PROCEEDINGS

OF THE

1996 INTERNATIONAL SYMPOSIUM ON ANTENNAS AND PROPAGATION

VOL. 1 : SEP. 24 (TUE)

SEPTEMBER 24-27, 1996
CHIBA, JAPAN

ANALYTICAL-NUMERICAL APPROACH FOR THE SOLUTION OF THE DIFFRACTION BY A RESISTIVE STRIP

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1. Introduction

The analysis of the scattering by resistive strips is an important subject in diffraction theory. This geometry can be regarded as a suitable model of thin dielectric slabs and coating of finite length which are often used for radar cross section (RCS) reduction. There have been several investigations on the scattering by resistive/impedance strips based on high-frequency and numerical methods [1, 2]. In this paper, we shall analyze the plane wave diffraction by a resistive strip using the analytical-numerical approach [3] which is entirely different from the previous methods employed to solve the impedance-related problems. Applying the boundary condition to an integral representation of the scattered field, the problem is formulated as an integral equation satisfied by the unknown current density function. Expanding the current density function in terms of the Gegenbauer polynomials by taking into account the edge condition, our problem is reduced to the solution of an infinite system of linear algebraic equations (SLAE) satisfied by the unknown expansion coefficients. These coefficients are determined numerically with high accuracy via truncation of the SLAE. The scattered field is evaluated asymptotically and the far field expression is derived. Numerical examples on the total scattering cross section and the monostatic RCS are presented and the far field scattering characteristics are discussed. Some comparisons with a high-frequency technique are also given to validate the present approach.

The time factor is assumed to be $e^{-i\omega t}$ and suppressed throughout this paper.

2. Formulation of the Problem

We consider a resistive strip as shown in Fig. 1, being illuminated by an E-polarized plane wave, where the strip occupies the region $|x| \leq l$ of the $y = 0$ plane and is assumed to be infinitely thin. The E polarization implies that the incident electric field is parallel to the z-axis.

Let the total electric field $E_z(x, y)$ be

$$E_z(x,y) = E_z^i(x,y) + E_z^s(x,y), \tag{1}$$

where $E_z^i(x,y)$ is the incident field of E polarization given by

$$E_z^i(x,y) = e^{-ik(x\cos\theta + y\sin\theta)}, \quad 0 < \theta < \pi \tag{2}$$

with $k\,[\equiv \omega(\varepsilon_0\mu_0)^{1/2}]$ being the free-space wavenumber. The total field satisfies the impedance-type boundary condition, as given by [4, 5]

$$E_z(x,+0) = E_z(x,-0) = -(\zeta Z/2)[H_x(x,+0) - H_x(x,-0)], \quad |x| < l, \tag{3}$$

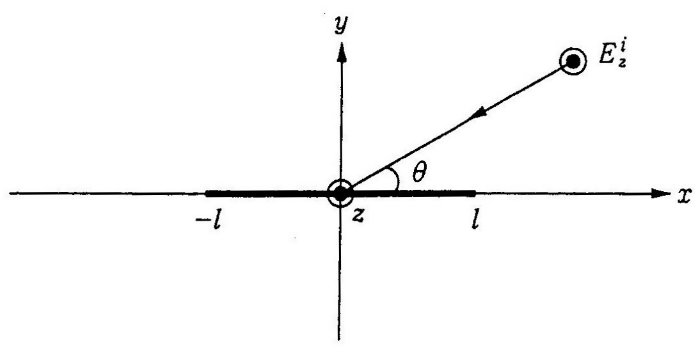

Fig. 1. Geometry of the problem.

where ζ is the resistivity and Z is the intrinsic impedance of free space. Using Green's formula, we can express the scattered field $E_z^s(x,y)$ as

$$E_z^s(x,y) = -\frac{i}{4}\int_{-l}^{l} f(x')H_0^{(1)}(k\sqrt{(x-x')^2+y^2})dx', \tag{4}$$

where $H_0^{(1)}(\cdot)$ denotes the Hankel function of the first kind, and $f(\cdot)$ is the unknown current density function defined as

$$f(x) = ikZ\,[H_x(x,+0) - H_x(x,-0)]. \tag{5}$$

Taking into account the boundary condition as given by (3), we obtain from (1), (2), and (4) that

$$-\frac{\zeta}{k}f(x) = 2ie^{-ikx\cos\theta} + \frac{1}{2}\int_{-l}^{l} f(x')H_0^{(1)}(k|x-x'|)dx'. \tag{6}$$

Equation (6) is the integral equation to this diffraction problem.

3. Solution of the Integral Equation

Using the integral representation of the Hankel function, it follows that

$$H_0^{(1)}(k|x-x'|) = \frac{1}{\pi}\int_{-\infty}^{\infty} e^{ik\alpha(x-x')}\frac{d\alpha}{\sqrt{1-\alpha^2}}, \tag{7}$$

where $\alpha = \mathrm{Re}\,\alpha + i\mathrm{Im}\,\alpha\ (\equiv \sigma + i\tau)$. The branch for $\sqrt{1-\alpha^2}$ is chosen such that $\mathrm{Im}\sqrt{1-\alpha^2} > 0$ as $|\sigma| \to \infty$. Substituting (7) into (6) and taking the finite Fourier transform of the resultant equation over the interval $|x| < l$, we obtain the following integral equation in the spectral domain:

$$-\zeta F(\beta) = 4i\frac{\sin\kappa(\beta+\cos\theta)}{\beta+\cos\theta} + \frac{1}{\pi}\int_{-\infty}^{\infty}\frac{\sin\kappa(\alpha-\beta)}{\alpha-\beta}\frac{F(\alpha)}{\sqrt{1-\alpha^2}}d\alpha, \tag{8}$$

where

$$F(\alpha) = \int_{-1}^{1} f(\eta)e^{-i\kappa\alpha\eta}d\eta, \quad \kappa = kl. \tag{9}$$

Taking into account the edge condition for a resistive half-plane [5], the current density function can be expanded in terms of the Gegenbauer polynomial $C_n^{1/2}(\cdot)$ as

$$f(\eta) = \sum_{n=0}^{\infty} f_n C_n^{1/2}(\eta), \quad |\eta| < 1, \tag{10}$$

where f_n for $n = 0, 1, 2, \cdots$ are unknown coefficients to be determined. Substituting (10) into (9) and applying some properties of the Weber-Schafheitlin discontinuous integrals to (8), we derive the infinite system of linear algebraic equations (SLAE) as in

$$-\zeta f_m = \gamma_m + \sum_{n=0}^{\infty} A_{mn} f_n, \quad m = 0, 1, 2, \cdots, \tag{11}$$

where

$$\gamma_m = -\sqrt{2\pi\kappa}(2m+1)(-i)^{m+1}\frac{J_{m+1/2}(\kappa\cos\theta)}{\sqrt{\cos\theta}}, \tag{12}$$

$$A_{mn} = [1+(-1)^{m+n}](m+1/2)(-i)^{n-m}I_{mn}, \tag{13}$$

$$I_{mn} = \frac{\kappa}{2}\left[\kappa^{2K}\sum_{p=0}^{\infty} d_p^{(1)}\kappa^{2p} + \frac{i}{\pi}\left(-u_N\sum_{p=0}^{K-1} d_p^{(2)}\kappa^{2p} + \kappa^{2K}\sum_{p=0}^{\infty} c_p d_p^{(1)}\kappa^{2p}\right)\right], \quad (m+n\text{: even}), \tag{14}$$

$$d_p^{(1)} = (-1)^p\Gamma(K+p+1/2)\Gamma(p+K+3/2) \\ /[\Gamma(p+1)\Gamma(p+m+3/2)\Gamma(p+n+3/2)\Gamma(p+2K+2)], \tag{15}$$

$$d_p^{(2)} = \Gamma(-p+K)\Gamma(p+1/2)\Gamma(p+3/2) \\ /[\Gamma(p-K+m+3/2)\Gamma(p-K+n+3/2)\Gamma(p+K+2)], \tag{16}$$

$$c_p = 2\ln\kappa + \Psi(p+K+1/2) + \Psi(p+K+3/2) - \Psi(p+1) \\ - \Psi(p+m+3/2) - \Psi(p+n+3/2) - \Psi(p+2K+2), \tag{17}$$

$$u_0 = 0, \quad u_K = 1 \text{ for } K \geqq 1 \text{ with } K = (m+n)/2. \tag{18}$$

In the above, $\Gamma(\cdot)$ and $J_{m+1/2}(\cdot)$ denote the gamma function and the Bessel function, respectively, and $\Psi(\cdot)$ is the psi function defined by

$$\Psi(z) = -C + \sum_{n=0}^{\infty}\left(\frac{1}{n+1} - \frac{1}{n+z}\right), \quad z \neq 0, -1, -2, \cdots \tag{19}$$

with C (= 0.57721566$\cdots$) being Euler's constant.

Our problem has now been reduced to the solution of the SLAE satisfied by the unknown coefficients f_n for $n = 0, 1, 2, \cdots$. By solving (11) numerically via appropriate truncation, these coefficients are determined with high accuracy.

4. Scattered Far Field

Substituting the asymptotic representation of the Hankel function into (4) and carrying out some manipulations with the aid of (10), we derive the scattered far field with the result that

$$E_z^s(r,\varphi) \sim \sqrt{\frac{2}{\pi kr}}\, e^{i(kr-\pi/4)}\Phi(\varphi), \quad kr \to \infty, \tag{20}$$

where (r,φ) is the cylindrical coordinate defined by $x = r\cos\varphi$, $y = r\sin\varphi$ for $-\pi \leqq \varphi \leqq \pi$, and

$$\Phi(\varphi) = \frac{\sqrt{2\pi}}{4}\sum_{n=0}^{\infty}(-i)^{n+1} f_n \frac{J_{n+1/2}(\kappa\cos\varphi)}{\sqrt{\kappa\cos\varphi}}. \tag{21}$$

5. Numerical Results and Discussion

Figure 2 illustrates the normalized total scattering cross section (TSCS) $\sigma_t/4l$ as a function of normalized frequency kl, where the incidence angle and the resistivity for numerical computation are chosen as $\theta = 45°$, $90°$ and $\zeta = 0.1 - i0.27$, respectively. The results for a perfectly conducting strip ($\zeta = 0$) have also been added for comparison. It is seen that the resistive strip gives a lower TSCS level than the perfectly conducting case over the whole frequency range in the figure. Shown in Fig. 3 is the monostatic RCS σ/λ [dB] as a function of incidence angle θ with λ being the free-space wavelength, where the results for the resistive strip with $\zeta = 0.1 - i0.27$ and the perfectly conducting strip ($\zeta = 0$) are again plotted. From the figure, we observe noticeable peaks at $\theta = 90°$ due to the specular reflection from the strip surface, and there appear some oscillations in the RCS characteristics for $2a = 4\lambda$. Comparing the results for the resistive and perfectly conducting strips, it is seen that the RCS is reduced for the resistive case. Figures 4 and 5 show comparisons with the results obtained by Herman and Volakis [2]. It is seen from the figures that the agreement between the two methods is excellent for $\theta \geqq 30°$ whereas there are some discrepancies near $\theta = 0°$. Herman and Volakis use Clemmow's approach together with the extended spectral ray method for analyzing the problem. The discrepancies near $\theta = 0°$ is perhaps due to the fact that the higher order diffraction between the edges of the strip is not taken into account in their analysis.

Acknowledgments

The authors would like to thank Professor A. Hamit Serbest of Çukurova University for helpful discussions. They are also indebted to Mr. Takahiro Ueki, a graduate school student at Chuo University, for assistance in numerical computations. This work was supported in part by the Institute of Science and Engineering, Chuo University.

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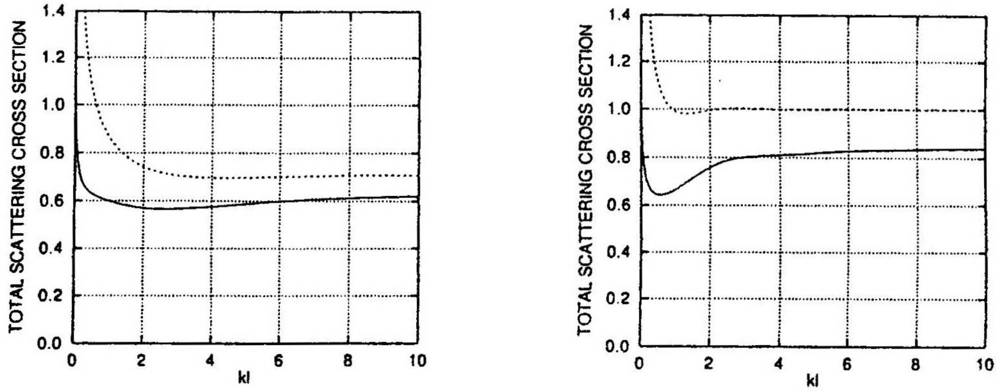
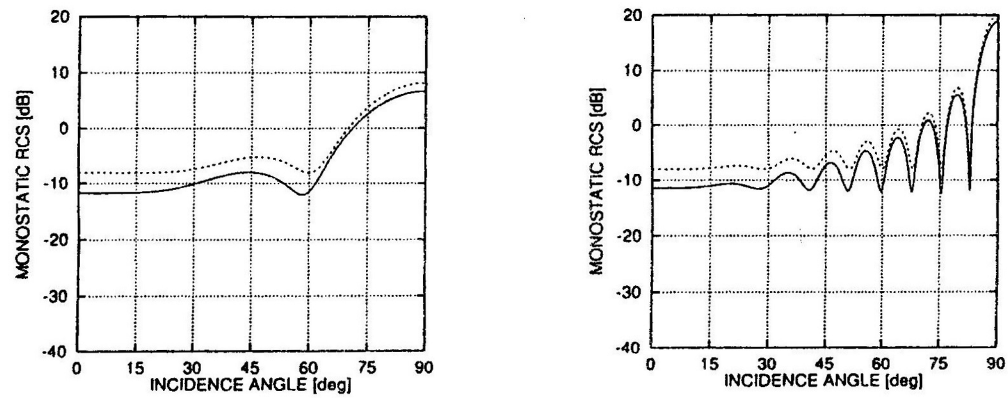


(a) $\theta = 45°$.

(b) $\theta = 90°$.

Fig. 2. Frequency dependences of the total scattering cross section $\sigma_t/4l$. Solid and dashed lines denote the results for the resistive strip ($\zeta = 0.1 - i0.27$) and the perfectly conducting strip ($\zeta = 0$), respectively.



(a) $2l = \lambda$.

(b) $2l = 4\lambda$.

Fig. 3. Monostatic RCS σ/λ [dB] versus incidence angle θ. Solid and dashed lines denote the results for the resistive strip ($\zeta = 0.1 - i0.27$) and the perfectly conducting strip ($\zeta = 0$), respectively.

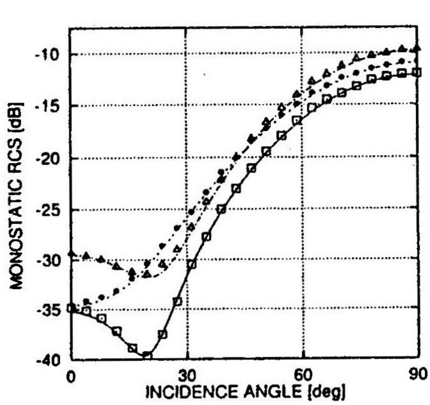


Fig. 4. Monostatic RCS σ/λ [dB] versus incidence angle θ for $2l = 0.5\lambda$ and its comparison with the results in [2]. ………… :$\zeta = -i4$; –·–·–·– :$\zeta = i4$; ———— :$\zeta = 4$ (this paper). • :$\zeta = -i4$; △ :$\zeta = i4$; ⊡ :$\zeta = 4$ (Herman and Volakis [2]).

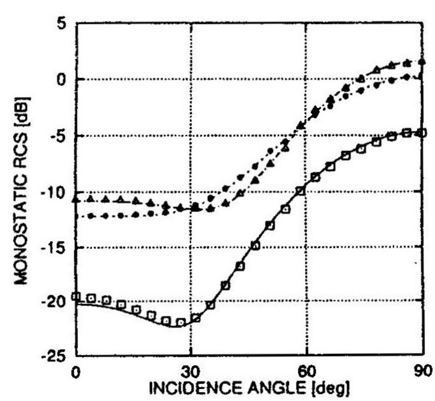


Fig. 5. Monostatic RCS σ/λ [dB] versus incidence angle θ for $2l = 0.5\lambda$ and its comparison with the results in [2]. ………… :$\zeta = 0.1 - i0.27$; –·–·–·–: $\zeta = 0.1 + i0.27$; ———— :$\zeta = 1.1$ (this paper). • :$\zeta = 0.1 - i0.27$; △ :$\zeta = 0.1 + i0.27$; ⊡ :$\zeta = 1.1$ (Herman and Volakis [2]).