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ANALYTICAL-NUMERICAL APPROACH FOR THE SOLUTION OF THE DIFFRACTION BY A RESISTIVE STRIP: THE CASE OF *H* POLARIZATION

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1. Introduction

The problem of diffraction by resistive strips has received much attention recently in connection with the radar cross section (RCS) reduction of targets. This structure serves as a suitable model of thin dielectric slabs and coating of finite length. Some of the resistive/impedance strip problems have been analyzed thus far by means of high-frequency and numerical techniques [1, 2]. In [3], we have rigorously solved the E-polarized plane wave diffraction by a resistive strip using the analytical-numerical approach [4] which is entirely different from the previous methods employed to treat the impedance-related problems. The purpose of this paper is to analyze the diffraction problem involving the same strip geometry as in [3] for the H-polarized plane wave incidence. The method of solution is again based on the analytical-numerical approach.

Applying the boundary condition to an integral representation of the scattered field, the problem is formulated as an integral equation satisfied by the unknown current density function. Expanding the current density function in terms of the Gegenbauer polynomials by taking into account the edge condition, our problem is reduced to the solution of an infinite system of linear algebraic equations (SLAE) satisfied by the unknown expansion coefficients. These coefficients are determined numerically with high accuracy via truncation of the SLAE. The scattered field is evaluated asymptotically and the far field expression is derived. Numerical results on the total scattering cross section as well as the monostatic and bistatic RCS are presented and the far field scattering characteristics are discussed.

The time factor is assumed to be $e^{-i\omega t}$ and suppressed throughout this paper.

2. Formulation of the Problem

We consider the H-polarized plane wave diffraction by a resistive strip of zero thickness as shown in Fig. 1, where the H polarization implies that the incident magnetic field is parallel to the z-axis. Let the total magnetic field $H_z(x,y)$ be

$$H_z(x,y) = H_z^i(x,y) + H_z^s(x,y), \tag{1}$$

where $H_z^i(x,y)$ is the incident field given by

$$H_z^i(x,y) = e^{-ik(x\cos\theta + y\sin\theta)}, \quad 0<\theta<\pi \tag{2}$$

with $k\,[\equiv \omega(\varepsilon_0\mu_0)^{1/2}]$ being the free-space wavenumber. The total field satisfies the impedance-type boundary condition, as given by [5, 6]

$$E_x(x,+0) = E_x(x,-0) = (\zeta Z/2)[H_z(x,+0) - H_z(x,-0)], \quad |x|<l, \tag{3}$$

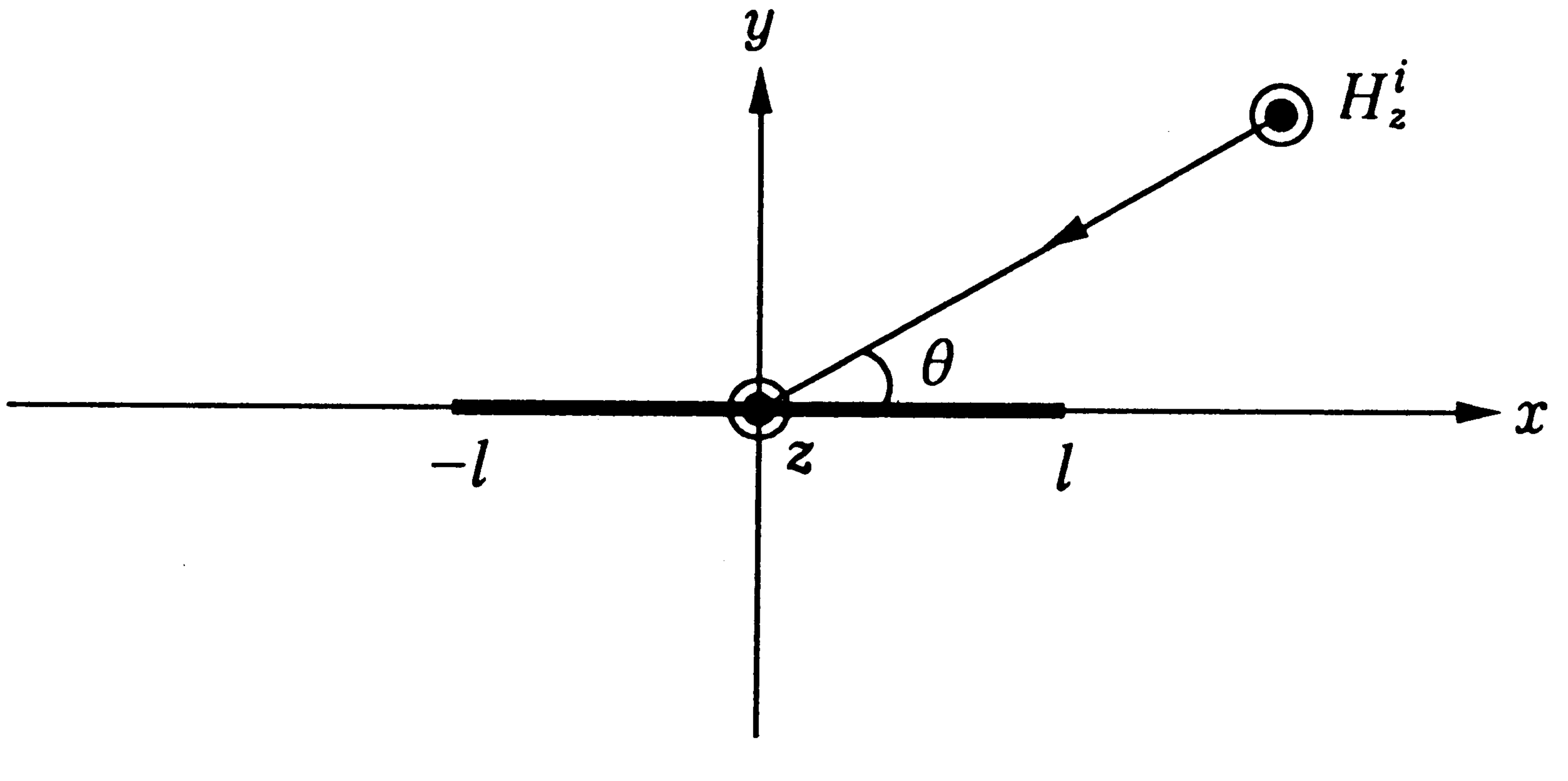

Fig. 1. Geometry of the problem.

where ζ is the resistivity and Z is the intrinsic impedance of free space. Using Green's formula, we can express the scattered field $H_z^s(x,y)$ in (1) as

$$H_z^s(x,y) = -\frac{i}{4}\int_{-l}^{l} f(x')\frac{\partial H_0^{(1)}(k\sqrt{(x-x')^2+y^2})}{\partial y}dx', \tag{4}$$

where $H_0^{(1)}(\cdot)$ denotes the Hankel function of the first kind, and $f(\cdot)$ is the unknown current density function defined as

$$f(x) = H_z(x,+0) - H_z(x,-0). \tag{5}$$

Taking into account the boundary condition as given by (3), we obtain from (1), (2), and (4) that

$$k\zeta f(x) = 2k\sin\theta e^{-ikx\cos\theta} + \frac{1}{2}\lim_{y\to 0}\frac{\partial}{\partial y}\int_{-l}^{l} f(x')\frac{\partial H_0^{(1)}(k\sqrt{(x-x')^2+y^2})}{\partial y}dx'. \tag{6}$$

Equation (6) is the integral equation to this diffraction problem.

3. Solution of the Integral Equation

Using the integral representation of the Hankel function, it follows that

$$H_0^{(1)}(k\sqrt{(x-x')^2+y^2}) = \frac{1}{\pi}\int_{-\infty}^{\infty}\exp\{ik[(x-x')\alpha+\sqrt{1-\alpha^2}|y|]\}\frac{d\alpha}{\sqrt{1-\alpha^2}}, \tag{7}$$

where $\alpha = \mathrm{Re}\,\alpha + i\mathrm{Im}\,\alpha\ (\equiv \sigma + i\tau)$. The proper branch for $\sqrt{1-\alpha^2}$ is chosen such that $\mathrm{Im}\sqrt{1-\alpha^2} > 0$ as $|\sigma| \to \infty$. Substituting (7) into (6) and taking the finite Fourier transform of the resultant equation over the interval $|x| < l$, we obtain the following integral equation in the spectral domain:

$$\zeta F(\beta) = 4\sin\theta\frac{\sin\kappa(\beta+\cos\theta)}{\kappa(\beta+\cos\theta)} - \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin\kappa(\alpha-\beta)}{\alpha-\beta}F(\alpha)\sqrt{1-\alpha^2}d\alpha, \tag{8}$$

where

$$F(\alpha) = \int_{-1}^{1} f(\eta)e^{-i\kappa\alpha\eta}d\eta, \quad \kappa = kl. \tag{9}$$

Taking into account the edge condition for a resistive half-plane [6], the current density function can be expanded in terms of the Gegenbauer polynomial $C_n^1(\cdot)$ as

$$f(\eta) = \sqrt{1-\eta^2}\sum_{n=0}^{\infty} f_n C_n^1(\eta), \quad |\eta| < 1, \tag{10}$$

where f_n for $n = 0, 1, 2, \cdots$ are unknown coefficients. Substituting (10) into (9) and applying some properties of the Weber–Schafheitlin discontinuous integrals to (8), we derive the infinite system of linear algebraic equations (SLAE) as in

$$\gamma_m = \sum_{n=0}^{\infty}(A_{mn} + \zeta B_{mn})f_n, \quad m = 0, 1, 2, \cdots, \tag{11}$$

where

$$\gamma_m = 4\tan\theta(-1)^m J_{m+1}(\kappa\cos\theta), \tag{12}$$

$$A_{mn} = [1+(-1)^{m+n}](-i)^n(n+1)I_{mn}, \tag{13}$$

$$B_{mn} = \frac{\kappa}{4}\frac{[1+(-1)^{m+n}](-i)^n(n+1)\Gamma\left(\frac{m+n+1}{2}\right)}{\Gamma\left(\frac{3}{2}-\frac{m-n}{2}\right)\Gamma\left(\frac{3}{2}+\frac{m-n}{2}\right)\Gamma\left(\frac{m+n+1}{2}+2\right)}, \tag{14}$$

$$I_{mn} = \frac{\kappa^2}{4}\left[\kappa^{2K}\sum_{p=0}^{\infty}d_p^{(1)}\kappa^{2p} + \frac{i}{\pi}\left(-\sum_{p=-1}^{K-1}d_p^{(2)}\kappa^{2p} + \kappa^{2K}\sum_{p=0}^{\infty}c_p d_p^{(1)}\kappa^{2p}\right)\right], \quad (m+n\text{: even}), \tag{15}$$

$$d_p^{(1)} = (-1)^p\Gamma(K+p+1/2)\Gamma(p+K+3/2)/[\Gamma(p+1)\Gamma(p+m+2)\Gamma(p+n+2)\Gamma(p+2K+3)], \tag{16}$$

$$d_p^{(2)} = \Gamma(-p+K)\Gamma(p+1/2)\Gamma(p+3/2)/[\Gamma(p-K+m+2)\Gamma(p-K+n+2)\Gamma(p+K+3)], \tag{17}$$

$$c_p = 2\ln\kappa + \Psi(p+K+1/2) + \Psi(p+K+3/2) - \Psi(p+1) - \Psi(p+m+2) - \Psi(p+n+2) - \Psi(p+2K+3) \tag{18}$$

with $K=(m+n)/2$. In the above, $\Gamma(\cdot)$ and $J_{m+1}(\cdot)$ denote the gamma function and the Bessel function, respectively, and $\Psi(\cdot)$ is the psi function defined by

$$\Psi(z)=-C+\sum_{n=0}^{\infty}\left(\frac{1}{n+1}-\frac{1}{n+z}\right),\quad z\neq 0,-1,-2,\cdots \tag{19}$$

with $C\,(=0.57721566\cdots)$ being Euler's constant.

Our problem has now been reduced to the solution of the SLAE satisfied by the unknown coefficients f_n for $n=0, 1, 2, \cdots$. By solving (11) numerically via appropriate truncation, these coefficients are determined with high accuracy.

4. Scattered Far Field

Taking into account the asymptotic expansion of the Hankel function for large argument, the scattered far field is found to be

$$H_z^s(r,\varphi)\sim\sqrt{\frac{2}{\pi kr}}e^{i(kr-\pi/4)}\Phi(\varphi),\quad kr\to\infty, \tag{20}$$

where (r,φ) is the cylindrical coordinate defined by $x=r\cos\varphi$, $y=r\sin\varphi$ for $-\pi\leqq\varphi\leqq\pi$, and

$$\Phi(\varphi)=\frac{\pi}{4}\tan\varphi\sum_{n=0}^{\infty}(-i)^{n+1}(n+1)f_nJ_{n+1}(\kappa\cos\varphi). \tag{21}$$

5. Numerical Results and Discussion

We shall now show numerical results on the total scattering cross section (TSCS) σ_t and the RCS σ to discuss the far field scattering characteristics. Figure 2 illustrates the normalized TSCS $\sigma_t/4l$ as a function of normalized frequency kl, where the incidence angle and the resistivity are chosen as $\theta=45°, 90°$ and $\zeta=0.1+i0.27$, respectively. The results for a perfectly conducting strip ($\zeta=0$) have also been added for comparison. It is seen from the figure that the resistive strip gives a lower TSCS level than the perfectly conducting case. Shown in Figs. 3 and 4 are numerical examples of the the monostatic RCS as a function of incidence angle θ and the bistatic RCS as a function of observation angle φ, respectively, where the normalized value σ/λ with λ being the free-space wavelength has been plotted in decibels. The results for the resistive strip with $\zeta=0.1+i0.27$ and the perfectly conducting strip ($\zeta=0$) are again presented and in the bistatic RCS computations, the incidence angle is fixed as $\theta=45°$. It is noted from the figures that the monostatic RCS and the bistatic RCS show the largest values along the reflected shadow boundaries at $\theta=90°$ and $\varphi=135°$, respectively. We also observe some oscillations in the RCS characteristics for the strip width $2a=4\lambda$. Comparing the results for the resistive and perfectly conducting strips, it is seen that the RCS is reduced for the resistive case.

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