THE KINETIC MODEL OF THE PRODUCTION LINE

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Abstract – The paper discusses methods of constructing the kinetic equation of the technology process. The article presents a model of the interaction of objects of labor with technological equipment, which is the basis for the derivation of the kinetic equation. To describe the state of the production line introduced numerical characteristics.

Keywords – kinetic equation, the production line, mass production, work in progress, balance equations.

I. Introduction

Modeling complex dynamic production processes is an effective method of research [1,2]. One of widespread classes form the production systems, in which character of the observable production processes has stochastic nature [3,4]. Regularities of inherent equilibrium states in production systems in many ways similar to those that take place in the physical (thermodynamic) systems. On the basis of the principles of the functioning of modern mass production it can be represented as a stochastic process, during which the manufacturing system changes from one state to another [4,5]. The production process state is determined by the state of the overall number N of items of work [4]. In transition of the object of labor from one state to another, there is a transformation of resources (raw material, materials, living labor) in the prepared product as a result of purposeful influence of equipment. State j-th object of labor in the phase space will be described by state parameters, $\vec{S}_j = (S_{j,1},...S_{j,A})$, $\vec{\mu}_j = (\mu_{j,1},...,\mu_{j,A})$ where $S_{j,\alpha}$ (USD) value of the transferred α - of the technological resource or part thereof for the j-th subject of work, $\mu_{j,\alpha}$ (USD/hour), the intensity of the transfer value of α- of the resource to the j-th subject of work, fig.1. The state of parameters of production process in some moment of time will be defined, if the parameters of the state of the object of labor are defined $(\vec{S}_1,\vec{\mu}_1,...,\vec{S}_N,\vec{\mu}_N)$ and the objective function $J(t,\vec{S}_j,\vec{\mu}_j)$ [1], and at any other time it is found from the equations of states of objects of labor. However, if the number of objects of labor N is much greater than unity, then decide System of N-equations second order is practically impossible [2,4]. The last clarification requires a transition from the object-process description (micro-description) to aggregated streaming (macroscopic) description with the elements of probabilistic nature.

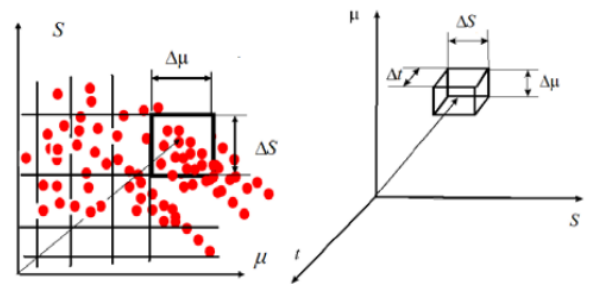

Fig. 1. The unit cell of phase technological space.

The main difficulty in this specification is to highlight the characteristics of the parameters of states objects of labor [2], which could be measured in the study of the actual production processes. Instead of considering the state of the production process with the parameters of the state of the objects of labor $(\vec{S}_1,\vec{\mu}_1,...,\vec{S}_N,\vec{\mu}_N)$, we will enter normalized discrete phase function of distribution of number N objects of labor in the phase space (t,S,μ) [4]. Each point in the space of states will set the state of the object of labor.

II. The construction of a kinetic equation of the production system

For the technological process, consisting of several hundred operations it is reasonable to go to the continuous description of the parameters, that describe the operation, considering along with [3,6] the main limit when $N \to \infty$ and the limiting case of $M >> 1$. Due to the fact that the product of $\chi(t,S,\mu) \cdot d\Omega$ is the number of items of work in the cell $d\Omega$ phase space with coordinates $S_j \in [S, S+\Delta S]$, $\mu_j \in [\mu, \mu+\Delta\mu]$ the integration over the volume Ω of the phase space (t,S,μ), gives the total number N of items of work that are in progress:

$$\int_0^{S_d}\int_0^{\infty} \chi(t,S,\mu) d\mu dS = N \text{ , } \Omega = \int_0^{S_d}\int_0^{\infty} d\mu dS \qquad (1)$$

where S_d (USD) - the cost of the product. The limits of integration $S = 0$ and $S = S_d$ specify the range of change in the coordinates $S, S \in [0, S_d]$ which determines the position of the object of labor along the technological route. We will accept (1) as a condition for the normalization of the phase distribution function $\chi(t,S,\mu)$ items of work over the states [2], which is the law of conservation of articles in the production process [4]. We introduce a numerical characteristics

$$\int_0^{\infty} \mu^k * \chi(t,S,\mu) d\mu = [\chi]_k \qquad (2)$$

that reflect the essential features of the distribution of the states of the objects of labor that are in progress. Features (2) for the distribution function $\chi(t,S,\mu)$, we will define as moments of k-th order. Often the problem can be solved using numerical characteristics, leaving aside the laws of distribution. Essential value in the models of the production process are the zero $[\chi]_0 = [\chi]_0(t,S)$ and the first $[\chi]_1 = [\chi]_1(t,S)$ moments of the distribution function of the states of the objects of labor which determine density of distribution on the positions of objects of labor and the rate of processing operations on objects of labor. The change of the distribution function $\chi(t,S,\mu)$ of the states of the of objects of labor is due to the stochastic nature of the interaction of objects with the equipment and with each other. This interaction is characterized by function $G(t,S,\mu)$ [6,7]

$$\frac{d\chi(t,S,\mu)}{dt} = G(t,S,\mu) \qquad (3)$$

The total derivative in (3) means differentiation along the phase trajectory of the object of labor. If the motion of the object of labor in the state space is deterministic and defined by the Euler equations for the objective function of the production system [2,3], the equation (3) by virtue of the Liouville theorem becomes the identity $G(t,S,\mu)=0$. We write a total derivative $\chi(t,S,\mu)$ in the form

$$\frac{\partial\chi(t,S,\mu)}{\partial t}+\frac{\partial\chi(t,S,\mu)}{\partial S}\cdot\mu+\frac{\partial\chi(t,S,\mu)}{\partial\mu}\cdot\frac{d\mu}{dt}=G(t,S,\mu) \quad (4)$$

Replace $d\mu/dt$ in (4) with the equations defining the normative trajectory of the object of labor in the cell $d\Omega$ [2,7],

$$\frac{d\mu}{dt}=\frac{\partial}{\partial t}\left(\frac{[\chi]_{1\psi}(t,S)}{[\chi]_0(t,S)}\right)+\frac{[\chi]_{1\psi}(t,S)}{[\chi]_0(t,S)}\frac{\partial}{\partial S}\left(\frac{[\chi]_{1\psi}(t,S)}{[\chi]_0(t,S)}\right)=f(t,S),$$

$$\mu=\frac{dS}{dt}, \quad (5)$$

where $[\chi]_{1\psi}(t,S)$ - the pace of processing of objects of labor equipment at the point of the technological route with coordinate *S*. Equation (5) connects the release of products in place technological route, specified by the coordinate *S* and the amount of necessary costs of technological resources to transform the object of labor. The ratio of the form (5) which provides the connection with the cost of manufacture can be determined as a function of the generalized manufacturing equipment located in the designated area of a technological route.

The number of objects of labor that are experienced per unit time the impact of the technological equipment and took random value in the range of $(\tilde{\mu},\tilde{\mu}+d\tilde{\mu})$ is the product of the transition probabilities $\varphi(t,S,\mu,\tilde{\mu})\cdot d\tilde{\mu}$ on the total number of objects of labor $\lambda_{Plant}(t,S)\cdot dS\cdot\chi(t,S,\mu)\cdot\mu\cdot d\mu$, experienced the impact of the equipment.

$$\varphi(t,S,\mu,\tilde{\mu})\cdot d\tilde{\mu}\cdot\lambda_{Plant}(t,S)\cdot dS\cdot\chi(t,S,\mu)\cdot\mu\cdot d\mu \quad (6)$$

Along with the departure of (6) objects of labor from the volume element $dS\cdot d\mu$ in the element of volume $dS\cdot d\tilde{\mu}$ objects of labor come from volume $dS\cdot d\tilde{\mu}$ in the amount of:

$$\varphi(t,S,\tilde{\mu},\mu)\cdot d\mu\cdot\lambda_{Plant}(t,S)\cdot dS\cdot\chi(t,S,\tilde{\mu})\}\cdot\tilde{\mu}\cdot d\tilde{\mu} \quad (7)$$

After the integration of the difference between (6) and (7) over a range of values of $\tilde{\mu}$, we obtain the change in the number of objects of labor in the volume element $dS\cdot d\mu$ per unit of time

$$\frac{\partial\chi}{\partial t}+\frac{\partial\chi}{\partial S}\mu+\frac{\partial\chi}{\partial\mu}f=$$

$$=\lambda_{Plant}\int_0^\infty\{\varphi(t,S,\tilde{\mu},\mu)\tilde{\mu}\chi(t,S,\tilde{\mu})-\varphi(t,S,\mu,\tilde{\mu})\mu\chi(t,S,\mu)\}d\tilde{\mu} \quad (8)$$

III. Conclusion

Integro-differential equation (9) is a kinetic equation that describes the processing of objects of labor during their movement on the technological route, it was first obtained in [8]. In the case where the intensity μ is slowly varying with time, $\mu=\mu_0\cong const$ (quasi-static process) [6], the kinetic equation (8) takes the form:

$$\frac{\partial\chi}{\partial t}+\frac{\partial\chi}{\partial s}\cdot\mu=$$

$$=\lambda_{Plant}\cdot\{\int_0^\infty[\varphi(t,S,\tilde{\mu},\mu)\cdot\tilde{\mu}\cdot\chi(t,S,\tilde{\mu})]\cdot d\tilde{\mu}-\mu\cdot\chi\}, \quad (9)$$

$$\frac{d\mu}{dt}=f(t,S)\cong 0, \quad (10)$$

which is used in the quasi-static description of the technological process [5]. From equation $f(t,S)\cong const$ (6) follows $[\chi]_{1\psi}(t,S)\cong const*[\chi]_0(t,S)$, which requires compliance with performance proportional to the number of items of work the equipment in its in-process storage. It is difficult to realize in practical terms, except for the case of synchronization the equipment $\frac{\partial}{\partial S}[\chi]_{1\psi}(t,S)\cong 0$. The kinetic equation of the form (8) can be used to construct models of synchronized production lines. Built on the basis of its balance equations is used to study the synchronized production lines for the production of semiconductor products. The right side of the kinetic equation (8,9) is written in general terms, it requires further in-depth study of the effect as the features of processing technology as well as layouts of components and assemblies inside the unit of generalized equipment.

IV. References

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