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Numerical-analytical methods in diffraction theory boundary-value problems

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I. Introduction.

Recent years in diffraction theory (electrodynamics) the three principal ways in solving the boundary-value problems have been conditionally tracked. They correspond to numerical, analytical and numerical-analytical methods.

The numerical methods are considered as the most perspective because of their univesality (in the scope aspect). Last years the series of problems on electromagnetic waves scattering from different obstacles has been solved due them. The starting-point of this approach is formulating an integral equation (IE) in unknowns of the search field or current density distribution on the surface of scatterer. Futher this IE is transformed by means of the moments method to infinite system of linear algebraic equations (ISLAE) which after using the truncation method becomes suitable for computer treatment. As the idea of this method is simple enough its implementation area is spacious.

In contrast to the numerical methods the scope of the analytical methods is not extensive. Unfortunately, the exact solutions can be achieved in not many cases but only when the variables separation technique is feasible. But although for certain class of problems the numerical methods have clear advantages over the analytical ones their applicability becomes questionable for the problems of complicated geometry because of troubles with the computations. The memory volume and computer time reguirements, the errors problems may bee too large. On the contrary, the analytical approach, if it is applicable, provides more effective calculations, gives more accurate results, makes easier their physical interpretation etc.

The numerical-analytical approach may be considered as a combination of analytical and numerical methods. It bases on analytical manipulations of the initial operator equation that the boundary-value problem is reduced to. The new ISLAE derived is much better for computering due to less size of inverted matrix, feasible estimation of the convergence criterion etc.

Although the numerical-analytical approach compares unfavourable with more universal in view of the scope numerical methods it synthesizes highly efficient algorithms for a certian class of problems. Besides, it can be considered as a tool to be used for the determination of different asymptotical methods validity range. When numerical-analytical methods are applicable then on minimum computer time expenses they output reliable data to be a standard for the testing of different numerical methods accuracy. In particular, such poweful methods as the modificated residue method [1], various modifications of the Viener-Hopf method [2], and so-called semi-inversion method [3-5] are the representatives of numerical-analytical class.

This lecture does not intend to be a presentation of an exhaustive survey of multitude of numerical-analytical methods (for this see the books [2,6]). Its aim is to introduce a general idea only of one of them.

It would seem reasonable to present the semi-inversion method because step-by-step it has been extending chiefly by the scientists of the Institute of Radiophysics and Electronics of Ukrainian Academy of Sciences over the past three decades. In Soviet times these results were familiar to the soviet specialists mainly. In spite of the times considerable number of original works on this subject (only the monographs' number is of a few dosens) they remain unknown to the scolars abroad. They were never translated from Russian.

Because of a large body of the works I wish to present only those having a methodological value and which obtaining I took part in.

It is worthwhile mentioning that the works [7–10] promoting the progress and implementation of the semi-inversion methods present the studies of numerous external and internal diffraction problems. Through the efficient well-conditioned computational algorithms the rigorous investigations have been carried out to study in detailes the features of the fields scattered on following obstacles: on confined axial-/plane-symmetry screens as a strips, circular waveguide cut, disk (Sologub [7], Veliev [8]); on periodic diffraction gratings of various profiles (Kirilenko, Masalov, Shestopalov, Sirenko [10], Veliev [8], Litvinenko [9],), echelette, strip grating, grating of circular/rectangular rods; on waveguide heterogeneities with the partial domains confined by the piece-linear boundaries (Kirilenko, Rud' [10]); on plane waveguide fractures, dielectric wedges in waveguide etc.

The brief scheme of the semi-inversion technique is as follows. Let an initial boundary-value problem be posed in a from of operator equation

$$Lx = B \tag{1}$$

in corresponding space of the functions (this is the space l_2 isually).The principal (singular) part L_0 should be separated from L operator.Then the equation (1) can be rewritten in form

$$(L_0 + L_1)x = B \tag{2}$$

Futher L_0 operator can be analytically inverted, i.e. the explicit form of L_0^{-1} operator is formulated. By acting L_0^{-1} operator on (2) we get the next equation

$$(I + Q)x = f; \qquad Q \Rightarrow L_0^{-1}L_1, \qquad f \Rightarrow L_0^{-1}B \tag{3}$$

Where I is unit operator.

If the right basis set has been found, then the equations (1) and (2) can be represented in a form of ISLAE. As a rule, the ISLAE matrix elements corresponding with the equation (1) are of a slow convergency. This means the computing of ISLAE of a large order, i.e. considerable waste of computer time.

In contrast, the equation (3) is the Fredholm equation of the second kind. And in case when L_0 operator has been found correctly the matrix operator Q can become a quite contiuous one with the fast-decreasing matrix elements. This circumstance provides a fast convergency for the approximate solutions either. Thus for (1)L_0^{-1} plays part of the regularization operator. It is easy to see that the right definition of L_0 operator is the critical point in this approach because this procedure conditions on the effectiveness of the final result. In particular, any operator confroming to the special and limiting values of parameters characterizing the scattering object may be chosen as L_0 operator.

The subsequent paragraphs are dedicated to thr proposed scheme realization for a specific class of boundary-value diffraction problems.

1. Dual series and integral equations.

Agranovich [11] was the first who proposed the semi-inversion method in capasity of effective method solution of dual series equations (DSE) with the kernel in trigonometrical functions appearing on solving the problem of wave diffractin by a flat grating of strips.

Let turn our attention to finding the regularizator for the most simple form of dual series equations (DSE) and dual integral equations (DIE):

$$\begin{cases} \sum\limits_{n=-\infty}^{\infty} \rho_n \gamma_n e^{in\theta\eta} = f_1(\eta), & |\eta| < 1 \\ \sum\limits_{n=-\infty}^{\infty} \rho_n e^{in\theta\eta} = 0, & |\eta| > 1 \end{cases} \tag{4}$$

$$\begin{cases} \int\limits_{-\infty}^{\infty} \rho(\alpha) k(\alpha) e^{i\varepsilon\alpha\eta} d\alpha = f_2(\eta), & |\eta| < 1 \\ \int\limits_{-\infty}^{\infty} \rho(\alpha) e^{i\varepsilon\alpha\eta} d\alpha = 0, & |\eta| > 1 \end{cases} \tag{5}$$

Here $\{\rho_n\}_{n=-\infty}^{\infty}$ and $\rho(\alpha)$ unknowns are the Fourier coefficients for the functions conforming to the electromagnitic field component or the surface current density; h is normalized coordinate; θ and ε are the geometrical parameters of the scattering object; $\{f_j\}_{j=1}^{2}$ are the given continuous functions specified by the incident field. The given complex values $\{\gamma_{|n|}\}_{n=-\infty}^{\infty}$ and $k(\alpha)$ may depend on wave-dimension of the object. Tables 1 and 2 list these parameters data corresponding to such problem of plane E-polarized wave

$$E^0 = e^{i(\alpha_0 kx + ky\sqrt{1-\alpha_0^2})}, \qquad k = \frac{2\pi}{\lambda}; \qquad \alpha = \cos\varphi_i \tag{6}$$

as scattering on the flat grating of strips, non-closed circular cylinder, flat strip, and for the problem of electrical dipole field excitation of circular waveguide cut respectively.

In particular, the only non-zero component of the electromagnetic field (for (b) problems see Table 1, for (c) see Table 2) can be put in the form [5,12]

$$E_z = E_z^0 + \sum_{n=\infty}^{\infty} \rho_n \begin{Bmatrix} J_n(ka) H_n^{(1)}(kr), \\ J_n(kr) H_n^{(1)}(ka), \end{Bmatrix} e^{in\theta\eta} \begin{matrix} r > a \\ r < a \end{matrix} \tag{7}$$

$$E_z = E_z^0 + \frac{i}{4\pi} \int\limits_{-\infty}^{\infty} \rho(\alpha) k(\alpha) e^{i\varepsilon[\alpha\eta + \xi\sqrt{1-\alpha^2}]} d\alpha \qquad \xi > 0 \tag{8}$$

Here

$$\rho_n = \int\limits_{-1}^{1} \rho(\eta) e^{-in\theta\eta} d\eta; \qquad \rho(\alpha) = \int\limits_{-1}^{1} \rho(\eta) e^{-i\varepsilon\alpha\eta} d\eta$$

where $\rho(\eta)$ is the current density function appearing due to the field incidence on the scattering screens surfaces; $J_n(x)$ and $H_n(x)$ are the Bessel and Hankel functions respectively; $\varepsilon = ka$

It is easy to see that the imposing of the Dirichlet boundary condition on the field (7,8) on the scattering screens surfaces yields DSE like (4) and DIE like (5).

Before setting forth an idea of the semi-inversion technique conformably to DSE (4) and DIE (5) it is noteworthy to emphasize that these equations correspond to integral

Table 1

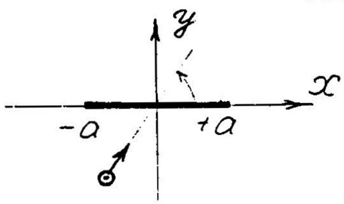
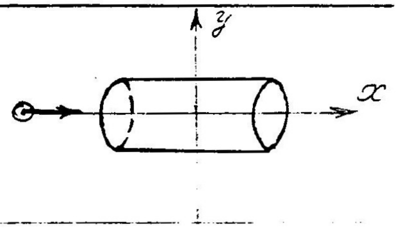
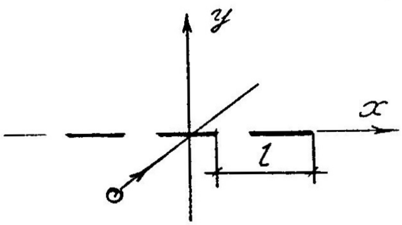
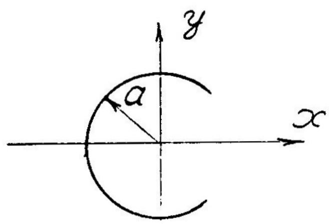
$K(\alpha) = (1-\alpha^2)^{-\frac{1}{2}}$ $= \frac{c}{\lvert\alpha\rvert}[1-\varepsilon(\lvert\alpha\rvert)]$	$K(\alpha) = g^2 J_0(g) H_0^{(1)}(g)$ $= \frac{i}{\pi}\lvert\alpha\rvert[1-\varepsilon(\lvert\alpha\rvert)]$; $g^2 = (ka)^2 - \alpha^2$
	

Table 2

$\gamma_n = k\sqrt{-1+(\frac{n}{\chi})^2}$ $= k\frac{\lvert n\rvert}{\chi}(1+\varepsilon_{\lvert n\rvert})$; $\chi = \frac{l}{\lambda}$	$\gamma_n = J_n(ka)H_n^{(1)}(ka)$ $= \frac{c}{\lvert n\rvert}[1+\varepsilon_{\lvert n\rvert}]$
	

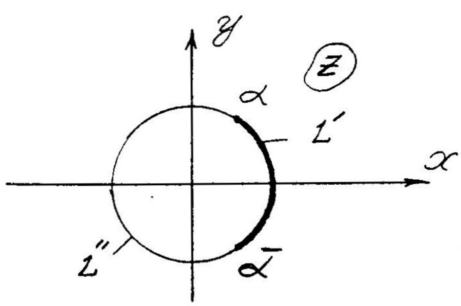


Fig.1

equations (IE) with the difference kernels. For the problems of wave scattering on a flat strip and nonclosed circular cylindrical screen these IE are respectively as following [12]:

$$\int_{-1}^{1} \rho(\xi) H_0^{(1)}\left(\varepsilon|\xi-\eta|\right) d\xi = f_1(\eta), \qquad \eta \in [-1,1] \tag{9}$$

$$\int_{-\alpha}^{\alpha} \rho(\varphi) H_0^{(1)}\left(2\varepsilon\left|\sin\frac{\varphi-\varphi_0}{2}\right|\right) d\varphi = f_2(\varphi_0), \qquad \varphi_0 \in [-\alpha,\alpha] \tag{10}$$

In order to go from IE (9,10) to DSE (4) and DIE (5) the unknown current functions $\rho(\xi)$ should be expanded in the Fourier-type integral and Fourier series, and it is necessery to use the following represenations for the kernels of IE (9,10) [12]:

$$H_0^{(1)}\left(\varepsilon|\xi-\eta|\right) = \frac{i}{\pi}\int_{-\infty}^{\infty} e^{i\varepsilon\alpha(\xi-\eta)}\frac{d\alpha}{\sqrt{1-\alpha^2}}$$

$$H_0^{(1)}\left(2\varepsilon\left|\sin\frac{\varphi-\varphi_0}{2}\right|\right) = \sum_{n=-\infty}^{\infty} J_n(\varepsilon) H_n^{(1)}(\varepsilon) e^{in(\varphi-\varphi_0)} \tag{11}$$

The functions $\rho(\xi)$ and $\rho(\varphi)$ are extended by zero outside the region of their definition, i.e. $\rho = 0$ if $|\xi| > 1$, $\rho = 0$ if $\varphi \notin [-\alpha,\alpha]$.

IE (9,10) belong to very significant class of IE, namely, to IE with the log-difference kernel. In view of the following definition for the zero-order Hankel function

$$H_0^{(1)}(x) = \frac{2i}{\pi}\ln x + N(x), \qquad x > 0 \tag{12}$$

the IE (9,10) may be rewritten as follows:

$$\int_{-1}^{1} \rho(\xi)\left(\ln|\xi-\eta| d\xi + N_1(\xi,\eta,\varepsilon)\right) d\xi = f_1(\eta), \qquad \eta \in [-1,1] \tag{13}$$

$$\int_{-\alpha}^{\alpha} \rho(\varphi)\left[\ln 2\left|\sin\frac{\varphi-\varphi_0}{2}\right| + N_2(\varphi,\varphi_0,\xi)\right] d\varphi = f_2(\varphi_0), \qquad \varphi_0 \in [-\alpha,\alpha] \tag{14}$$

Let proceed now to finding the regularizations for DSE (4) and DIE(5). For the account simlisity reason the development will be continued for DSE(4) only. Let $\gamma_{|n|}$ values be in the form:

$$\gamma_n = \frac{C}{|n|}\left[1 + \varepsilon_{|n|}\right], \qquad \varepsilon_{|n|} \underset{n\to\infty}{\sim} O(N^{-1-s}) \tag{15}$$

The ε data are listed in Table 1 in an instance of (a) and (b) cases. The explicit form of ε shows that in order to get a concrete expsession of (15) for the particular scattering problem we need in asymptotes of γ when $|n| \to \infty$.

On account of (15) the DSE (4) may be rewritten:

$$\begin{cases} \sum_{n\neq 0}^{\infty} \frac{\rho_n}{|n|} e^{in\theta\eta} = \frac{1}{c}(f_1(\eta) - \rho_0\gamma_0) \sum_{n\neq 0}^{\infty} \frac{\rho_n}{|n|} \varepsilon_n e^{in\theta\eta}, & |\eta| < 1 \\ \sum_{n=-\infty}^{\infty} \rho_n e^{in\theta\eta} = 0, \quad |\eta| > 1 \end{cases} \tag{16}$$

Notice that if DSE (4) is equivalent to IE (10), then DSE (16) is equivalent to (14). This can be proved. Then the splitting of the DSE (4) operator with reference to γ values on using (15) is equivalent to the separation of the singular (log-difference) part from the operator of IE (10). This brings us a conclusion that in the semi-inversion technique for DSE(4) and DIE(5) L_0 operator is equivalent to the singular parts of the corresponding equations operators. The from of the inverted operator L_0^1 for DSE(16) can be effectively synthesized by the Riemann-Hilbert problem method [3].

Proceeding in this way let us prove that DSE(16) can formulate the Riemann-Hilbert problem on the reconstruction of an analytical function of complex variable via the limiting data of this function belonging some controur. Let us prove this sttement in the following way.

After differentiation of equation (16a) with respect to η and atseparate considera-tion of $\eta = 0$ case we get the new system:

$$\begin{cases} \sum_{n\neq 0} \frac{|n|}{n} \rho_n e^{in\theta\eta} = \Gamma'(\eta) & |\eta| < 1 \\ \sum_{n\neq 0} \frac{\rho_n}{|n|} = \Gamma(0) & \eta = 0 \\ \sum_{n=-\infty}^{\infty} \rho_n e^{in\theta\eta} = 0 & |\eta| > 1 \end{cases} \tag{17}$$

where

$$\Gamma(\eta) = \frac{1}{c}(f_1(\eta) - \rho_0\gamma_0) - \sum_{n\neq 0} \frac{\rho_n}{n} \rho_n \varepsilon_n e^{in\eta} = \sum_{n=-\infty}^{\infty} \Gamma_n e^{in\theta\eta} \tag{18}$$

Let now introduce the ancillary functions

$$X^+(z) = \sum_{n>0} \rho_n z^n, \qquad X^-(z) = -\sum_{n<0} \rho_n z^n$$

that are holomorphic functions inside and outside unit circle $|z = 1|$ on complex variable plane $z = je^{i\psi}$. According to (17) the following relations

$$\begin{cases} X^+(e^{i\psi}) + X^-(e^{i\psi}) = \Gamma^+(e^{i\psi}) & \psi = \theta\eta, \quad \psi \in L' \\ X^+(e^{i\psi}) - X^-(e^{i\psi}) = 0 & \psi \in L'' \end{cases} \tag{19}$$

are valid

So, the limiting values of X^+ and X^- are the Same for L'' arc (Fig.1) of the unit circle. This means that one of them analytically becomes another one over L'' arc, i.e. both of them are the same analytical al function $X(z)$. On the added L'' arc the limited values of $X(z)$ function are connected with one anoter through the relationship (19) that is the relationship for $X(z)$ function reconstruction [13]

$$X(z) = \frac{1}{2\pi i}\frac{1}{R(z)}\int\limits_{L'}\frac{\Gamma(t)R^+(t)}{t-z}dt + \frac{C}{R(z)}, \tag{20}$$

where C is a constant,

$$R(z) = \begin{cases} 0, & z \in L'' \\ \sqrt{(t-\alpha)(t-\alpha^*)}, & \alpha = e^{i\theta}, \quad \alpha^* = e^{-i\theta} \quad z \in L' \end{cases} \tag{21}$$

$$R(z)^{\pm} = \pm R(z), \qquad z \to z_0 \in L'$$

As follows from (19) and (20) the function $X^+(e^{i\psi}) - X^-(e^{i\psi}) = \sum\limits_{(n)}\rho_n e^{in\psi}$ has the rootsingularity at the end points of L' controur. Because of proportionality of this function value to the surface current density for the problems under consideration it may be sure that the function X satisfies the edge condition.

In order to obtain the Fourier coefficients for the function $\Gamma(\eta)$ it is recommended to emply the plemelj-Sokhotskii formulas [13] for the extreme values of functions represented by the Cauchy-type integrals.

As a result of the C constant deduction from (20) and on using the Fourierexpansion of $\Gamma(\eta)$ function (18) and equation (17b) we obtain ISLAE of the second kind in $\rho(\eta)\}_{n=-\infty}^{\infty}$:

$$\rho_n = \sum_{(m)}\rho_m Q_{mn} + b_n, \qquad n = 0, \pm 1\ldots \tag{22}$$

where

$$Q_{mn} \sim \frac{\varepsilon_m}{\gamma_n}T_{mn}(\eta), \quad . \quad b_n \sim \frac{1}{\gamma_n}\sum_m f_m T_{mn}(\eta)$$

$$T_{mn}(\eta) = \begin{cases} \frac{1}{m}V_{m-1}^{n-1}(\eta), & m \neq 0 \\ \frac{1}{n}V_{n-1}^{-1}(\eta), & m = 0, n \neq 0, \qquad U = \cos\theta \\ -2\ln\frac{1+U}{2}, & m = n = 0 \end{cases}$$

$$V_m^n = \frac{1}{2\pi}\int\limits_{L'}\frac{V_n(e^{i\psi},\theta)}{R^+(e^{i\psi})}e^{-im\psi}; \qquad V_n(t_o,\theta) = \frac{1}{i\pi}P.V.\int\limits_{L'}\frac{R^+(t)t^n}{t-t_0}dt$$

$$V_m^n(U) = \frac{m+1}{2(m-n)}\left[P_m(U)P_{n+1}(U) - P_{m+1}(U)P_n(U)\right] \tag{23}$$

where $P_m(\eta)$ are the Legendre polynomials.

ISLAE (22) is equivalent to DSE(16) and is a result of the application of the regularization procedure to DSE(16). This system of equations can be effectively solved by the reduction method.

It seems reasonable to address the questions: What is the most important feature of the Riemann-Hilbert problem method. The way of answering this question is to turn to the fact thatDSE(16) is equivalent to IE(11). It is known that this class of IE can be reduced to the IE of Fredholm the second kind by means of the inversion procedure imposed on the log-difference part of the operator. This procedure is known enough in mathematics [13]. In the useal way this is a formal procedure and the obtained equations kernels are intricated enough and represented by the integrals in the sense of the main value. In the contrast of the methods the Riemann-Hilbert problem method brings us the ISLAE of the second kind with matrix elements characterized by the fact that all the integrals in their expressions can be caleulated though the quadrature technique and have a simple apperance. This allows to synthesize highly efficient algorithms for calculations. This is a clear advantage of this method.

As has been pointed before the scattering characteristics of numerous structures had been calculated on the basis of theRiemann-Hilbert problem method. As an example the frequency dependencies of total scattering cross-section σ and near- and far-field distributions both for the flat and cylindrical strips are shown in Fig. 2,3,4.

2. Method of orthogonal polynomials (O.P.)

2.1. Method of O.P. for IE with log-difference kernel.

Let try now to solve IE (13), (14) by means of more general and quite symple method. Let call it the method of O.P. This is the particular implentation of general scheme of the moments method (M.M.) because it differs from the last by two considerations. It needs firstly to investigate preliminary the structure of solution near the edge point of the domain of integration and secondly to construct spectral expressions for singular parts of the kernels with orthogonal polyunomials as eigenfunctions.

The method of O.P. is widely used for solution of problems of the theory of elastisity and continuous media mechanics [14,15]. Nevertheless, it has not yet found applications in electromagnetic theory.

It will be shown below that the method of O.P. has the power to formulate the solutions to DSE (4) and DIE (5).

At first let us consider IE (13). Assume that the solution structure be such that $\rho(\xi)$ satisfies the condition

$$\rho(\xi) \underset{\xi_1 \pm 1}{\sim} (1-\xi^2)^{-\frac{1}{2}} \tag{24}$$

That means $\rho(\xi)$ has square-root singularity at the ends of interval $[-1,1]$. It is well known that condition (24) follows from Meixner condition in diffraction problems.

Let us unite the unknown function $\rho(\xi)$ as follows

$$\rho(\xi) = (1-\xi^2)^{-\frac{1}{2}} \sum_{n=-\infty}^{\infty} \rho_n T_n(\xi) \tag{25}$$

where $\{T_n(\xi)\}_{n=0}^{\infty}$ are Chebyshev polynomials of the 1-st kind, ρ_n are the coefficients to be found out.

The expansion (25) for $rho(\xi)$ is caused by the fact that Chebyshev polynomials $T_n(\xi)$ are eigen-functions of the following intefral operator (I.O.) [14,15]

$$-\frac{1}{\pi}\int_{-1}^{1} \frac{T_n(\xi)}{\sqrt{1-\xi^2}} \ln|\xi-\eta| d\xi = \omega_n T_n(\eta), \qquad |\eta| \le 1 \tag{26}$$

Here $\{\omega_n\}_{n=0}^{\infty}$ are eigenvalues of I.O. and are given by

$$\omega_n = \begin{cases} \ln 2, & n=0 \\ \dfrac{1}{n}, & n=1,2,\ldots \end{cases} \tag{27}$$

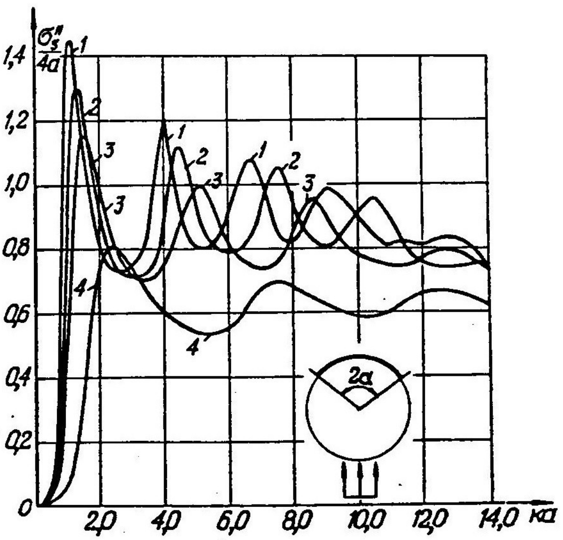


Fig.2 1. $\alpha = 80^o$; 2. $\alpha = 70^o$; 3. $\alpha = 60^o$; 4. $\alpha = 40^o$

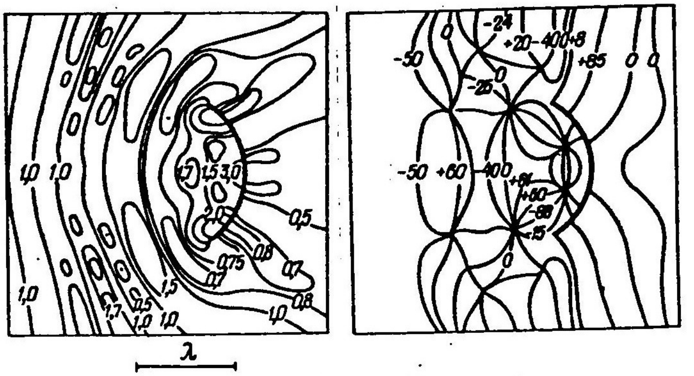


Fig.3 $\alpha = 60^o$; $ka = 5.0$ ($H_z = const$)

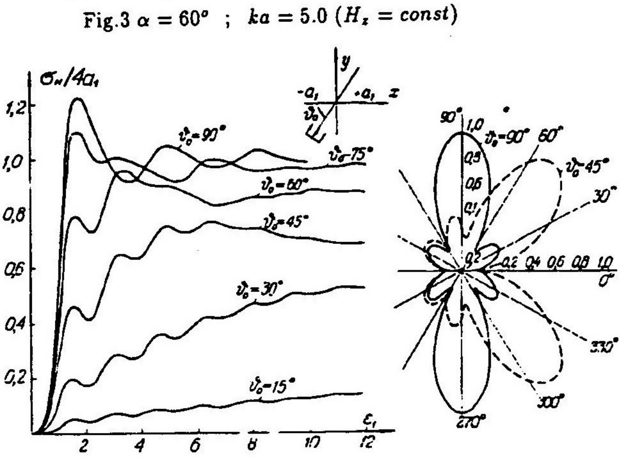


Fig. 4

For further derivations we need orthogonally condition for Chenyshev polynomials given by

$$\int_{-1}^{1} T_n(\eta)T_n(\eta)\frac{d\eta}{\sqrt{1-\eta^2}} = \frac{1}{\beta_n}\pi\delta_{kn}; \qquad \beta = \begin{cases} 1, & n=0 \\ 2, & n \neq 0 \end{cases} \tag{28}$$

Using (26) and (28) one can show the kernel of (26) to have a convergent in definite sense bilimear expansion

$$\frac{1}{\pi}\ln\frac{1}{|\xi-\eta|} = \frac{1}{\pi}\sum_{n=0}^{\infty}\omega_n T_n(\xi)T_n(\eta) \tag{29}$$

Substituting (25) into I.E. (13), let us take into account spectral expression (26) Then assuming the continuous function to $F_1(n)$ be represented by the series expression

$$f_1(\eta) = \sum_{n=0}^{\infty} f_n T_n(\eta) \tag{30}$$

and using orthogonality condition (28) for coefficients ρ_k seeking, we obtain infinite system of linear algebraic equations (SLAE)

$$\rho_k - \sum_{n=0}^{\infty} a_{kn}\rho_n = \gamma_k, \qquad k = 0, 1 \tag{31}$$

Here

$$a_{kn} = \frac{\beta_k}{2\omega_k}\int_{-1}^{1}\int_{-1}^{1}\frac{T_n(\xi)T_n(\eta)}{\sqrt{1-\xi^2}\sqrt{1-\eta^2}}N_1(\xi,\eta,\varepsilon)d\eta d\xi \tag{32}$$

$$\gamma_k = \frac{\pi f_k}{(2\omega_k)}$$

It may be shown that $\{\gamma_k\}_{k=0}^{\infty} \in l_2$ and besides

$$\sum_{n=0}^{\infty}\sum_{k=0}^{\infty}|a_{kn}|^2 < \infty \tag{33}$$

It means that matrix $\{a_{kn}\}_{k=0}^{\infty}$ produced a fully continuous operator A, from l_2 to l_2. Consequently, this operator may be approximated by a finite-dimensional one, in other words SLAE (31) may be treated by means of truncation.

Let proceed to IE (14). For the simplicity reason the unknown function $\rho(\varphi)$ and the given function $f_2(\varphi)$ are assumed to be even.

Then for the functions $\rho(\varphi)$ we can write

$$\rho(\varphi) = \frac{\cos\frac{\varphi}{2}}{\sqrt{2(\cos\varphi - \cos\alpha)}}\sum_{n=0}^{\infty} x_{2n}T_{2n}\left(\frac{\sin\frac{\varphi}{2}}{\sin\frac{\alpha}{2}}\right), \qquad \varphi \in [-\alpha, \alpha] \tag{34}$$

As before, the form of expansion (22) is caused by the fact that Chebyshev polynomials $T_{2n}\left(\sin\frac{\varphi}{2}/\sin\frac{\alpha}{2}\right)$ are eigen functions of I.O. corresponding to the singular part of the IE kernel (14). As has been shown in there exist the following spectral expressions

$$\frac{1}{\pi}\int_{-\alpha}^{\alpha}\ln\frac{1}{2|\sin\frac{\varphi-\varphi_0}{2}|}\frac{T_{2n}\left(\frac{\sin\frac{\varphi}{2}}{\sin\frac{\alpha}{2}}\right)}{\sqrt{2(\cos\varphi-\cos\alpha)}}\cos\frac{\varphi}{2}d\varphi=\sigma_{2n}T_{2n}\left(\frac{\sin\frac{\varphi_0}{2}}{\sin\frac{\alpha}{2}}\right) \tag{35}$$

Here $\{\sigma_{2n}\}_{n=0}^{\infty}$ are eigenvalues of I.O., equal to

$$\sigma_{2k}=\begin{cases}-\ln\sin\frac{\alpha}{2}, & n=0\\ \frac{1}{2n}, & n=1,2,\dots\end{cases} \tag{36}$$

Besides it shown be noted that for Chebyshev polynomials $T_{2n}\left(\sin\frac{\varphi}{2}/\sin\frac{\alpha}{2}\right)$ the following orthogonality conditions are valid

$$\int_{-\alpha}^{\alpha}T_n\left(\frac{\sin\frac{\varphi}{2}}{\sin\frac{\alpha}{2}}\right)T_k\left(\frac{\sin\frac{\varphi}{2}}{\sin\frac{\alpha}{2}}\right)\frac{\cos\frac{\varphi}{2}}{\sqrt{(\cos\varphi-\cos\alpha)}}d\varphi=\frac{1}{\beta_k}\pi\delta_{nk}, \tag{37}$$

The krenels of IE (35) may be shown to have the following bilinear expansions

$$\ln\frac{1}{2|\sin\frac{\varphi-\varphi_0}{2}|}=\sum_{n=0}^{\infty}\sigma_{2n}\beta_nT_{2n}\left(\frac{\sin\frac{\varphi}{2}}{\sin\frac{\alpha}{2}}\right)T_{2n}\left(\frac{\sin\frac{\varphi_0}{2}}{\sin\frac{\alpha}{2}}\right) \tag{38}$$

Now let us substitute expressions (34) into IE (14). Taking into account spectral expressions (35) and orthogonality (37) we obtain an infinite SLAE for seeking unknown coefficients

$$x_{2k}+\sum_{n=0}^{\infty}b_{2k,2n}x_{2n}=f_{2k}^{+},\qquad k=0,1,\dots \tag{39}$$

Here we have denoted

$$b_{2k,2n}=$$
$$=\frac{\beta_k}{8\sigma_{2k}}\int_{-\alpha}^{\alpha}\int_{-\alpha}^{\alpha}\frac{\cos\frac{\varphi}{2}\cos\frac{\varphi_0}{2}T_{2k}\left(\frac{\sin\frac{\varphi}{2}}{\sin\frac{\alpha}{2}}\right)T_{2n}\left(\frac{\sin\frac{\varphi_0}{2}}{\sin\frac{\alpha}{2}}\right)}{\sqrt{(\cos\varphi-\cos\alpha)(\cos\varphi_0-\cos\alpha)}}N_2(\varphi,\varphi_0;\varepsilon)d\varphi d\varphi_0$$

$$f_{2k}^{+}=\frac{\beta_k}{4\sigma_{2k}}\int_{-\alpha}^{a}\frac{f^{+}(\varphi_0)\cos\frac{\varphi_0}{2}}{\sqrt{2(\cos\varphi_0-\cos\alpha)}}T_{2k}\left(\frac{\sin\frac{\varphi}{2}}{\sin\frac{\alpha}{2}}\right)d\varphi_0$$

It may be shown that

$$\sum_{k=0}^{\infty}\sum_{n=0}^{\infty}|b_{2k,2n}|^2<\infty;\qquad\sum_{k=0}^{\infty}|f_{2k}^{+}|^2<\infty \tag{40}$$

In other words, matrix operator B produced by the matrix $\{b_{2k,2n}\}_{k=0}^{\infty}$ is an absolutely continuous one and may be approximated by finite-dimensional operator. Then the solution of infinite SLAE (39) may be obtained with any given prefixed accuracy by the truncation method.

The universality of the method of O.P. resides in the opportunity to construct the spectral relationships for different singular kernels where O.P. play part of the eigenfunctions. As an example it seems reasonable to cite the following spectral relationships [14,15] :

$$\frac{2}{\pi}\int_{-1}^{1} \ln\frac{1}{|x-y|}\frac{U_n(y)}{(1-y^2)^{\frac{1}{2}}}dy = \begin{cases} \ln 2 - \frac{1}{2}T_2(x), & n=0 \\ \frac{T_n(x)}{n} - \frac{T_{n+2}(x)}{n+2}, & n>0 \end{cases} \tag{41}$$

$$\frac{d^2}{dx^2}\int_{-1}^{1}\sqrt{1-y^2}\ln\frac{1}{|x-y|}U_n(y)dy = -\pi(n+1)U_n(x) \tag{42}$$

Here $U_n(y)$ are the Chebyshev's polynomials of the second kind.

$$\frac{i}{2}\int_{-1}^{1}\frac{Ce_n(\arccos\xi,g)}{\sqrt{1-\xi^2}}H_0^{(2)}(\varepsilon|\xi-\xi'|)\,d\xi = \frac{Me_n^{(2)'}(0,q)}{Me_n^{(2)}(0,q)}Ce_n(\arccos\xi',g) \tag{43}$$

$q=\frac{\varepsilon^2}{4}$, $\{Ce_n(\arccos\xi,g\}_{n=0}^{\infty}$ are the Mathieu's functions.

$$\int_{0}^{\infty}\frac{K_0(|x-y|)}{\sqrt{y}e^y}L_n^{-\frac{1}{2}}(2y)dy = \frac{\pi}{\sqrt{2}}\frac{\Gamma(n+\frac{1}{2})}{\Gamma(n+1)}L_n^{-\frac{1}{2}}(2x)e^{-x} \tag{44}$$

Here $\{L_n^{-\frac{1}{2}}(2x)e^{-x}\}_{n=0}^{\infty}$ are the Lagerr polynomials; $K_0(x)$ is the MacDonald's function.

$$\int_{-1}^{1}\frac{C_n^{\frac{\nu}{2}}(y)dy}{|x-y|^y\sqrt{(1-y^2)^{1-\nu}}} = \pi\frac{\Gamma(n+\nu)}{\Gamma(n+1)}\frac{C_n^{\frac{\nu}{2}}}{\Gamma(\nu)\cos\frac{\pi\nu}{2}} \tag{45}$$

These spectral relation ship may be used in solving the different scattering problems with both the Dirichlet and Neumann boundary conditions. In particular, in view of (44) IE of the Viener-Hopf type can be solved in explicit form.

2.2 Solution of DSE and DIE by the method of O.P.

Let show that DSE(4) and DIE (5) can be solved by the method of O.P. For simplicity reason it is worth considering only DIE (5). Solution of this system is formulated on more general assumptions. Namely, $\rho(\alpha)$ function is to be the Fourier transform of $\rho(h)$ function that satisfies the condition

$$\rho(\eta) \underset{\eta\to\pm1}{\sim} (1-\eta^2)^{\nu-1}, \qquad \frac{1}{2}\le\nu<1 \tag{46}$$

At $\nu = \frac{1}{2}$ function ρ_n satisfies the classical Meiksner condition for infinitesimally thin screens. In particular, condition (46) appears in the problem of wave scattering on cylindrical object with the edges.

So, to DIE (5) is formulated on the following assumptions:

i) the unknown function $\rho(\alpha)$ is the Fourier transform of $\rho(\eta)$ function satisfying condition (46):

ii) $k(\alpha)$ function may be represented in the form:

$$k(\alpha) = \frac{c}{|\alpha|}\left[1 - \delta(|\alpha|)\right], \qquad \delta(|\alpha|) \underset{|\alpha|\to\infty}{\sim} O|\alpha|^{-1-s}, \qquad s > 0 \tag{47}$$

iii) the given function $f_2(\eta)$ is continuous and $f'(\eta) \in L_2[-1,1;(1-\eta^2)^{\nu-1}]$; here $L_2[-1,1;(1-\eta^2)^{\nu-1}]$ is the Hilbeert space where scalar product is defined with weight factor $(1-\eta^2)^{\nu-1}$.

In order to satisfy (46) the function $\rho(\eta)$ is represented as uniformly converging series

$$\rho(\eta) = (1-\eta^2)^{\nu-1}\sum_{m=0}^{\infty}\rho_m C_m^{\nu-\frac{1}{2}}(\eta), \qquad \frac{1}{2} \le \nu < 1 \tag{48}$$

where ρ_m are the un,nowns coefficients; $C_m^{\nu-\frac{1}{2}}(\eta)$ are the Gegenbaier polynomials which set a basis in $L_2[-1,1;(1-\eta^2)^{\nu-1}]$.

In view of (48) we get the following representation for the Forier transform:

$$\rho(\alpha) = \int_{-1}^{1}\rho(\eta)e^{-i\varepsilon\alpha\eta}d\eta =$$

$$= \frac{2\pi}{\Gamma(\nu-\frac{1}{2})}\sum_{m=0}^{\infty}(-i)^m\rho_m\beta_m^{(\nu-\frac{1}{2})}\frac{J_{m+\nu-1/2}(\varepsilon\alpha)}{(2\varepsilon\alpha)^{\nu-1/2}}, \qquad \nu > \frac{1}{2} \tag{49}$$

where

$$\beta_m^{(\nu-1/2)} = \frac{\Gamma(m+2\nu-1)}{\Gamma(m+1)}$$

Let substitute (49) and (47) into DIE (5) taking inti account that the continuous functions $e^{i\varepsilon\alpha\eta}$ and $f_2(\eta)$ in this equation may be represented as expansions in terms of Gegenbauer polynomials

$$e^{i\varepsilon\alpha\nu} = \left(\frac{2}{\varepsilon\alpha}\right)^{\nu-1/2}\Gamma\left(1-\frac{1}{2}\right)\sum_{k=0}^{\infty}i^k\left(k+\nu-\frac{1}{2}\right)J_{k+\nu-\frac{1}{2}}(\varepsilon\alpha)C_k^{\nu-\frac{1}{2}}(\eta); \quad \frac{1}{2} < \nu < 1$$

$$f_2(\eta) = \sum_{k=0}^{\infty}f_k C_k^{\nu-\frac{1}{2}}(\eta) \tag{50}$$

Then after interchanging the orders of summation and integration in deduced equation and using the discontinuous integrals by Weber-Schafheitlin [16] we may conclude that, first, uniform equation of DIE system (5) is satistied identically and, second, for definition of unknown $\{\rho_n\}_{n=0}^{\infty}$ the infinite SLAE of the following taking place:

$$\sum_{m=0}^{\infty}Z_m\left[1+(-1)^{k+m}\right]N_{km}^{(\nu-\frac{1}{2})} = \Gamma_k, \qquad k = 1,2,\ldots, \tag{51}$$

The following notation

$$Z_m = (-i)^m \rho_m \beta_m^{(\nu-\frac{1}{2})} \frac{2\pi}{\Gamma\left(\nu-\frac{1}{2}\right)}; \qquad K_{\nu-\frac{1}{2}}(\varepsilon) = \left(\frac{2}{\varepsilon}\right)^{2\nu-1} \frac{2\Gamma^2\left(\nu+\frac{1}{2}\right)}{\Gamma(2\nu)}$$

$$N_{km}^{(\nu-\frac{1}{2})} = \begin{cases} N_{00}^{(\nu-\frac{1}{2})} = 2\int\limits_0^\infty J^2_{\nu-\frac{1}{2}}(\varepsilon\alpha)\frac{d\alpha}{\alpha^{2\nu-1}\sqrt{1-a^2}}, & k=m=0 \\ i\left(C_{km}^{(\nu-\frac{1}{2})} - d_{km}^{(\nu-\frac{1}{2})}\right), & k+m\neq 0 \end{cases}$$

$$C_{km}^{(\nu-\frac{1}{2})} = K_{\nu-\frac{1}{2}}(\varepsilon)\int\limits_0^\infty J_{k+\nu-\frac{1}{2}}(\varepsilon\alpha)J_{m+\nu-\frac{1}{2}}(\varepsilon\alpha)\frac{d\alpha}{\alpha^{2\nu}} =$$

$$= \frac{\Gamma^2\left(\nu+\frac{1}{2}\right)\Gamma\left(\frac{k+m}{2}\right)}{\Gamma\left(\nu+\frac{m-k+1}{2}\right)\Gamma\left(\nu-\frac{k-m+1}{2}\right)\Gamma\left(2\nu+\frac{k+m}{2}\right)}, \qquad k+m\neq 0$$

$$d_{km}^{(\nu-\frac{1}{2})} = K_{\nu-\frac{1}{2}}(\varepsilon)\int\limits_0^\infty \delta(\alpha)J_{k+\nu-\frac{1}{2}}(\varepsilon\alpha)J_{m+\nu-\frac{1}{2}}(\varepsilon\alpha)\frac{d\alpha}{\alpha^{2\nu}}, \qquad k+m\neq 0$$

$$\Gamma_k \sim \frac{f_k}{k-\nu-\frac{1}{2}}$$

It can be proved that the ISLAE (51) is the Fredholm equation of the second kind, and its approximate solution can be obtained by the truncation method to an arbitrary accuracy.

Conclusions.

Comparision of the nethods solutions to DSE (4) and DIE (5) on the basis of the Riemann-Hilbert problem method with the method of O.P. gives a clear evidence that the last is simple enough and more general. This shows its worth through, in particular, the fact that the first is the method solution only for the scattering problems where $\rho(\eta)$ function satisfies the condition (46) at $\nu = \frac{1}{2}$.

Pay attention that the problem of wave scattering on polygonal cylinder has been investigated over a wide range of the parameters data by means of the method of O.P.

As well the method of O.P. holds a key to solution of IE with the kernel of the Bessel functions. These equations appear in the problems of wave scattering on the screens like a disk.

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