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## DIFFRACTION OF A PLANE WAVE BY A THIN MATERIAL STRIP: SOLUTION BY THE ANALYTICAL-NUMERICAL APPROACH

Eldar I. Veliev<sup>1)</sup>, Shoichi Koshikawa<sup>2)</sup>, and Kazuya Kobayashi<sup>3)</sup>

<sup>1)</sup>Institute of Radiophysics and Electronics, National Academy of Sciences, Ulitsa Proskury,  
12, Kharkov 310085, Ukraine

E-mail: veliev@dut.kharkov.ua

<sup>2)</sup>Antenna Giken Co., Ltd., 4-72 Miyagayato, Omiya 330-0011, Japan

E-mail: skoshi@mail.raidway.ne.jp

<sup>3)</sup>Department of Electrical, Electronic, and Communication Engineering, Chuo University  
1-13-27 Kasuga, Bunkyo-ku, Tokyo 112-8551, Japan

E-mail: kazuya@kazuya.elect.chuo-u.ac.jp

### ABSTRACT

The plane wave diffraction by a thin material strip is analyzed for both  $E$  and  $H$  polarizations using a new analytical-numerical approach together with approximate boundary conditions. The problem is reduced to the solution of infinite systems of linear algebraic equations. The final results are valid provided that the thickness of the strip is small compared with the wavelength. The scattered field is evaluated asymptotically and a far field expression is derived. Illustrative numerical examples on the radar cross section are presented and the far field scattering characteristics are discussed. Some comparisons with the other existing method are also given.

### INTRODUCTION

Analysis of the scattering by imperfectly conducting and absorbing strips is an important subject in electromagnetic theory, and it is relevant to many engineering applications such as antenna and radar cross section (RCS) studies. A resistive sheet can be regarded as a suitable model of thin material layers and there have been investigations on the scattering by resistive strips based on some analytical methods [1,2]. It is also known that a thin material layer with arbitrary permittivity and permeability can be modeled more accurately by a pair of modified resistive and conductive sheets each satisfying given boundary conditions [3-5].

In a previous paper [6], we have analyzed the plane wave diffraction by a thin dielectric strip using a new analytical-numerical approach [7] based on the orthogonal polynomial expansion in conjunction with the Fourier transform, where an efficient solution has been obtained for the strip thickness small compared with the wavelength. In this paper, we shall consider a thin strip consisting of a homogeneous material with arbitrary permittivity and permeability as a generalization to our previous geometry in [6], and analyze the plane wave diffraction for both  $E$  and  $H$  polarizations. Applying the approximate boundary conditions to an integral representation of the scattered field, the problem is formulated as two integral equations satisfied by the unknown current density functions. Expanding the current density functions in terms of the orthogonal polynomials, our problem is reduced to the solution of two infinite systems of linear algebraic equations (SLAE) satisfied by the unknown expansion coefficients. These coefficients are determined numerically with high accuracy via appropriate truncation of the SLAE. The scattered field is evaluated asymptotically and a far field expression is derived.

We shall present illustrative numerical examples on the RCS and discuss the far field scattering characteristics in detail. Some comparisons with Volakis [4] are also given to validate the present method. In the following, the analysis procedure is presented only for the  $E$ -polarized

case, but numerical results are given for both polarizations. The time factor is assumed to be  $e^{-i\omega t}$  and suppressed throughout this paper.

### SOLUTION BASED ON THE ANALYTICAL-NUMERICAL APPROACH

Let us consider the diffraction of an  $E$ -polarized plane wave by a thin material strip as shown in Fig.1, where the relative permittivity and permeability of the strip are denoted by  $\epsilon_r$  and  $\mu_r$ , respectively. Let the total field be  $E_z(x, y) = E_z^i(x, y) + E_z^s(x, y)$ , where  $E_z^i(x, y) [= e^{-ik(x\alpha_0 + y\sqrt{1-\alpha_0^2})}]$  is the incident field with  $\alpha_0 = \cos\theta_0$  for  $0 < \theta < \pi/2$  and  $k(=\omega\sqrt{\epsilon_0\mu_0})$  being the free-space wavenumber, and  $E_z^s(x, y)$  is the unknown scattered field. The material strip is replaced by a strip of zero thickness satisfying the boundary condition corresponding to modified resistive and conductive sheets [5] under the condition that the strip thickness is small compared with the wavelength. Then the total field satisfies the boundary condition on the strip as given by

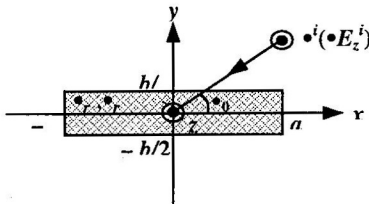


Fig.1. Geometry of the problem.

$$H_x(x, +0) + H_x(x, -0) + 2R_m[E_z(x, +0) - E_z(x, -0)] = 0 \tag{1a}$$

$$\left[ \frac{1}{2R_e} + \frac{1}{2\tilde{R}_m} \left( 1 + \frac{1}{k^2} \frac{\partial^2}{\partial y^2} \right) \right] [E_z(x, +0) + E_z(x, -0)] = H_x(x, +0) - H_x(x, -0), \tag{1b}$$

where  $R_e = iZ/[kb(\epsilon_r - 1)]$ ,  $R_m = iY/[kb(\mu_r - 1)]$ , and  $\tilde{R}_m = iZ\mu_r/[kb(\mu_r - 1)]$  with  $Z(=1/Y)$  being the intrinsic impedance of free space. Using Green's formula, we can express the scattered field as

$$E_z^s(x, y) = \frac{kZ}{4} \int_{-a}^a \left[ J_z(x') - \frac{iY}{k} J_x^*(x') \frac{\partial}{\partial y} \right] H_0^{(1)} \left[ k\sqrt{(x-x')^2 + y^2} \right] dx' \tag{2}$$

with  $H_0^{(1)}(\square)$  being the Hankel function of the first kind, where  $J_z(x) [\equiv H_x(x, +0) - H_x(x, -0)]$  and  $J_x^*(x) [\equiv E_z(x, +0) - E_z(x, -0)]$  are the unknown electric and magnetic current density functions, respectively. Taking into account the boundary condition as given by (1a,b), we obtain from (2) that

$$2ZR_m J_x^*(x) = 2\sqrt{1-\alpha_0^2} e^{-ikx\alpha_0} + \lim_{y \rightarrow 0} \frac{1}{2k} \frac{\partial^2}{\partial y^2} \int_{-a}^a J_x^*(x') H_0^{(1)} \left[ k\sqrt{(x-x')^2 + y^2} \right] dx', \tag{3a}$$

$$J_z(x) = - \left( \frac{1}{R_e} + \frac{\alpha_0^2}{\tilde{R}_m} \right) e^{-ikx\alpha_0} - \lim_{y \rightarrow 0} \frac{kZ}{2} \left[ \frac{1}{2R_e} + \frac{1}{2\tilde{R}_m} \left( 1 + \frac{1}{k^2} \frac{\partial^2}{\partial y^2} \right) \right] \int_{-a}^a J_z(x') H_0^{(1)} \left[ k\sqrt{(x-x')^2 + y^2} \right] dx'. \tag{3b}$$

Equations (3a,b) are the integral equations to this diffraction problem. Taking the finite Fourier transform of (3a,b) over the interval  $|x| < a$  and using the integral representation of the Hankel function, we are led to

$$2ZR_m F_x(\beta) = 4\sqrt{1-\alpha_0^2} \frac{\sin \kappa(\beta + \alpha_0)}{\kappa(\beta + \alpha_0)} - \frac{1}{\pi} \int_{-\infty}^{\infty} F_x(\alpha) \sqrt{1-\alpha^2} \frac{\sin \kappa(\alpha - \beta)}{\alpha - \beta} d\alpha, \tag{4a}$$

$$F_z(\beta) = -2 \left( \frac{1}{R_e} + \frac{\alpha_0^2}{\tilde{R}_m} \right) \frac{\sin \kappa(\beta + \alpha_0)}{\kappa(\beta + \alpha_0)} - \frac{Z}{2\pi} \left( \frac{1}{R_e} + \frac{1}{\tilde{R}_m} \right) \int_{-\infty}^{\infty} \frac{F_z(\alpha)}{\sqrt{1-\alpha^2}} \frac{\sin \kappa(\alpha - \beta)}{\alpha - \beta} d\alpha + \frac{Z}{2\pi \tilde{R}_m} \int_{-\infty}^{\infty} F_z(\alpha) \sqrt{1-\alpha^2} \frac{\sin \kappa(\alpha - \beta)}{\alpha - \beta} d\alpha \tag{4b}$$

with  $\kappa = ka$ , where

$$F_x(\alpha) = \int_{-1}^1 J_x^*(\alpha x') e^{-i\kappa \alpha x'} dx', \quad F_z(\alpha) = \int_{-1}^1 J_z(\alpha x') e^{-i\kappa \alpha x'} dx'. \tag{5}$$

It can be shown that the unknown functions  $J_x^*(x)$  and  $J_z(x)$  are expanded in the form

$$J_x^*(x) = \sqrt{1 - \left(\frac{x}{a}\right)^2} \sum_{n=0}^{\infty} J_n^x U_n \left(\frac{x}{a}\right), \quad J_z(x) = \frac{1}{a\sqrt{1 - (x/a)^2}} \left[ J_0^z + 2 \sum_{n=1}^{\infty} \frac{J_n^z}{n} T_n \left(\frac{x}{a}\right) \right], \tag{6}$$

where  $T_n(\square)$  and  $U_n(\square)$  denote the Chebyshev polynomial of the first and second kinds, respectively. In (6),  $J_n^{x,z}$  for  $n=0,1,2,\dots$  are unknown coefficients to be determined. Substituting (6) into (4a,b) and applying some properties of the Weber-Schafheitlin discontinuous integrals, we derive the two infinite systems of linear algebraic equations (SLAE) as in

$$\sum_{n=0}^{\infty} (A_{mn} + 2ZR_m B_{mn}) J_n^x = \gamma_m^x, \quad \sum_{n=0}^{\infty} (-C_{mn} - D_{mn} + E_{mn}) X_n^z = \gamma_m \tag{7}$$

for  $m=0,1,2,\dots$ , where  $X_0^z = J_0^z$ ,  $X_n^z = 2(-i)^n J_n^z/n$  for  $n=1,2,3,\dots$ . In (7),  $A_{mn}$ ,  $B_{mn}$ ,  $C_{mn}$ ,  $D_{mn}$ ,  $E_{mn}$ ,  $\gamma_m^x$ , and  $\gamma_m$  are known coefficients. The unknowns  $J_n^{x,z}$  are determined with high accuracy by solving (7) numerically via truncation.

**NUMERICAL RESULTS AND DISCUSSION**

We shall now present illustrative numerical examples of the RCS for both  $E$  and  $H$  polarizations to discuss the far field scattering characteristics of the strip in detail. Figure 2 shows the monostatic RCS as a function of incidence angle, where the strip width is  $2a = 2\lambda$ ,  $3\lambda$ , and

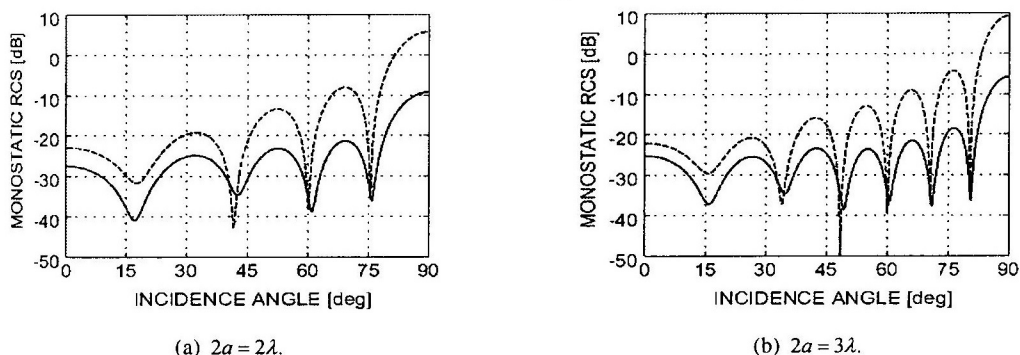


Fig. 2. Monostatic RCS versus incidence angle for  $b = 0.05\lambda$ ,  $\epsilon_r = 2.5 + i1.25$ ,  $\mu_r = 1.6 + i0.8$ .  $\bullet\bullet$  :  $E$  polarization; - - - -

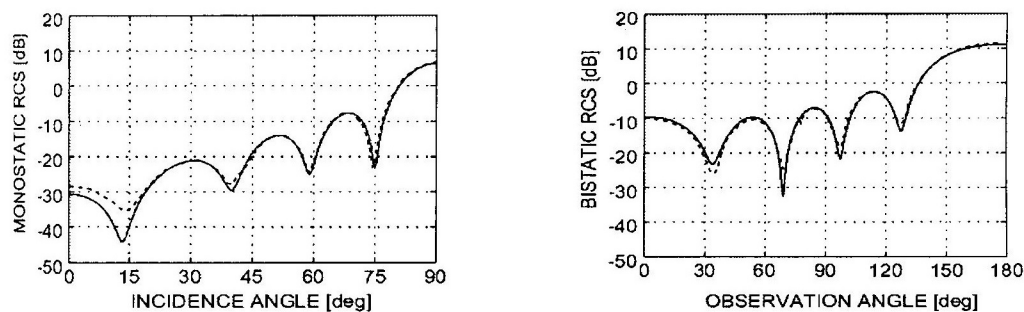


Fig. 3. Monostatic RCS versus incidence angle for  $H$  polarization,  $2a = 2\lambda$ ,  $b = 0.05\lambda$ ,  $\epsilon_r = 4.0$ ,  $\mu_r = 1.0$  and its comparison with Volakis [4].  $\bullet\bullet$  : this paper; - - - - : Vo-  
 Fig. 4. Bistatic RCS versus observation angle for  $H$  polarization,  $\theta_0 = 1^\circ$ ,  $2a = 2\lambda$ ,  $b = 0.05\lambda$ ,  $\epsilon_r = 1.5 + i0.1$ ,  $\mu_r = 1.0$  and its comparison with Volakis [4].  $\bullet\bullet$  : this paper; - - - - :



the strip thickness is  $b = 0.05\lambda$ .

As an example of existing lossy materials, we have chosen the ferrite with  $\epsilon_r = 2.5 + i1.25$  and  $\mu_r = 1.6 + i0.8$  in numerical computation. It is obvious that the peaks at  $90^\circ$  in the figure correspond to the specular reflection from the upper surface of the strip. We also notice that the RCS shows sharp oscillation with an increase of the strip width as can be expected. It is interesting to note that the RCS level for the  $E$  polarization is lower than that for the  $H$  polarization over the whole range of the incidence angle.

This problem has previously been analyzed by Volakis [4] for the  $H$ -polarized case based on the plane wave spectrum method together with the extended spectral ray method. In the formulation, Volakis employed the same approximate boundary conditions as in this paper. Figures 3 and 4 show comparisons with the results obtained by Volakis, where the monostatic RCS as a function of incidence angle and the bistatic RCS as a function of observation angle are illustrated. It is seen from the figure that our results agree reasonably well with Volakis' results.

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