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MATHEMATICAL MODEL OF A SPHERICAL FRACTAL EMITTER

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Abstract. Results of mathematical simulation of a spherical fractal emitter for the first time represented. The application of fractional calculation for mathematical model generation is justified. The α-characteristic of the magnetic field component of a symmetric spherical emitter and current distribution on a surface is retrieved. The properties of a spherical fractal emitter are defined. The outputs are confirmed by matching of the obtained formulas and graphs with known data.

Introduction

Rise of interest to designing different devices of UHF of a range with fractal structure (see, e.g., [1],) boosted application of fractional calculation (see, e.g., [2]) in the electromagnetic theory [3,4]. Results on learning physical and geometrical properties of fractal fields now are obtained [5,6], in particular, in the radiation theory [7,8].

In [6] is shown, that the α-characteristics of a field, which are generated α-dimensional by a Hausdorff's measure of the fractal object, can be constructed with the application of fractional integration $\alpha-1$ measure (or fractional derivation of $\alpha+1$ measure). The enumeration of outcomes of simulation of a spherical emitter further represented.

Formulation

The attempts of measurement of a usual current with density $j(P)$ on a fractal surface unit dS^{α} inevitably reduce in necessity of input of the fractal α-characteristic $d^{\alpha}J(P)$ on a unit dS. Thus,

$$d^{\alpha}J(r)d^{\alpha}S = j(r)l^{-\alpha}\frac{\mu(\alpha)}{r^{1-\alpha}}dS = j^{\alpha}(r)dS \,, \qquad (1)$$

where $\mu(\alpha)=\Gamma(\alpha)\frac{2\pi^{\alpha/2}}{\Gamma(\alpha/2)}$ is defined by sort of a coating unit; $\Gamma(.)$ - Euler's gamma-function.

Let us consider an emitter as a sphere. Let its surface has fractal properties. A fractal current we set as a thin spherical stratum with radius a. The surface current density $j^{\alpha}(r,\theta)$ does not depend on a lateral angle. Dependence of a current density on a meridional corner θ we set by the α_{θ}-characteristic $D^{\alpha_{\theta}}\Theta(\theta)$, and dependence on radius by the α_r-characteristic $D^{\alpha_r}R(r)$:

$$\vec{j}^{\alpha}(r,\theta)=D^{\alpha_r}R(r)D^{\alpha_{\theta}}\Theta(\theta).$$

At such symmetry the electromagnetic field will have three components, equal to zero, E_φ, H_θ and H_r. On an emitter surface a current density is $j^\alpha(a,\theta) = H_\varphi^\alpha(a,\theta)$, and complete current is $J^\alpha(a,\theta) = 2\pi a \sin\theta\, j^\alpha(a,\theta)$.

For determination of a current density $j^\alpha(r,\theta)$ from Maxwell equations, in view of the indicated symmetry, we obtain the α-characteristic of the magnetic component

$$D^\alpha H_\varphi^\alpha = {}_aD_r^{\alpha_r} R^\alpha(kr)\, {}_0D_\theta^{\alpha_\theta} \Theta^\alpha(\theta),\ {}_aD_r^{\alpha_r} R^\alpha(kr) = \frac{C_n}{\sqrt{r}} H^{(2)}_{n+\frac{1}{2}}(kr),\ {}_0D_\theta^{\alpha_\theta} \Theta^\alpha(\theta) = \frac{d}{d\theta} P_n(\cos\theta)$$

(here: C_n - harmonics amplitude; $H^{(2)}_{n+\frac{1}{2}}(kr)$ - half-integer order Hankel's function; $P_n(\cos\theta)$ – Legendre polynomials.

According to designed model, the electromagnetic field (2) spherical fractal emitters, as well as in case of a classical emitter, represents a space *n*-order harmonic of an electrical wave. At *n*=0 the field is identically equal to zero. It means, that for the indicated model the emitter is impossible to create which isotropicly radiates on all directions.
The field of an emitter with a space first order harmonic comes nearer to the field diagram of a linear dipole fractal emitter with length equal to radius of an orb [7, 8].
In the total, density of a fractal current looks like

$$j^\alpha(r,\theta) = \sum_{n=1}^{\infty} C_n\, {}_0I_{kr}^{\alpha_r}\left(\frac{C_n}{\sqrt{r}} H^{(2)}_{n+\frac{1}{2}}(kr)\right) {}_0I_\theta^{\alpha_\theta}\left(\frac{d}{d\theta} P_n(\cos\theta)\right).$$

RESULTS AND DISCUSSION

In Fig. 1 the radial part $R^\alpha(kr)$ is shown, and in Fig. 2 - meridional part $\Theta^\alpha(\theta)$, which determine a current density on a surface of an ideal spherical emitter (dashed line) and on a surface of an emitter with fractal properties (solid line).

We score the following: 1) offset of maximas of $\Theta^\alpha(\theta)$ curves in the side of major corners θ, that reduces in rotational displacement of a polar pattern (it depends on the value of a scaling metric α); 2) considerable increase of wattless currents, and it reduces in decrease of an efficiency and emitter passband; 3) power reallocation of 2^α-poles in emitters petals with

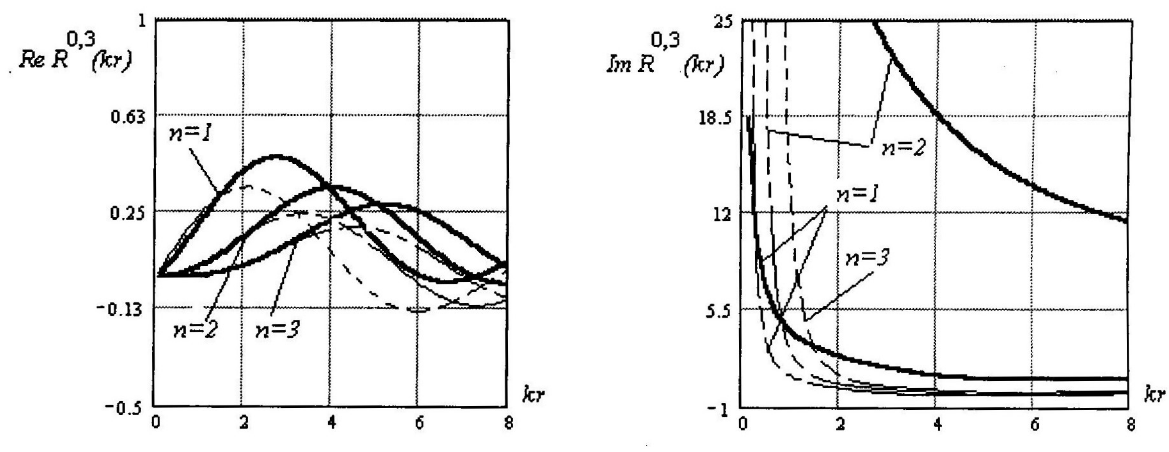

Fig. 1

a higher order space harmonic in comparison with classical multipoles; 4) confirmation of known experimental datas about offset of the characteristics of an actual emitter, that is stipulated by existence 2^{α} –poles in actual physical systems.

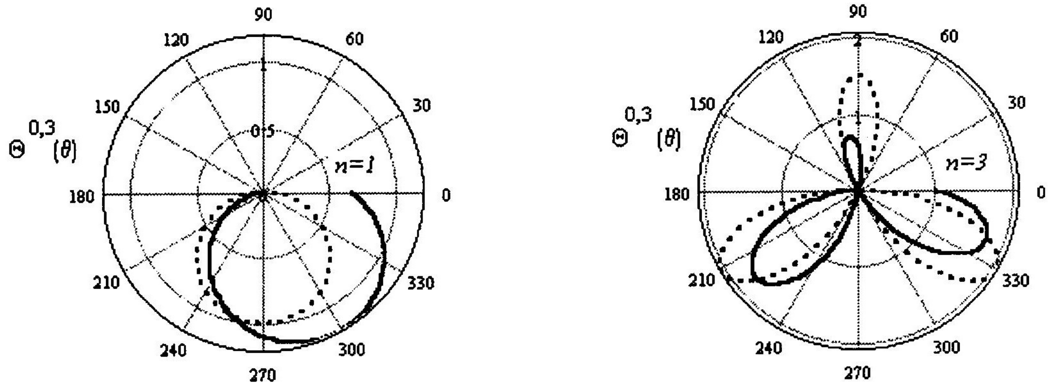

Fig. 2

CONCLUSION

The discussion of calculations results on designed radiation model of a sphere with fractal structure of surface currents allocation points presence of known properties of a classical emitter (alternation of phase-alternating currents; values fissile and wattles currents). There are also new effects (rotational displacement of a polar pattern, power reallocation a 2^{α} –poles etc.). The obtained results allow explaining influence of 2^{α} –poles to the form of a polar pattern, on the value of an antenna factor, and also possibility of designing pencil-beam and isotropic emitter.

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