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DIFFRACTION BY A STRIP WITH DIFFERENT SURFACE IMPEDANCES: THE CASE OF H POLARIZATION

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1. INTRODUCTION

The analysis of the scattering by imperfectly conducting and absorbing strips is an important subject in antenna and radar cross section (RCS) studies since this geometry serves as a suitable model of finite metal-backed dielectric layers and dielectric-coated wires. The diffraction by strips with impedance and related approximate boundary conditions has been investigated thus far using function-theoretic and high-frequency methods [1–3]. In [4], we have considered a two-dimensional (2-D) strip with different impedances on its two surfaces, and solved the E-polarized plane wave diffraction rigorously using the analytical-numerical approach [5]. This approach is based on the orthogonal polynomial expansion in conjuction with the Fourier transform, and is entirely different from the methods employed previously for analyzing scattering problems related to the impedance strip. In this paper, we shall analyze the diffraction problem involving the same impedance strip as in [4] for the H-polarized plane wave incidence by means of the analytical-numerical approach.

Applying the boundary condition to an integral representation of the scattered field, the problem is formulated as simultaneous integral equations satisfied by the electric and magnetic current density functions. The integral equations are reduced to two infinite systems of linear algebraic equations (SLAE) using a method similar to that employed for the E-polarized case [4], which are solved numerically with high accuracy via a truncation procedure. Physical quantities are then expressed in terms of the solution of the SLAE. Illustrative numerical examples on the monostatic and bistatic RCS are presented, and the far field scattering characteristics are discussed. Some comparisons with Tiberio *et al.* [3] are given to validate the present method.

The time factor is assumed to be $e^{-i\omega t}$ and suppressed throughout the following analysis.

2. SOLUTION BASED ON THE ANALYTICAL-NUMERICAL APPROACH

We consider a 2-D impedance strip of zero thickness illuminated by an H-polarized plane wave, as shown in Fig. 1, where ζ_1 and ζ_2 denote the normalized impedance of the upper and lower surfaces of the strip, respectively. Let the total magnetic field be $H_z(x,y) = H_z^i(x,y) + H_z^s(x,y)$, where $H_z^i(x,y)$ $[= e^{-ik\left(x\alpha_0 + y\sqrt{1-\alpha_0^2}\right)}]$ is the incident field with $\alpha_0 = \cos\theta$ for $0 \le \theta \le \pi$ and $k = \omega\sqrt{\mu_0\epsilon_0}$ being the free-space wavenumber. Here, $H_z^s(x,y)$ is the unknown scattered field. The total field satisfies the boundary condition as given by $\partial H_z(x,\pm 0)/\partial y \pm (ik\zeta_{1,2})H_z(x,\pm 0) = 0$ for $|x| < a$. Using Green's formula, we can express the scattered field as

$$H_z^s(x,y) = -\frac{i}{4}\int_{-a}^{a}\left[f_1(x') + f_2(x')\frac{\partial}{\partial y}\right] H_0^{(1)}\left(k\sqrt{(x-x')^2 + y^2}\right) dx' \tag{1}$$

with $H_0^{(1)}(\cdot)$ being the Hankel function of the first kind, where $f_1(x)$ $[\equiv \partial H_z(x,+0)/\partial y - \partial H_z(x,-0)/\partial y]$ and $f_2(x)$ $[\equiv H_z(x,+0) - H_z(x,-0)]$ are the unknown magnetic and electric current density functions, respectively. Taking into account the boundary condition on the strip, we obtain from (1) that

$$-\frac{Z_1}{k}f_1(x) - Z_2 f_2(x) = 2ie^{-ikx\alpha_0} + \frac{1}{2}\int_{-a}^{a} f_1(x') H_0^{(1)}(k|x-x'|)\, dx', \tag{2a}$$

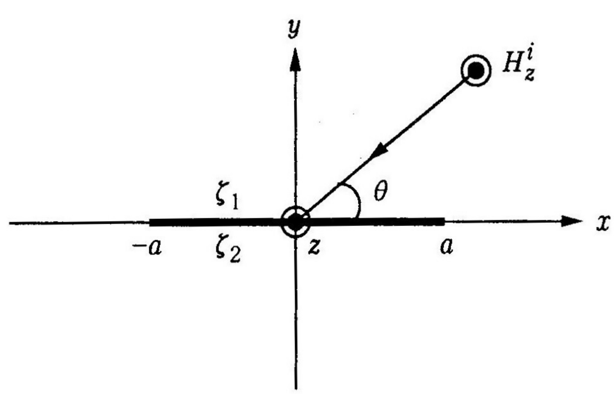

Fig. 1. Geometry of the problem.

$$- Z_2 f_1(x) + kZ_3 f_2(x) = 2k\sqrt{1-\alpha_0^2}e^{-ikx\alpha_0} + \frac{1}{2}\lim_{y\to 0}\frac{\partial^2}{\partial y^2}\int_{-a}^{a} f_2(x')H_0^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right)dx', \tag{2b}$$

where $Z_1 = 2/(\zeta_1+\zeta_2)$, $Z_2 = i(\zeta_1-\zeta_2)/(\zeta_1+\zeta_2)$, and $Z_3 = 2\zeta_1\zeta_2/(\zeta_1+\zeta_2)$.

Taking the finite Fourier transform of (2a,b) over the interval $|x| < a$ and using the integral representation of the Hankel function, we are led to

$$-\frac{Z_1}{\kappa}F_1(\beta) + iZ_2F_2(\beta) = 4i\frac{\sin\kappa(\alpha_0+\beta)}{\kappa(\alpha_0+\beta)} + \frac{1}{\pi}\int_{-\infty}^{\infty} F_1(\alpha)\frac{\sin\kappa(\alpha-\beta)}{\kappa(\alpha-\beta)}\frac{d\alpha}{\sqrt{1-\alpha^2}}, \tag{3a}$$

$$\frac{Z_2}{\kappa}F_1(\beta) + Z_3F_2(x) = 4\sqrt{1-\alpha_0^2}\frac{\sin\kappa(\alpha_0+\beta)}{\kappa(\alpha_0+\beta)} - \frac{1}{\pi}\int_{-\infty}^{\infty} F_2(\alpha)\frac{\sin\kappa(\alpha-\beta)}{\alpha-\beta}\sqrt{1-\alpha^2}d\alpha, \tag{3b}$$

where

$$F_{1,2}(\alpha) = \int_{-a}^{a} f_{1,2}(x')e^{-ikx'\alpha}dx', \quad \kappa = ka. \tag{4}$$

It can be shown that the unknown functions $f_{1,2}(x)$ in (4) are expanded in the form

$$f_1(x) = \frac{1}{a\sqrt{1-(x/a)^2}}\left[f_0^1 + 2\sum_{n=1}^{\infty}\frac{f_n^1}{n}T_n\left(\frac{x}{a}\right)\right], \quad f_2(x) = \sqrt{1-\left(\frac{x}{a}\right)^2}\sum_{n=0}^{\infty} f_n^2 U_n\left(\frac{x}{a}\right), \tag{5}$$

where $T_n(\cdot)$ and $U_n(\cdot)$ denote the Chebyshev polynomial of the first and second kinds, respectively. In (5), $f_n^{1,2}$ for $n = 0, 1, 2, \cdots$ are unknown coefficients. Substituting (5) into (3a,b) and carrying out some manipulations, we arrive at the two infinite systems of linear algebraic equations (SLAE) as in

$$-\sum_{n=0}^{\infty} x_n^1 (Z_1 A_{mn} + B_{mn}) - Z_2\sum_{n=0}^{\infty} x_n^2 C_{mn} = 4i\gamma_m, \tag{6a}$$

$$Z_2\sum_{n=0}^{\infty} x_n^1 A_{mn} + \sum_{n=0}^{\infty} x_n^2 (Z_3 C_{mn} + D_{mn}) = 4\sqrt{1-\alpha_0^2}\gamma_m \tag{6b}$$

for $m = 0, 1, 2, \cdots$, where $x_0^1 = f_0^1$, $x_n^1 = 2(-i)^n f_n^1/n$ for $n = 1, 2, 3, \cdots$, and $x_n^2 = (-i)^n(n+1)f_n^2$ for $n = 0, 1, 2, \cdots$. In (6a,b), A_{mn}, B_{mn}, C_{mn}, D_{mn}, and γ_m are known coefficients. The unknowns $f_n^{1,2}$ can be determined with high accuracy by solving (6a,b) numerically via appropriate truncation.

3. NUMERICAL RESULTS AND DISCUSSION

We shall now discuss the far field scattering characteristics of the strip based on numerical examples of the RCS. Figure 2 illustrates the monostatic RCS as a function of incidence angle for $ka = 5.0, 15.0$. In order to investigate the effect of the surface impedance on the scattered far field, four different cases have been considered as in $(\zeta_1, \zeta_2) = (1.5, 3.0), (1.5, 1.5), (3.0, 3.0), (0.0, 0.0)$, where $\zeta_{1,2} = 0.0$ corresponds to a perfectly conducting strip. Comparing the results between the impedance and perfectly conducting strips, the RCS reduction is observed for the impedance case as expected. In view of the three RCS curves for the impedance strip, the backscattered field is not affected by the impedance of the strip surface in the shadow region. Shown in Fig. 3 is the bistatic RCS as a function of observation angle for $\theta = 60°$ and $ka = 5.0, 15.0$. It is seen that the RCS level of the impedance strip is lower than the perfectly conducting case for $x > 0$, whereas all the four curves are close to each other for $x < 0$ and hence the RCS does not much depend on the surface impedance in that region. Figure 4 shows comparison with the results obtained by Tiberio *et al.* [3] using the geometrical theory of diffraction (GTD) together with the Maliuzhinetz method, where the bistatic RCS is illustrated as a function of observation angle for $\theta = 180°$, $ka = 10.0$, and $(\zeta_1, \zeta_2) = (4.0, 0.0)$. We see from the figure that our RCS results agree quite well with the results presented in [3].

ACKNOWLEDGMENTS

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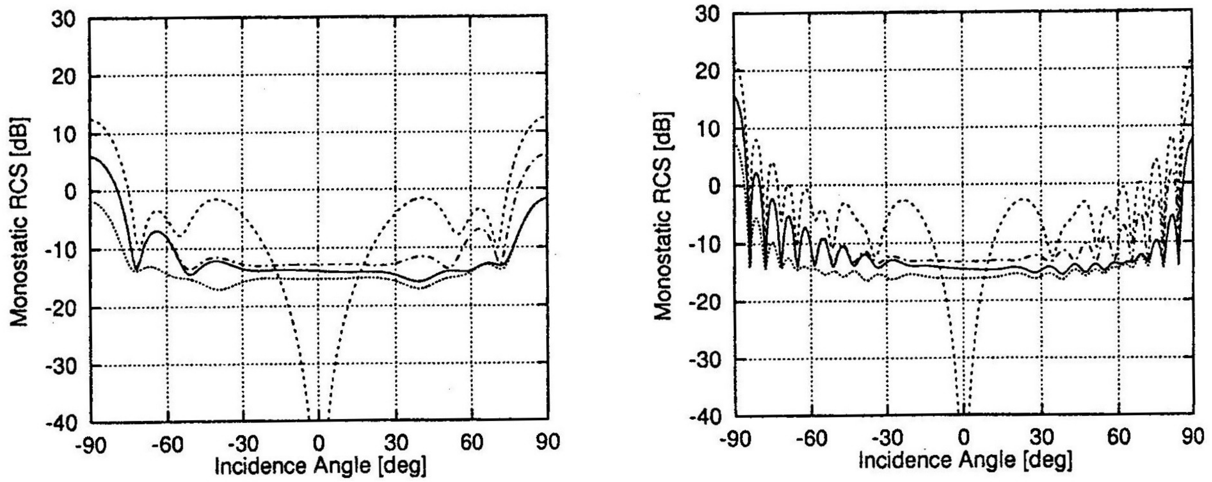
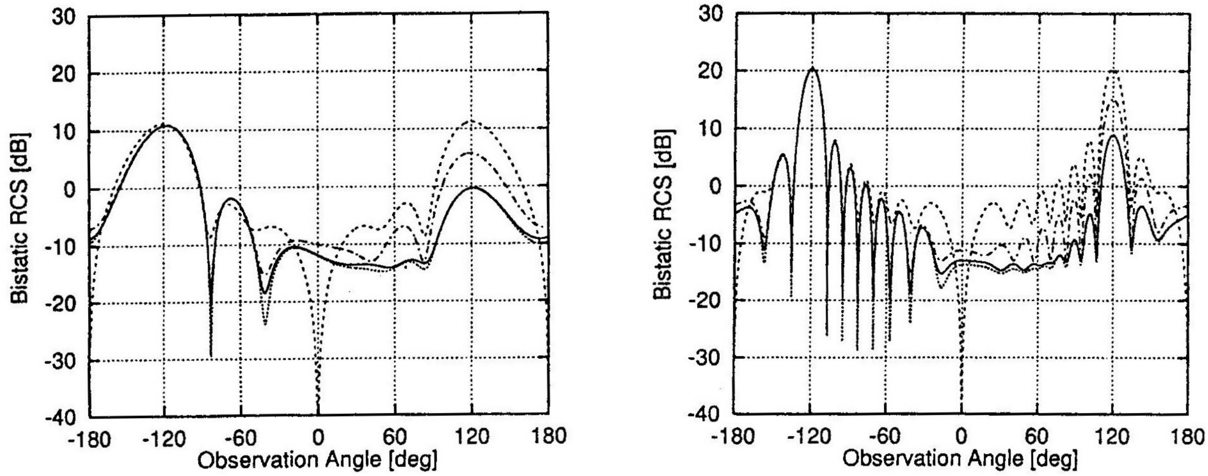


(a) $ka = 5.0$. (b) $ka = 15.0$.

Fig. 2. Monostatic RCS [dB]. - - - - - -: $\zeta_1 = \zeta_2 = 0.0$; ————: $\zeta_1 = 1.5$, $\zeta_2 = 3.0$; ·········: $\zeta_1 = \zeta_2 = 1.5$; - · - · - ·: $\zeta_1 = \zeta_2 = 3.0$.



(a) $ka = 5.0$. (b) $ka = 15.0$.

Fig. 3. Bistatic RCS [dB] for $\theta = 60°$. - - - - - -: $\zeta_1 = \zeta_2 = 0.0$; ————: $\zeta_1 = 1.5$, $\zeta_2 = 3.0$; ·········: $\zeta_1 = \zeta_2 = 1.5$; - · - · - ·: $\zeta_1 = \zeta_2 = 3.0$.

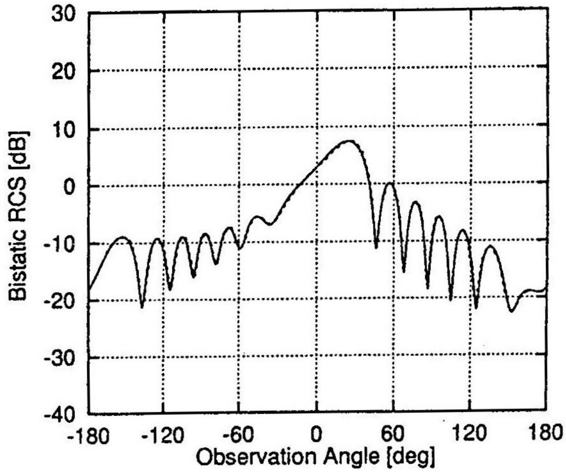


Fig. 4. Bistatic RCS [dB] for $\theta = 180°$, $ka = 10.0$, $\zeta_1 = 4.0$, $\zeta_2 = 0.0$ and its comparison with Tiberio *et al.* [3]. ————: this paper; - - - - - -: Tiberio et al. [3].