and equals accordingly [15;10;15], [2,c.910]. Each of the sections is a conveyor, that provides a constant speed of subjects of labor. Conveyors are located sequentially one after another. In the work [7] given the numerical solution of the system (1)–(4), analysis of the solution for specified initial and boundary conditions is given. The initial condition determines the condition of the subjects of labor at each processing step at time t=0, and boundary condition sets the intensity of the receipt of the subjects of labor for processing, according to the available orders for production. In the work [8] tasked an optimal control of multiple threads, which are connected into a single stream. Despite the fairly large number of publications, dedicated to the design of control systems, production lines with the use of PDE-models [1,2,7,8], the analysis of model of the conveyor line, in our opinion, not enough attention is given to. In the same time, conveyor type production is a common way of organizing the production process. A characteristic feature of this way of organizing production is, that the all subjects of labor located on the conveyor line are moving with the same speed. This fact allows to simplify the system of equations (1)–(4).

2. Statement of the problem and formulation of the purpose of the article

The purpose of this article is to build an analytical solution of the system (1)–(4) in the case of movement of the subjects of labor along the technological route at a constant speed. An important and separate task along with the analysis of the obtained solution is to show the dependence of solutions from initial and boundary conditions, that determine the functioning of the conveyor line. Special attention by solving the system of equations (1)–(4) deserve problems as condition parameters determining a conveyor line for an arbitrary point in time, as well as the calculation of the duration of the production cycle of manufacturing of products [9–11]. The production cycle of manufacturing of products is one of the most important characteristics of the production system.

3. A single-node conveyor line model at a constant speed movement of subjects of labor

Consider a production line consisting of M units of technological equipment, located on technological positions with the coordinate $S = S_m$ (pic.1). Subjects of labor are moving on technological route, consisting of M units of technological positions with a constant speed a. The system of equations for describing the parameters of the production line in a conveyor model has the form

$$\frac{\partial[\chi]_0(t,S)}{\partial t} + \frac{\partial[\chi]_1(t,S)}{\partial S} = 0, \quad (5)$$

$$[\chi]_1(t,S) = a \cdot [\chi]_0(t,S), \quad (6)$$

a – conveyor speed.

For clarity of presentation of research results, we introduce dimensionless variables τ, ξ and the dimensionless density of the subjects of labor $[\theta]_0(\tau,\xi)$ on technological positions. Then it is possible to present:

$$t = t_d \cdot \tau, \qquad S = S_d \cdot \xi,$$
$$[\chi]_0(t,S) = [\theta]_0(\tau,\xi) \cdot \Theta, \ \Theta = \max\{\Psi(S), \Phi(t)\}. \quad (7)$$

$$\Psi(S) = \Theta \cdot \psi(\xi), \ \Phi(t) = \Theta \cdot \vartheta(\tau) \quad (8)$$

By substituting (6) in (5) and, taking into account (7), system of equations (1)–(4) can be written in dimensionless form:

$$\Theta \cdot \frac{\partial[\theta]_0(\tau,\xi)}{\partial \tau} + \Theta \cdot \frac{a \cdot t_d}{S_d} \frac{\partial[\theta]_0(\tau,\xi)}{\partial \xi} = 0, \quad (8)$$

$$[\theta]_0(\tau_0,\xi) = \psi(\xi), \quad (9)$$

$$[\theta]_0(\tau,\xi_0) = \vartheta(\tau), \quad (10)$$

The value of the characteristic time t_d is determined by the operating conditions of production. Its choice determines the value of the constant dimensionless speed of a conveyor line g:

$$g = \frac{a \cdot t_d}{S_d} = \frac{t_d}{T_d}. \quad (11)$$

If the characteristic time t_d corresponds to the duration of the production cycle of manufacturing products $T_d = \frac{S_d}{a}$ (period of time during which the subject of labor is treated, starting with the first technological operation and ending with the last technological operation), then in that case g=1 and the dimensionless time $\tau = \frac{t}{T_d}$ will characterize the ratio of the total time τ to the production cycle time T_d. Taking into account (11) system of equations (8)–(10) takes the form

$$\frac{\partial[\theta]_0(\tau,\xi)}{\partial \tau} + g\frac{\partial[\theta]_0(\tau,\xi)}{\partial \xi} = 0, \quad (12)$$

$$[\theta]_0(\tau_0,\xi) = \psi(\xi), \quad (13)$$

$$[\theta]_0(\tau,\xi_0) = \vartheta(\tau), \quad (14)$$

which corresponds to the system of characteristics:

$$\frac{d\xi}{d\tau} = g, \ \xi(\tau_0) = \xi_0, \quad (15)$$

$$\frac{d[\theta]_0}{d\tau} = 0, \quad [\theta]_0(\xi,\tau_0) = \psi(\xi) \quad (16)$$

Integrating (15), we will get

$$\xi = g\tau + C, \qquad C = \text{const}, \tag{17}$$

where $\xi(\tau)$ – dimensionless coordinate. The function $\xi(\tau)$ can be interpreted as a position of the subject of labor on the conveyor line at time τ, which at time τ_0 has held position ξ_0.

Wherein movement of the conveyor runs at a constant dimensionless speed g (11). The equation (17), that determine the possible trajectories of movement of the subjects of labor in the phase coordinate space [12], is a first integral of the equation (15). Taking into account initial condition $\xi(\tau_0) = \xi_0$, we can write:

$$\xi = \xi_0 + g(\tau - \tau_0), \; C = \xi_0 - g\tau_0 . \tag{18}$$

The general solution of the equation (12) is a function of the first integral (17):

$$[\theta]_0(\tau, \xi) = [\theta]_0(\xi - g\tau) . \tag{19}$$

Due to the fact that:

$$\frac{\partial[\theta]_0(\xi - g\tau)}{\partial\tau} = -g\frac{\partial[\theta]_0(\xi - g\tau)}{\partial(\xi - g\tau)},$$
$$\frac{\partial[\theta]_0(\xi - g\tau)}{\partial\xi} = \frac{\partial[\theta]_0(\xi - g\tau)}{\partial(\xi - g\tau)}, \tag{20}$$

by direct substitution of expressions (20) in equation (12) we obtain the identity

$$-g\frac{\partial[\theta]_0(\xi - g)}{\partial(\xi - g\tau)} + g\frac{\partial[\theta]_0(\xi - g\tau)}{\partial(\xi - g\tau)} = 0 . \tag{21}$$

Taking into account initial condition (13) and the boundary condition (14), solution (19) takes the form

$$[\theta]_0(\tau, \xi) = [\theta]_0(\xi - g\tau) = \psi(r + \xi_0),$$
$$[\theta]_0(\tau, \xi) = [\theta]_0(\xi - g\tau) = \vartheta\left(\tau_0 - \frac{r}{g}\right), \tag{22}$$

$$\psi(r + \xi_0) = \vartheta\left(\tau_0 - \frac{r}{g}\right), \tag{23}$$

where we have introduced the variable r

$$r(\xi \;, \tau) = (\xi - \xi_0) - g(\tau - \tau_0) . \tag{24}$$

The substitution $\tau = \tau_0$ in (22) and $\xi = \xi_0$ in (23) gives the identical equality, that corresponds to the initial and boundary conditions:

$$[\theta]_0(\tau_0, \xi) = [\theta]_0(\xi - g\tau_0) = \psi(\xi),$$
$$r(\xi \;, \tau_0) = (\xi - \xi_0), \tag{25}$$

$$[\theta]_0(\tau, \xi_0) = [\theta]_0(\xi_0 - g\tau) = \vartheta(\tau),$$
$$r(\xi_0, \tau) = -g(\tau - \tau_0), \tag{26}$$

$$[\theta]_0(\xi_0 - g\tau_0) = \psi(\xi_0) = \vartheta(\tau_0),$$
$$r(\xi_0, \tau_0) = 0, \; \psi(\xi_0) = \vartheta(\tau_0). \tag{27}$$

Thus, for the construction of the solution that does not have discontinuity points, we must specify the distribution of interoperable backlog $\psi(\xi)$ along the conveyor line at time τ_0 or a change in the state of interoperable backlog $\vartheta(\tau)$ in time on technological position, that is characterized by the coordinate ξ_0. Due to the equation (6) function $g \cdot \psi(\xi)$ can be interpreted as a dimensionless rate of movement subjects of labor on the conveyor line for the technological position, which is characterized by the coordinate ξ, at time τ_0, and $g \cdot \vartheta(\tau)$ – as a dimensionless rate of processing of the subjects of labor at time τ on the technological position, which is characterized by the coordinate ξ_0. The dimensionless function $\psi(\xi)$ and $\vartheta(\tau)$ related to the density distribution of subjects of labor along the conveyor line $[\chi]_0(t,S)$ and to the pace of their movement $[\chi]_1(t,S)$ by the ratios (6)–(8). It is reasonable to expect that a dimensionless value of interoperable backlog of subjects of labor $\vartheta(\tau)$ at the position ξ_0 at time τ must be associated with a dimensionless quantity of the interoperable backlogs $\psi(\xi)$ at time τ_0 on the technological position, which is characterized by the coordinate ξ. Let us show the relationship between a functions $\psi(\xi)$ and $\vartheta(\tau)$.

For technological position with the coordinate $\xi = \xi_0$ we have the equality:

$$[\theta]_0(\xi_0 - g\tau) = \psi(r(\xi_0, \tau) + \xi_0) = \tag{28}$$
$$= \vartheta\left(\tau_0 - \frac{r(\xi_0, \tau)}{g}\right), r(\xi_0, \tau) = -g(\tau - \tau_0),$$

$$\psi(\xi_0 - g\tau + g\tau_0) = \vartheta(\tau), \psi(\xi_0) = \vartheta(\tau_0). \tag{29}$$

We use the notation

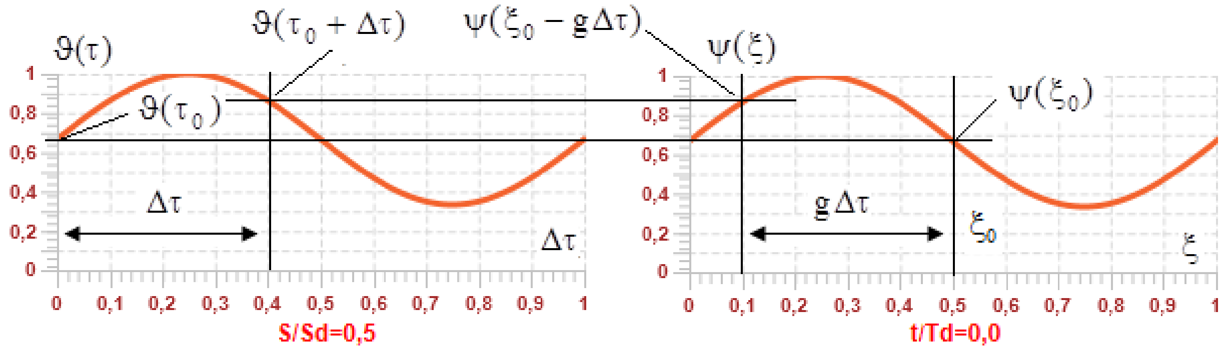
$$\Delta\tau = \tau - \tau_0, \tag{30}$$

taking into account that, the equality (29) can be rewritten as follows:

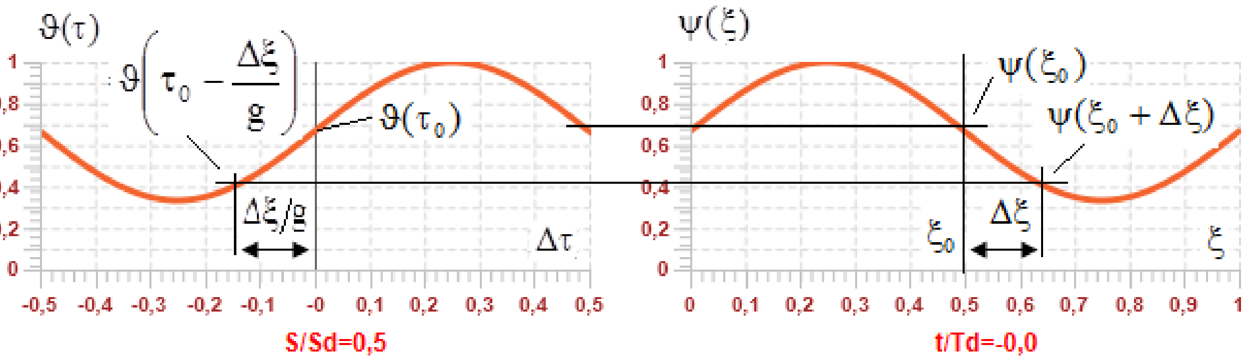
$$\psi(\xi_0 - g\Delta\tau) = \vartheta(\tau_0 + \Delta\tau), \psi(\xi_0) = \vartheta(\tau_0). \tag{31}$$

The relationship of functions $\psi(\xi)$ and $\vartheta(\tau)$ for different instants of time $\tau = \tau_0 + \Delta\tau$ is demonstrated at pic.2. Clearly showed that the value of interoperable backlog $\vartheta(\tau_0 + \Delta\tau)$ for equipment in technological position ξ_0 at time $\tau = (\tau_0 + \Delta\tau)$ is determined by the state of the interoperable backlogs on the technological position $\xi = (\xi_0 - g\Delta\tau)$ at time $\tau = \tau_0$.

Thus if we know initial distribution of the subjects of labor along the conveyor line at time $\tau = \tau_0$ (dimensionless speed of the conveyor g), it is possible to uniquely determine the number of the subjects of labor that will be located on the technological position ξ_0 at time $\tau = \tau_0 + \Delta\tau$. During the $\Delta\tau$ since the start of the conveyor $\tau = \tau_0$ the subjects of labor are moving with the speed of the conveyor g from the technological position $\xi = (\xi_0 - g\Delta\tau)$ to the technological position ξ_0. This relationship is important to the operation of conveyor type production lines [13]. The distribution of technological resources of production in time on the technological operations can be determined from the current state of the distribution of the subjects of labor along the



Pic.2. The relationship of functions $\psi(\xi)$ and $\vartheta(\tau)$ for different instants of time $\tau = \tau_0 + \Delta\tau$



Pic.3. The relationship of functions $\psi(\xi)$ and $\vartheta(\tau)$ for the coordinate values $\xi = \xi_0 + \Delta\xi$

conveyor using a fairly simple equation of the form (31). The current distribution of subjects of labor along the line determines the plan of distribution of technological resources in future times. For example it is possible to determine in advance how many employees you need to use to perform those or other operations in future times, or to determine the exact time of the supply of raw materials for technological position of the conveyor line.

On the other hand, regarding the point in time $\tau = \tau_0$ it is possible to write the relation

$$[\theta]_0(\xi - g\tau_0) = \psi(r(\xi,\tau_0) + \xi_0) = \\ = \vartheta\left(\tau_0 - \frac{r(\xi,\tau_0)}{g}\right), \quad (32)$$

$$r(\xi\ ,\tau_0) = (\xi - \xi_0),$$

$$\psi(\xi) = \vartheta\left(\tau - \frac{\xi}{g} + \frac{\xi_0}{g}\right), \quad \psi(\xi_0) = \vartheta(\tau_0). \quad (33)$$

We use the notation

$$\Delta\xi = \xi - \xi_0, \quad (34)$$

which allows the expression (33) represent as follows:

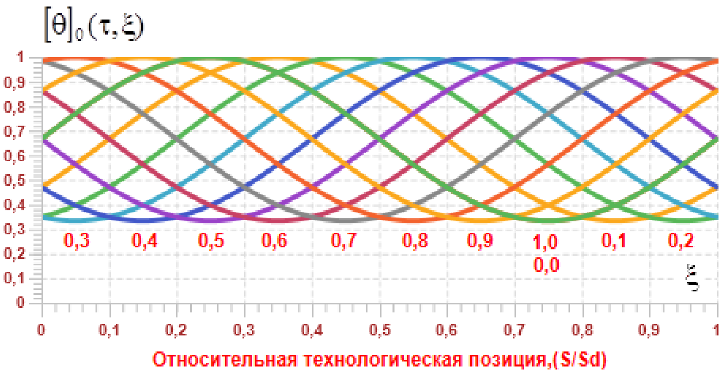
$$\psi(\xi_0 + \Delta\xi) = \vartheta\left(\tau_0 - \frac{\Delta\xi}{g}\right), \quad \psi(\xi_0) = \vartheta(\tau_0). \quad (35)$$

Pic.3 shows the relationship of functions $\psi(\xi)$ and $\vartheta(\tau)$ for different values of the coordinate $\xi = \xi_0 + \Delta\xi$. The size of interoperable backlogs for the technological equipment, located on an arbitrary technological position $\xi = \xi_0 + \Delta\xi$ based on the state of the interoperable backlogs for the equipment with technological position ξ_0 at time $\tau = \left(\tau_0 - \frac{\Delta\xi}{g}\right)$. Time interval $\Delta\tau = \frac{\Delta\xi}{g}$ determines the necessary time for which subjects of labor, which are on the technological position ξ_0 will move to the position $\xi = \xi_0 + \Delta\xi$ with the dimensionless conveyor speed g. The equation (35) shows the relationship of value of interoperable backlogs between technological positions ξ and ξ_0.

The collection of curves $[\theta]_0(\tau,\xi)$, each of which defines the state of interoperable backlogs for the technological position of the conveyor at the time moments $\tau = (t/T_d)$ $= \{0.0;0.1;0.2;\ldots.0.9;1.0\}$ respectively, is shown in Pic.4. The curves are shifted relative to each other with a step $\Delta\xi = g\Delta\tau$ with values g=1 and $\Delta\tau = 0.1$. The solution $[\theta]_0(\tau,\xi)$ is obtained for the case of the initial conditions $\psi(\xi) = \frac{1}{3}(2 + \sin(2\pi\xi))$ with $\tau = 0$:

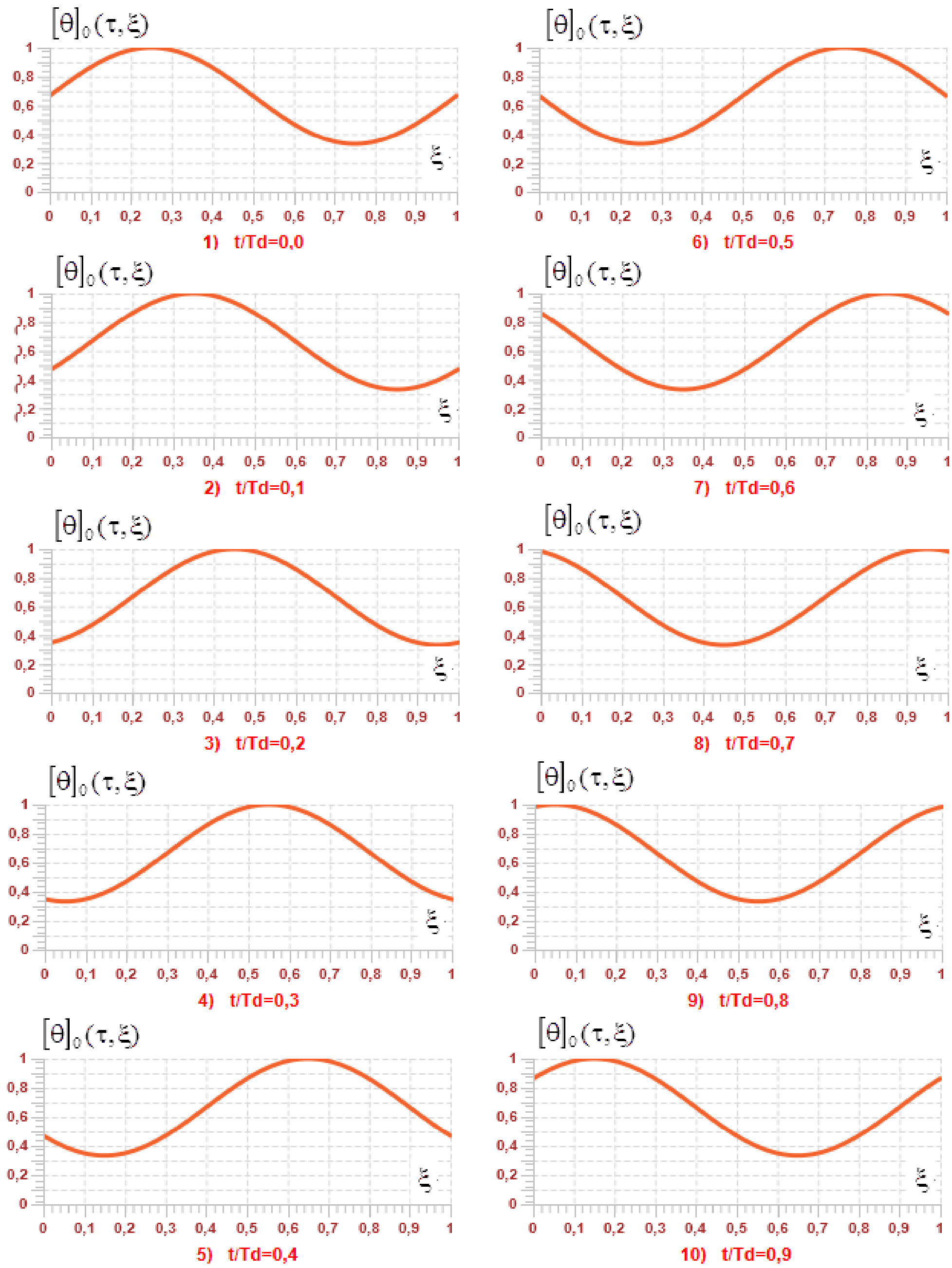
$$[\theta]_0(\tau,\xi) = \frac{1}{3}(2 + \sin(2\pi(\xi - g\tau))), \text{ g=1.} \quad (36)$$

The boundary condition (14) is absent. The graph shows the shift of the function $\psi(\xi) = \frac{1}{3}(2 + \sin(2\pi\xi))$ with the step $\Delta\xi = g\Delta\tau$ for time moments τ_i (pic.4.).



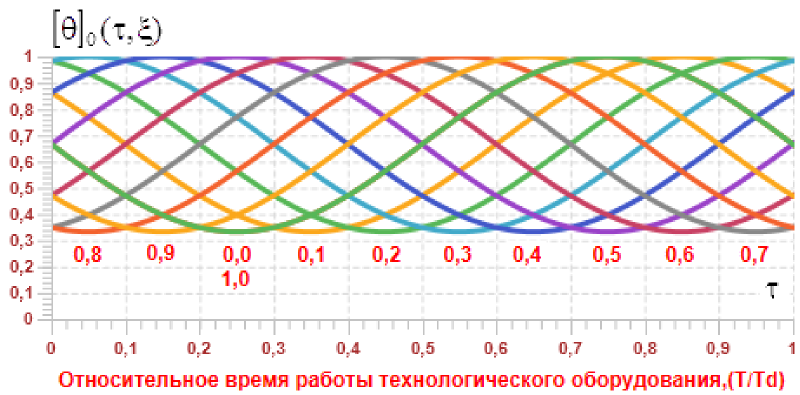
Pic.4. The distribution of subjects of labor $[\theta]_0(\tau,\xi)=\frac{1}{3}(2+\sin(2\pi(\xi-g\tau)))$

on technological positions of conveyor for $\tau=(t/T_d)=\{0.0;0.1;0.2;....0.9;1.0\}$, ***g=1***



Pic.5. The distribution of subjects of labor $[\theta]_0(\tau,\xi)=\frac{1}{3}(2+\sin(2\pi(\xi-g\tau)))$

on technological positions of conveyor for $\tau=(t/T_d)=\{0.0;0.1;0.2;....0.9\}$, **g=1 (Expanded view)**



Pic.6. The dependence of the number of subjects of labor $[\theta]_0(\tau,\xi)=\frac{1}{3}(2+\sin(2\pi(\xi-g\tau)))$ **from time for conveyor position** $\xi=(S/S_d)=\{0.0;0.1;0.2;....0.9;1.0;\}$**, g=1**

It is shown a consistent movement of the subjects of labor along the route on a conveyor line, which moves with the dimensionless speed g. Pic.5 displays the functions individually $[\theta]_0(0,\xi)$, $[\theta]_0(0.1,\xi)$,..., $[\theta]_0(0.8,\xi)$, $[\theta]_0(0.9,\xi)$ at the time moments $\tau=(t/T_d)=\{0.0;0.1;0.2;....;0.8;0.9\}$. Equally important from a practical point of view the information presented in pic.6, which gives information about the size of interoperable backlogs for given technological positions and about desired tempo of processing subjects of labor, which provides a smooth non-stop conveyor work [14,15].

Pic.7 shows the change in value of interoperable backlogs $[\theta]_0(\tau,0.0)$, $[\theta]_0(\tau,0.1)$,..., $[\theta]_0(\tau,0.8)$, $[\theta]_0(\tau,0.9)$ for technological positions $\xi=(S/S_d)=\{0.0;0.1;....;0.8;0.9\}$. Non-stationary time-dependent, the rate of processing of subjects of labor $[\theta]_1(\tau,\xi)=g\cdot[\theta]_0(\tau,\xi)$ on the same technological position at constant conveyor speed g is determined by the difference in the density of subjects of labor at the working technological line segments. With the value of the dimensionless speed of the conveyor $g=1$ the pace of processing subjects of labor takes the form $[\theta]_1(\tau,\xi)=[\theta]_0(\tau,\xi)$. Pic.8 represents considerable practical interest for the design of the conveyor line with displaying family of characteristics $\xi-g\tau=C_i$ (17). Each of the characteristics $\xi-g\tau=C_i$ connects values ξ and τ, for which value of interoperable backlogs, and accordingly, the pace of processing subjects of labor is constant and equal to:

$$[\theta]_0(\tau,\xi)=[\theta]_0(C_i)=\psi(C_i+g\tau_0),\quad r(\xi\ ,\tau)=C_i-(\xi_0-g\tau_0) \qquad (37)$$

$$[\theta]_0(\tau,\xi)=[\theta]_0(C_i)=\vartheta\left(\frac{\xi_0-C_i}{g}\right), \qquad (38)$$

In the characteristics (17) the value of interoperable backlogs and the pace of processing of subjects of labor are kept constant. The profile, that define the distribution of subjects of labor (or the rate of processing subjects of labor) on technological operations, moves along the conveyor line with constant speed g. Characteristics cover the whole plane (τ,ξ) and not intersect each other. The equation of characteristics can be used to track location of specific batch details in process of treatment in order to determine its status and conducting sampling inspection of party details, and to manage production inventory [16].

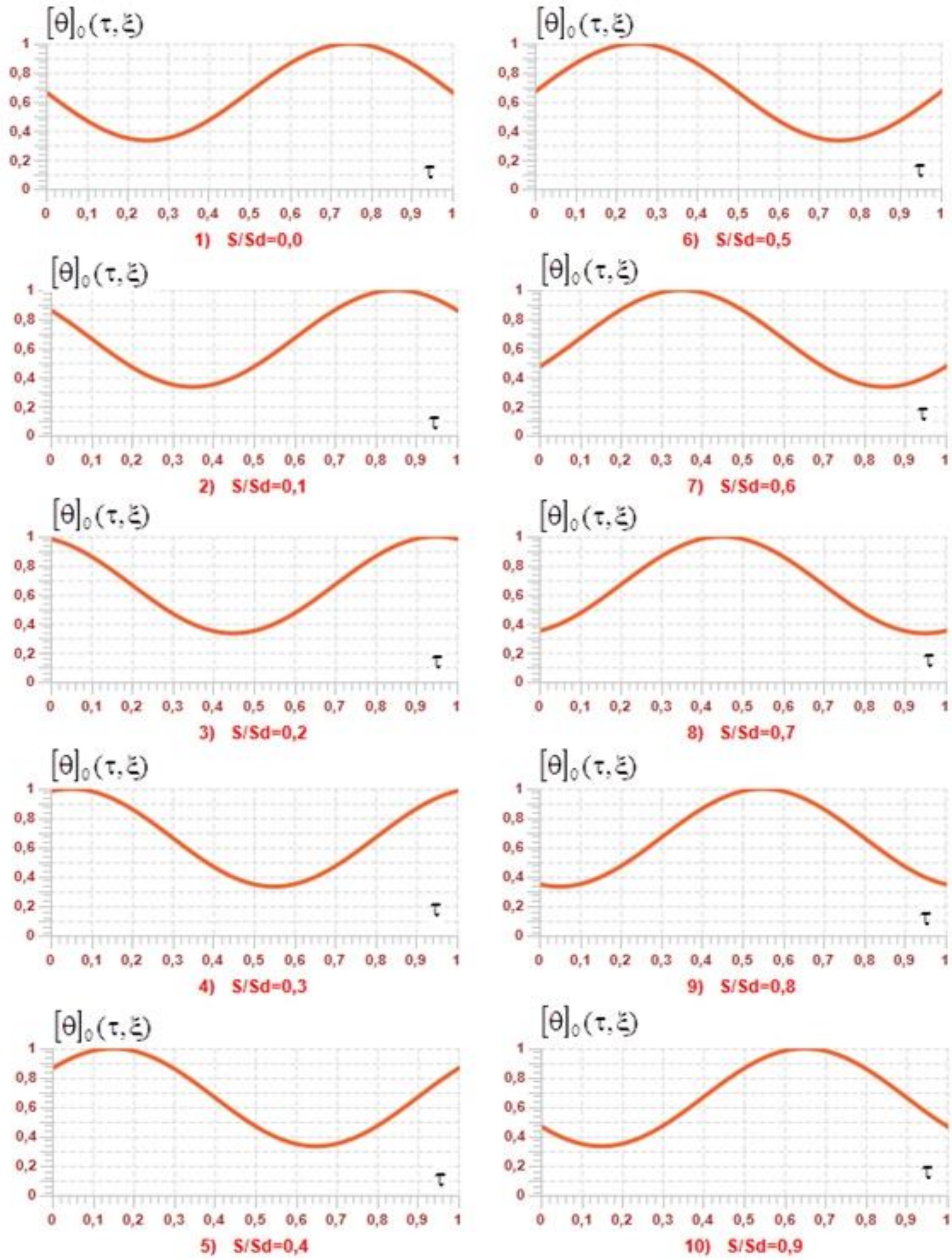
The equations (17) are the equations of motion of individual subjects of labor on technological route along the conveyor line. As expected, movement of a separate subject of labor on the trajectory (17) carried out with a constant speed, wich is equal to the speed of the conveyor line [17]. The value of the dimensionless speed g (11) is determined by selecting value of the characteristic time t_d to study the parameters of the production line. The increase in value of the characteristic time t_d towards relative to the amount of the production cycle T_d leads to a stretching of the graph with the decision $[\theta]_0(\tau,\xi)$ along the axis OS (transformation pic.4 into pic.9) and to its compression along the axis Ot (transformation pic.6 into pic.10)). At the same time chart with characteristics is compressing (transformation pic.8 into pic.11)). This allows, during the transition from one value of the characteristic time t_d to another value of characteristic time t_d use in a manufacturing enterprise the same current graphic and analytical materials.

4. The calculation of the duration of the production cycle of the conveyor line

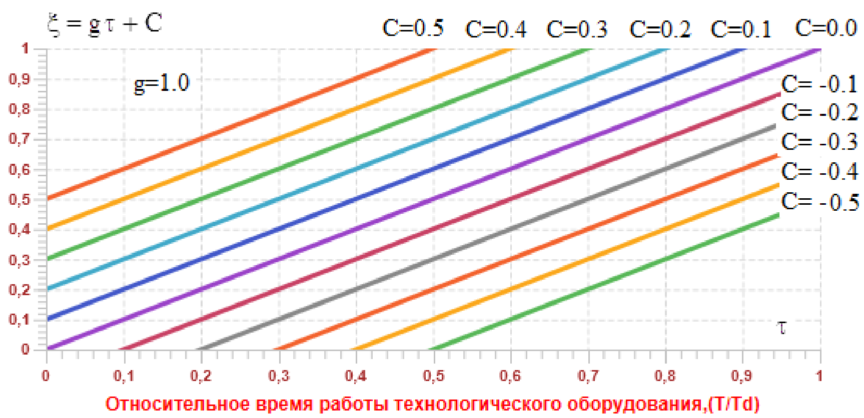
The duration of the production cycle is equal to the length of time, during which the subject of labor goes from the first technological position to the last. The calculation of the production cycle for production systems with the in-line way of organizing production is given in [9–11]. The value of duration of the production cycle of the conveyor type production line can be determined as follows: $T_d=\int_0^1\frac{dS}{a}$.

We introduce the notation for the dimensionless quantity, wich characterizes the duration of production cycle $\tau_d=\frac{T_d}{t_d}$. Using the notation (7),(8),(11), we obtain

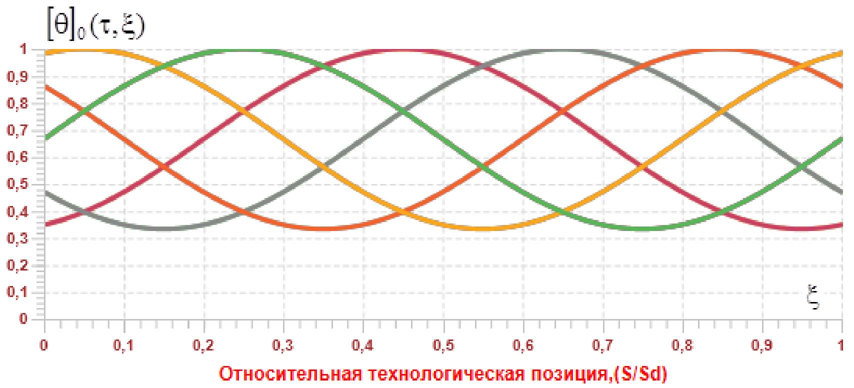
$$\tau_d=\frac{T_d}{t_d}=\frac{1}{t_d}\int_0^1\frac{S_d d\xi}{a}==\frac{S_d}{t_d a}\int_0^1 d\xi=\frac{1}{g}.$$



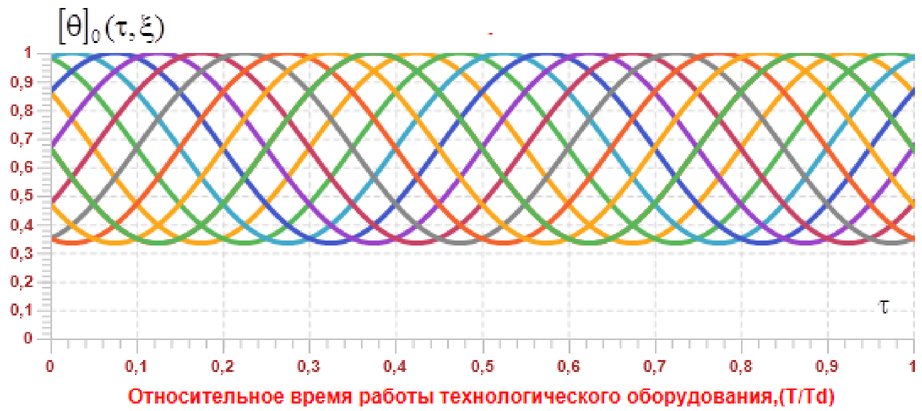
Pic.7. The dependence of the number of subjects of labor $[\theta]_0(\tau,\xi)=\frac{1}{3}(2+\sin(2\pi(\xi-g\tau)))$ **from time to time for conveyor position** $\xi=(S/S_d)=\{0.0;0.1;0.2;\ldots.0.9;1.0;\}$**, g=1. (Expanded view)**



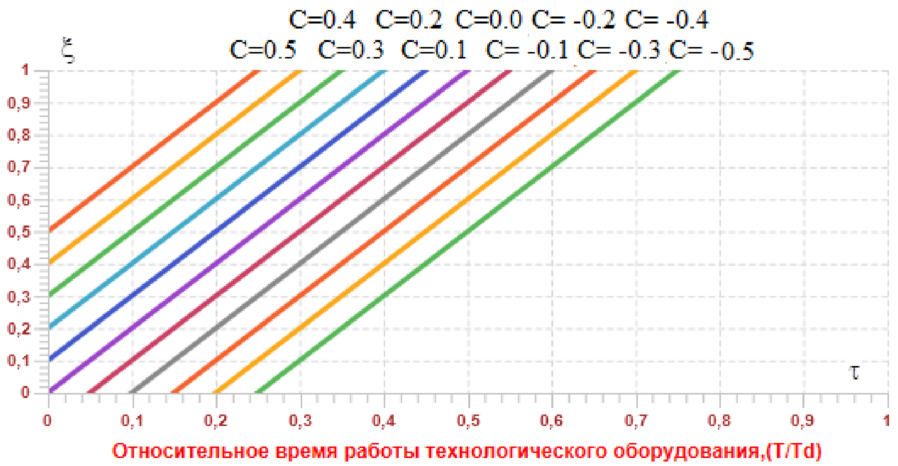
Pic.8. The family of characteristics $\xi=g\tau+C$, $g=1.0$, $C=\{0.5;0.4;0.3;\ldots.-0.4;-0.5;\}$



Pic.9. The distribution of subjects of labor $[\theta]_0(\tau,\xi)=\frac{1}{3}(2+\sin(2\pi(\xi-g\tau)))$ **on technological positions of conveyor for** $\tau=(t/T_d)=\{0.0;0.1;0.2;\dots.0.9;1.0\}$**, g=2**



Pic.10. The dependence of the number of subjects of labor $[\theta]_0(\tau,\xi)=\frac{1}{3}(2+\sin(2\pi(\xi-g\tau)))$ **from time for conveyor position** $\xi=(S/S_d)=\{0.0;0.1;0.2;\dots.0.9;1.0;\}$**, g=2**



Pic.11. The family of characteristics $\xi=g\tau+C$, $g=2.0$, $C=\{0.5;0.4;0.3;\dots.-0.4;-0.5;\}$

The products, that had gone into production in a dimensionless point in time τ_1, come out as a finished product at the time τ_2 with delay $\tau_d=(\tau_2-\tau_1)$, equal to the cycle time. Delay $(\tau_2-\tau_1)$ does not depend on the time of order receipt in production, is a constant, is inversely proportional to the dimensionless speed of the conveyor line $\tau_d=\frac{1}{g}$. The output state of parameters of a flow line is determined by the state of production lines at the input with the constant value delay.

Conclusions

The paper considers the model of the production line of conveyor type with a constant speed of subjects of labor. This method of organizing production is quite common, but the issue of analysis of PDE- model of conveyor production line in modern literature are not given the necessary attention. Despite the fact that the model of conveyor type has a fairly simple form, wich was obtained in the course of its analysis in the present study have important practical significance for the design of control systems. The advantage of the work is that the solution was obtained in an analytical form. This allowed to carry out a detailed analysis of the obtained solution. Clearly demonstrated the relationship between the parameters of various technological operations. Violation of this relationship leads to violations of uninterrupted mode of production, and also to the appearance of instabilities in the behavior of the production line parameters.

Duration of the production cycle have been calculated in this paper. It is shown that in a conveyor way of organizing production with a constant speed of movement of subjects of labor production cycle does not depend on the size of workpieces parties and from the sequence of their arrival in production. Prospects for further research is the analysis of the model of conveyor type production line with the speed of movement of subjects of labor, which depends on the time with a random admission orders in production.

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ПОСТРОЕНИЕ МОДЕЛИ ОДНОУЗЛОВОЙ КОНВЕЙЕРНОЙ ЛИНИИ С ПОСТОЯННОЙ СКОРОСТЬЮ

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Аннотация. В данной статье основное внимание уделяется модели производственной линии конвейерного типа. Записана в одномоментном приближении система уравнений для модели конвейерной линий с постоянной скоростью движения предметов труда и получено ее решение в аналитическом виде, что позволило рассчитать параметры системы в произвольный момент времени. Показано, что решение определяется видом начальных и граничных условий. Получено выражение для расчета производственного цикла конвейерной линии.
Ключевые слова: поточная линия, PDE-модель, предмет труда, система управления, технологический процесс, полупроводниковое производство, модель конвейера.

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