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PLANE WAVE DIFFRACTION BY A HALF-PLANE: A NEW ANALYTICAL APPROACH

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Abstract–In this paper, a new analytical method essentially different from the others is developed for obtaining a rigorous solution of a half-plane diffraction problem.

1. INTRODUCTION

As is known, the half-plane is the simplest structure for the illustration of the fundamentals of various analytical or numerical methods. There are two basic analytical methods commonly used for the solution of half-plane type boundary-value problems, namely, the Wiener-Hopf method and the Maliuzhinetz method. Although both methods have well-established standard procedures, a common disadvantage of both of them is the mathematical complexity of the solutions. In this paper, a new analytical method essentially different from the others is developed for obtaining a rigorous solution of diffraction problem by an infinitesimally thin perfectly conducting half-plane.

2. FORMULATION OF THE PROBLEM E-POLARIZED CASE

Let an infinitesimally thin perfectly conducting half-plane located in the $0XY$ plane ($y = 0,\ x \geq 0$) be excited by an E_z^i polarized plane wave, $E_z^i = \exp(-ik(x\alpha_0 + \sqrt{1-\alpha_0^2 y}))$, where $k = 2\pi/\lambda$, $\alpha_0 = cos\theta$ with θ being the incidence angle. The time factor is assumed to be $\exp(-iwt)$.

The single nonzero component of the total electrical field is considered as a superposition of the incident and scattered fields as shown below [1]:

$$E_z(x,y) = E_z^i + E_z^s(x,y) \tag{1}$$

An integral representation of the scattered field is given by [1]

$$E_z^s(x,y) = -\frac{i}{4}\int_0^\infty j_E(x')H_0^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right)dx' \tag{2}$$

where $j_E(x')$ is the surface current density, $H_0^{(1)}(x)$ denotes the zero order Hankel function of the first kind, which is the two-dimensional free space Green's function.

As is well known, the boundary condition for a perfectly conducting half-plane is given by:

$$E_z(x,0) = 0, \qquad x\epsilon(0,\infty) \tag{3}$$

In this equation by using (1) and (2), we obtain

$$\int_0^\infty j_E(x')H_0^{(1)}\left(k \mid x-x' \mid\right)dx' = -4ie^{-ikx\alpha_0} \tag{4}$$

This is the integral equation (IE) of our problem, which will be considered to determine the unknown current density function $j_E(x')$.

3. METHOD OF SOLUTION OF THE IE

Let us try to solve the IE in (4) by means of a more general and quite simple method named the method of orthogonal polynomials (OP) [2, 3]. This is a special implementation of the general scheme of the method of moments (MM), but it differs from it in two points. First, it is necessary to investigate the structure of the solution near the edge points of the domain of integration. Second, the spectral expressions for singular parts of the kernels with OP as eigenfunction are to be constructed. We would like to note that the method of OP is widely used for the solution of problems in the theory of elasticity and continuous media mechanics [2]. Nevertheless, it has not yet found an application in electromagnetic theory.

Let us rewrite the IE of (4) in the form

$$\int_0^\infty j_E\left(\frac{\zeta'}{k'}\right)K_0\left(\mid \zeta-\zeta' \mid\right)d\zeta' = 2\pi k' e^{\zeta\alpha_0} \tag{5}$$

where

$$k' = -ik, \qquad \zeta = k'x \tag{6}$$

and $K_0(x) = \frac{i\pi}{2} H_0^{(1)}(ix)$ is the MacDonald function.

From the Meixner's edge condition [1] it follows that the unknown function $j_E(x) = j_E(\frac{\zeta'}{k'})$ must behave as

$$j_E\left(\frac{\zeta'}{k'}\right) \sim O\left(\zeta^{-\frac{1}{2}}\right), \quad \text{as} \quad \zeta' \to 0 \tag{7}$$

By taking this asymptotic behavior into account, we can express the unknown function as follows:

$$j_E\left(\frac{\zeta}{k'}\right) = \frac{e^{-\zeta}}{\sqrt{\zeta}} \sum_{n=0}^{\infty} j_n^E L_n^{-\frac{1}{2}}(2\zeta), \qquad \zeta \geq 0 \tag{8}$$

Here, $L_n^{-\frac{1}{2}}(2\zeta)$ are the Laguerre polynomials and j_n^E are the unknown coefficients.

It is important to explain why the Laguerre polynomials are used for the representation of the function $j_E\left(\frac{\zeta}{k'}\right)$. In general, the choice of the type of the polynomials relies on two points:

i) The type of the polynomial chosen should involve the asymptotic behavior implied by the edge condition and also should satisfy the orthogonality condition together with the weight function. In this case, the weight function is $[\exp(-\zeta)/\sqrt{\zeta}]$ and the orthogonality condition is expressed as [4]

$$\int_0^{\infty} \frac{e^{-2\zeta}}{\sqrt{\zeta}} L_n^{-\frac{1}{2}}(2\zeta) L_k^{-\frac{1}{2}}(2\zeta) d\zeta = \frac{\gamma_n^E}{\sqrt{2}} \delta_{nk} \tag{9}$$

Here, δ_{nk} is the Kronecker deltas and

$$\gamma_n^E = \frac{\Gamma\left(n + \frac{1}{2}\right)}{\Gamma(n+1)} \sim O\left(n^{-\frac{1}{2}}\right), \quad \text{as} \quad n \to \infty, \tag{10}$$

with $\Gamma(x)$ being the Gamma function.

ii) Since the purpose of this approach is to solve an IE, the polynomial type should be the eigenfunction of the kernel. In this case, as has been shown by G. Popov [2], the Laguerre polynomials turn out to be the eigenfunctions for the following integral operator

$$\int_0^\infty e^{-\zeta'} \frac{K_0\left(|\zeta-\zeta'|\right)}{\sqrt{\zeta'}} L_n^{-\frac{1}{2}}\left(2\zeta'\right) d\zeta' = \frac{\pi}{\sqrt{2}}\gamma_n^E L_n^{-\frac{1}{2}}(2\zeta)e^{-\zeta} \tag{11}$$

Now let us substitute (8) into the IE of (5) and take into account the spectral expression in (11). Then, we have

$$\frac{\pi}{\sqrt{2}}\sum_{n=0}^{\infty} j_n^E \gamma_n^E L_n^{-\frac{1}{2}}(2\zeta)e^{-\zeta} = 2\pi k' e^{\zeta\alpha_0} \tag{12}$$

Here, by using the orthogonality condition (9), we can find the unknown coefficients j_n^E in explicit form as

$$j_n^E = \frac{4k' f_n^E}{\left(\gamma_n^E\right)^2}, \qquad n = 0, 1, 2, \ldots \tag{13}$$

where

$$f_n^E = \int_0^\infty \frac{e^{-\zeta(1-\alpha_0)}}{\sqrt{\zeta}} L_n^{-\frac{1}{2}}(2\zeta)d\zeta. \tag{14}$$

It can be shown that (see Appendix A)

$$f_n^E = \frac{\gamma_n^E}{\sqrt{1-\alpha_0}}q^n, \quad q = \frac{\alpha_0+1}{\alpha_0-1}, \quad \alpha_0 < 1, \tag{15}$$

and as a consequence of this expression we obtain

$$j_n^E = \frac{4k'}{\gamma_n^E}\frac{q^n}{\sqrt{1-\alpha_0}} \tag{16}$$

If we substitute (16) into (8) we have

$$j_E\left(\frac{\zeta}{k'}\right) = \frac{4k' e^{-\zeta}}{\sqrt{\zeta(1-\alpha_0)}}\sum_{n=0}^{\infty}\frac{q^n}{\gamma_n^E}L_n^{-\frac{1}{2}}(2\zeta) \tag{17}$$

Series in (17) is convergent. Thus, by using the explicit form of this sum (see Appendix B) in (17) we obtain

$$j_E\left(\frac{\zeta}{k'}\right) = 2k' \frac{e^{-\zeta}}{\sqrt{\pi\zeta}}\sqrt{1-\alpha_0}\,{}_1F_1\left(1, \frac{1}{2}; (1+\alpha_0)\zeta\right) \tag{18}$$

where ${}_1F_1(a,b;z)$ is the Kummer hypergeometric function. This function may be represented with the error function $erf(x)$ as done in [4]

$${}_1F_1\left(1,\frac{1}{2};(1+\alpha_0)\zeta\right)=1+\sqrt{\pi(1+\alpha_0)\zeta}\,erf\left(\sqrt{(1+\alpha_0)\zeta}\right)e^{(1+\alpha_0)\zeta}. \tag{19}$$

From (18) and (19) we see that the current density function $j_E(\frac{\zeta}{k'})$ may be given as

$$j_E\left(\frac{\zeta}{k'}\right)=2k'\frac{e^{-\zeta}}{\sqrt{\pi\zeta}}\sqrt{1-\alpha_0}\left(1+\sqrt{\pi(1+\alpha_0)}e^{(1+\alpha_0)\zeta}erf\left(\sqrt{(1+\alpha_0)\zeta}\right)\right) \tag{20}$$

By using the asymptotical behavior for the function ${}_1F_1(a,b;x)$ (see Appendix B) and taking into account the notations in (6), we obtain the following asymptotic expressions:

a) $x\to 0$

$$j_E(x)=2k\sqrt{\frac{2}{\pi kx}}e^{ikx-i\frac{\pi}{4}}\sin\frac{\theta}{2}\left(1-4i(kx)\cos^2\frac{\theta}{2}-\frac{16}{3}(kx)^2\cos^4\frac{\theta}{2}\right)+O\left((kx)^{\frac{5}{2}}\right) \tag{21}$$

b) $x\to\infty$

$$j_E(x)=2ke^{-ikx\cos\theta-i\frac{\pi}{2}}\sin\theta\left(1+O\left(\frac{1}{kx\cos^2\dfrac{\theta}{2}}\right)\right),\quad \theta\neq\pi. \tag{22}$$

Note that the representations for the surface current density in (20) and (21) coincide with the results previously obtained by other authors [5] using the method of dual integral equations closely related to the Wiener-Hopf method.

4. REPRESENTATION OF THE DIFFRACTED FIELD

Now we will investigate the scattered far field. For this purpose, we first note that the asymptotic behavior of the Hankel function $H_0^{(1)}\cdot\left(k\sqrt{(x-x')^2+y^2}\right)$ for $kr\to\infty$ is

$$H_0^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right)\sim\sqrt{\frac{2}{\pi kr}}e^{ikr-i\frac{\pi}{4}}e^{-ikx'\cos\varphi}+O\left(\frac{1}{(kr)^{\frac{3}{2}}}\right). \tag{23}$$

Here (r, φ) denotes the cylindrical polar coordinates, i.e.,

$$x = r\cos\varphi, \quad y = r\sin\varphi, \quad r = \sqrt{x^2 + y^2}$$

Substitution of (23) into (2) yields

$$E_z^s(r,\varphi) = \sqrt{\frac{2}{\pi k r}} e^{ikr - i\frac{\pi}{4}} \Phi_E(\varphi) \tag{24}$$

where

$$\Phi_E(\varphi) = -\frac{i}{4}\int_0^\infty j_E(x')e^{-ikx'\cos\varphi}dx' = -\frac{i}{4}J_E(\cos\varphi) \tag{25}$$

where $J_E(\cos\varphi)$ is the Fourier transform of the function $j_E(x')$.

In order to obtain $J_E(\cos\varphi)$, let us substitute (18) into (25) and take into account the definition of (6). Then, we obtain

$$\begin{aligned} J_E(\cos\varphi) &= \frac{1}{k'}\int_0^\infty j_E\left(\frac{\zeta}{k'}\right) e^{\zeta\cos\varphi} d\zeta \\ &= \frac{2}{\sqrt{\pi}}\sqrt{1-\alpha_0}\int_0^\infty \frac{e^{-\zeta(1-\cos\varphi)}}{\sqrt{\zeta}}\, {}_1F_1\left(1, \frac{1}{2}; (1+\alpha_0)\zeta\right) d\zeta \end{aligned} \tag{26}$$

As was shown in [6], the integral in (26) can be expressed in the explicit form (see Appendix B)

$$J_E(\cos\varphi) = -2\frac{\sqrt{1-\cos\varphi}\sqrt{1-\alpha_0}}{\cos\varphi + \alpha_0}. \tag{27}$$

Consequently, we obtain the following as the far field representation

$$\Phi_E(\varphi) = \frac{i}{2}\frac{2\sin\frac{\varphi}{2}\sin\frac{\theta}{2}}{\cos\varphi + \cos\theta} = \frac{i}{4}\left[\frac{1}{\cos\left(\frac{\varphi+\theta}{2}\right)} - \frac{1}{\cos\left(\frac{\varphi-\theta}{2}\right)}\right], \tag{28}$$

which is the well known result [1, 5].

5. H-POLARIZED CASE

In this section, we consider the diffraction of an H-polarized plane wave. We will construct the solution of this problem by following a method similar to that discussed in the previous section. Let the nonzero component of the total magnetic field H_z be given as

$$H_z(x, y) = H_z^i(x, y) + H_z^s(x, y) \tag{29}$$

An integral representation of the scattered field is now given by

$$H_z^s(x, y) = -\frac{i}{4}\int_0^{\infty} j_H(x')\frac{\partial}{\partial y}H_0^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right)dx' \tag{30}$$

where $j_H(x')$ is the surface current density. The boundary condition in this case is given by

$$\left.\frac{\partial H_z(x, y)}{\partial y}\right|_{y=0} = 0 \tag{31}$$

Substituting (29) and (30) into (31) we have

$$\frac{\partial^2}{\partial y^2}\int_0^{\infty} j_H(x')H_0^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right)dx'\bigg|_{y=0}$$
$$= -4k\sqrt{1-\alpha_0}e^{-ik\alpha_0 x} \tag{32}$$

where

$$\frac{\partial^2}{\partial y^2} = -\left(\frac{\partial^2}{\partial x^2} + k^2\right) \tag{33}$$

is obtained from Helmholtz equation. This yields

$$\left(\frac{d^2}{dx^2} + k^2\right)\int_0^{\infty} j_H(x')H_0^{(1)}\left(k \mid x - x' \mid\right)dx' = -4k\sqrt{1-\alpha_0}e^{-ik\alpha_0 x}. \tag{34}$$

This is the integro-differential equation (IDE) which needs to be solved for the H-polarized case.

6. METHOD OF SOLUTION OF THE IDE

We will construct the solution for the IDE of (34) by following the method of OP. Let us rewrite the IDE of (34) in the form

$$\left(\frac{d^2}{d\zeta^2} - 1\right)\int_0^{\infty} j_H\left(\frac{\zeta'}{k'}\right)K_0\left(\mid \zeta - \zeta' \mid\right)d\zeta' = -2\pi\sqrt{1-\alpha_0^2}e^{\alpha_0 x}. \tag{35}$$

In this case, Meixner's edge condition [1] requires the unknown function $j_H(\frac{\zeta'}{k})$ to behave as

$$j_H\left(\frac{\zeta}{k'}\right) \sim O\left(\zeta^{\frac{1}{2}}\right), \quad \text{for} \quad \zeta \to 0. \tag{36}$$

By taking into account this asymptotic behavior, the $j_H(\frac{\zeta}{k'})$ can be expressed as follows:

$$j_H\left(\frac{\zeta}{k'}\right) = \sqrt{\zeta}e^{-\zeta}\sum_{n=0}^{\infty} j_n^H L_n^{\frac{1}{2}}(2\zeta) \tag{37}$$

where $L_n^{\frac{1}{2}}(2\zeta)$ are the Laguerre polynomials. It can be shown that (see Appendix C) the expression (37) follows from the fact that the polynomials $L_n^{\frac{1}{2}}(2\zeta)$ are the eigenfunctions for the following integral operator

$$\left(\frac{d^2}{d\zeta^2} - 1\right)\int_0^{\infty} \sqrt{\zeta'}e^{-\zeta'} K_0\left(\mid \zeta - \zeta' \mid\right) L_n^{\frac{1}{2}}\left(2\zeta'\right) d\zeta'$$
$$= -\pi\sqrt{2}\gamma_n^H L_n^{\frac{1}{2}}(2\zeta)e^{-\zeta} \tag{38}$$

where $\gamma_n^H = \Gamma(n + \frac{3}{2})/\Gamma(n+1)$ are the eigenvalues of the integral operator in (38). Let us substitute (37) into the IDE of (35) and take into account the spectral expression in (38). Then we have

$$\pi\sqrt{2}\sum_{n=0}^{\infty} j_n^H \gamma_n^H L_n^{\frac{1}{2}}(2\zeta)e^{-\zeta} = 2\pi\sqrt{1-\alpha_0^2}e^{\alpha_0\zeta}. \tag{39}$$

Note that the orthogonality condition for $L_n^{\frac{1}{2}}(2\zeta)$ is given by [4]

$$\int_0^{\infty} e^{-2\zeta}\sqrt{\zeta}L_x^{\frac{1}{2}}(2\zeta)L_n^{\frac{1}{2}}(2\zeta)d\zeta = \frac{\gamma_n^H}{2\sqrt{2}}\delta_{kn} \tag{40}$$

which leads to

$$j_n^H = 4\sqrt{1-\alpha_0^2}\frac{f_n^H}{(\gamma_n^H)^2}; \qquad f_n^H = \int_0^{\infty} \sqrt{\zeta}e^{-(1-\alpha_0)\zeta} L_n^{\frac{1}{2}}(2\zeta)d\zeta \tag{41}$$

It can be shown that (see Appendix B)

$$j_n^H = \frac{\gamma_n^H}{(1-\alpha_0)^{\frac{3}{2}}} q^n, \qquad q = \frac{\alpha_0+1}{\alpha_0-1} \tag{42}$$

Now if (41) and (42) are substituted into (37), the following analytical expressions are obtained for the current density function $j_H(\frac{\zeta}{k'})$ (see Appendix B):

$$\begin{aligned} j_n^H\left(\frac{\zeta}{k'}\right) &= 4\frac{\sqrt{1+\alpha_0}}{1-\alpha_0}\sqrt{\zeta}e^{-\zeta}\sum_{n=0}^{\infty}\frac{q^n}{\gamma_n^H}L_n^{\frac{1}{2}}(2\zeta) \\ &= 4\sqrt{\frac{\zeta}{\pi}(1+\alpha_0)}e^{-\zeta}{}_1F_1\left(1,\frac{3}{2};\zeta(1+\alpha_0)\right) \\ &= 2e^{\zeta\alpha_0}erf\left(\sqrt{\zeta(1+\alpha_0)}\right). \end{aligned} \tag{43}$$

By using the asymptotical behavior for the ${}_1F_1(a,b,z)$ (see Appendix B), we obtain

$$\begin{aligned} j_H(x) = 4\cos\frac{\theta}{2}\sqrt{\frac{2kx}{\pi}}e^{i(kx-\frac{\pi}{4})}\left[1-\frac{4}{3}ikx\cos^2\frac{\theta}{2}-\frac{32}{15}\left(kx\cos^2\frac{\theta}{2}\right)^2\right] \\ +O\left((kx)^{\frac{5}{2}}\right) \end{aligned} \tag{44}$$

for $x \to 0$;

$$j_H(x) = 2e^{-ikx\cos\theta}\left[1+O\left(\frac{1}{kx\cos^2\dfrac{\theta}{2}}\right)\right], \qquad \theta \neq \pi \tag{45}$$

for $x \to \infty$.

For the definition of the far field, it is necessary to use the asymptotic expression for the Hankel function of zeroth order. It may be shown that

$$H_z^s(r,\varphi) = \sqrt{\frac{2}{\pi kr}}e^{i(kr-\frac{\pi}{4})}\Phi_H(\varphi), \tag{46}$$

with

$$\Phi_H(\varphi) = \frac{k}{4}\sin\varphi J_H(\cos\varphi); \qquad J_H(\cos\varphi) = \int_0^\infty j_H(x')e^{-ikx'\cos\varphi}dx' \tag{47}$$

where $J_H(\cos\varphi)$ is the Fourier transform of the function $j_H(x')$.

If we substitute (43) into (46) we obtain

$$\begin{aligned} J_H(\cos\varphi) &= \frac{1}{k'}\int_0^\infty j_H\left(\frac{\zeta}{k'}\right) e^{\zeta\cos\varphi} d\zeta \\ &= \frac{4}{k'}\sqrt{\frac{1+\alpha_0}{\pi}}\int_0^\infty \sqrt{\zeta} e^{-\zeta(1-\cos\varphi)} {}_1F_1\left(1,\frac{3}{2};(1+\alpha_0)\zeta\right) d\zeta. \end{aligned} \tag{48}$$

The integral in (48) can be evaluated in explicit form (see Appendix C) as

$$J_H(\cos\varphi) = -\frac{2}{k'}\frac{\sqrt{1+\alpha_0}}{\sqrt{1-\cos\varphi}}\cdot\frac{1}{\cos\varphi+\alpha_0} \tag{49}$$

Finally, for the far field we have

$$\begin{aligned} \Phi_H(\varphi) &= -\frac{i}{2}\frac{\sqrt{1+\alpha_0}\sqrt{1+\cos\varphi}}{\cos\varphi+\alpha_0} \\ &= -i\frac{\cos\frac{\varphi}{2}\cos\frac{\theta}{2}}{\cos\varphi+\cos\theta} \\ &= -\frac{i}{4}\left[\frac{1}{\cos\left(\frac{\varphi+\theta}{2}\right)}+\frac{1}{\cos\left(\frac{\varphi-\theta}{2}\right)}\right] \end{aligned} \tag{50}$$

It should be noted that the present method is also applicable to half-plane structures with impedance boundary conditions.

7. CONCLUSIONS

As we have mentioned before, the method of OP is a more general and quite simple method for the solution of IE, which arises in various diffraction problems. The universality of the method of OP resides in the opportunity to construct the spectral relationships for different singular kernels where OP plays the role of the eigenfunctions. As an example it seems reasonable to cite the following spectral relationships [2, 8]:

$$-\frac{1}{\pi}\int_{-1}^{1}\frac{T_n(\zeta)}{\sqrt{1-\zeta^2}}\ln|\zeta-\eta|\,d\eta = \omega_n T_n(\eta),\quad |\eta|\le 1\left\{\begin{matrix}\omega_n=\ln 2, & n=0\\ \omega_n=\frac{1}{n}, & n\neq 0\end{matrix}\right\} \tag{51}$$

$$\frac{1}{\pi}\int_{-\alpha}^{\alpha}\ln\frac{1}{2\left|\sin\dfrac{\phi-\phi_0}{2}\right|}\frac{T_{2n}\left(\dfrac{\sin\dfrac{\phi}{2}}{\sin\dfrac{\alpha}{2}}\right)}{\sqrt{2(\cos\phi-\cos\alpha)}}\cos\frac{\phi}{2}d\phi=\sigma_{2n}T_{2n}\left(\dfrac{\sin\dfrac{\phi_0}{2}}{\sin\dfrac{\alpha}{2}}\right) \tag{52}$$

Here $\{\sigma_{2n}\}_{n=0}^{\infty}$ are eigenvalues of integral operator equal to

$$\sigma_{2n}=\left\{\begin{array}{ll}-\ln\sin\dfrac{\alpha}{2}, & n=0\\ \dfrac{1}{2n}, & n\neq 0\end{array}\right\}$$

$$\frac{d^2}{dx^2}\int_{-1}^{1}\sqrt{1-y^2}\ln\frac{1}{|x-y|}U_n(y)dy=-\pi(n+1)U_n(x) \tag{53}$$

Here $U_n(y)$ are the Chebyshev's polynomials of the second kind.

$$\frac{i}{2}\int_{-1}^{1}\frac{Ce_n(\arccos\zeta,\gamma)}{\sqrt{1-\zeta^2}}H_0^{(2)}(\epsilon\left|\zeta-\zeta'\right|)d\zeta=\frac{Me_n^{(2)'}(0,g)}{Me_n^{(2)}(0,g)}Ce_n(\arccos\zeta',g) \tag{54}$$

where $g=\frac{\epsilon^2}{4}$, $\{Ce_n(\arccos\zeta,g)\}_{n=0}^{\infty}$ are the Mathieu's functions.

$$\int_{-1}^{1}\frac{C_n^{\frac{\nu}{2}}(y)dy}{|x-y|^{\nu}\sqrt{(1-y^2)^{1-\nu}}}=\pi\frac{\Gamma(n+\nu)}{\Gamma(n+1)}\frac{C_n^{\frac{\nu}{2}}(x)}{\Gamma(\nu)\cos\dfrac{\pi\nu}{2}} \tag{55}$$

These spectral relationships can be used to solve various scattering problems with both the Dirichlet and Neumann boundary conditions.

It should be noted that the problem of wave scattering by polygonal cylinders and cylindrical structures has been investigated by the method of OP [9, 10].

APPENDIX A

For the calculation of integrals in (13) and (38) it is necessary to use the following representation [5]

$$\int_0^{\infty}x^{\lambda}e^{-px}L_n^{\lambda}(cx)dx=\frac{\Gamma(\lambda+n+1)(p-c)^n}{\Gamma(n+1)p^{\lambda+n+1}},$$
$$Re(p)\geq 0,\quad Re(\lambda)\geq -1 \tag{A.1}$$

In our case $\lambda = \pm\frac{1}{2}$; $p = 1 - \alpha_0$; $c = 2$.

APPENDIX B

As shown in [5]

$$\frac{\Gamma(\lambda)}{\Gamma(b)} \sum_{k=0}^{\infty} \frac{\Gamma(k+b)}{\Gamma(k+\lambda+1)} t^k L_k^{\lambda}(x) = (1-t)^{-b}\, {}_1F_1\left(b; \lambda+1; \frac{xt}{1-t}\right), \quad |\, t \,| < 1. \tag{B.1}$$

In our case $\lambda = \pm\frac{1}{2}$; $b = 1$; $t = \frac{\alpha_0+1}{\alpha_0-1}$; $x = 2\zeta$.

As it is well known, the function ${}_1F_1(a;b;x)$ may be represented as follows [7]

$${}_1F_1(a,b;x) = \frac{\Gamma(b)}{\Gamma(a)} \sum_{x=0}^{\infty} \frac{\Gamma(k+a)}{\Gamma(k+b)} \frac{x^k}{\Gamma(k+1)}. \tag{B.2}$$

Using this equality we can obtain the asymptotical expression for the function ${}_1F_1(a,b;x)$ when $x \to 0$

$${}_1F_1(a,b;x) = \frac{\Gamma(b)}{\Gamma(a)} \left[\frac{\Gamma(a)}{\Gamma(b)} + \frac{\Gamma(a+1)}{\Gamma(b+1)} x + \frac{\Gamma(a+2)}{\Gamma(b+2)} x^2 \right] + O(x^3). \tag{B.3}$$

When x tends to $+\infty$, we can use the asymptotical expression [7]

$${}_1F_1(a,b;x) = \frac{\Gamma(b)}{\Gamma(a)} e^x x^{a-b} [1 + O(x^{-1})]. \tag{B.4}$$

APPENDIX C

As was shown in [6], for the integrals in (26) and (48) the following representation can be used:

$$\int_0^{\infty} e^{-px} x^{b-1}\, {}_1F_1(a,b;cx)dx = \Gamma(b) p^{a-b} (p-c)^{-a}, \quad Re(p), \quad Re\,(b)\,, \quad Re(p-c) \geq 0 \tag{C.1}$$

In our case
$b = \frac{1}{2}$ or $\frac{3}{2}$; $p = 1 - cos\varphi$; $c = 1 + \alpha_0$.

APPENDIX D

Here we will prove equation (38). For this purpose it is necessary to give a new representation for the Laguerre polynomials $L_n^{-\frac{1}{2}}(2\zeta)$ in (11) via the recurrence formula [4]

$$L_n^{-\frac{1}{2}}(2\zeta) = \frac{2\zeta}{n+\dfrac{1}{2}} L_n^{\frac{1}{2}}(2\zeta) + \frac{n+1}{n+\dfrac{1}{2}} L_{n+1}^{-\frac{1}{2}}(2\zeta). \tag{D.1}$$

This leads to

$$\int_0^\infty \sqrt{\zeta} e^{-\zeta'} K_0\left(\mid \zeta - \zeta' \mid\right) L_n^{\frac{1}{2}}(2\zeta') = \frac{\pi e^{-\zeta}}{2\sqrt{2}}\left(n+\frac{1}{2}\right) \cdot \gamma_n^E \left[L_n^{-\frac{1}{2}}(2\zeta) - L_{n+1}^{-\frac{1}{2}}(2\zeta)\right] \tag{D.2}$$

Here, if we use the other recurrence formula for $L_n^{-\frac{1}{2}}(2\zeta)$ [4]

$$L_{n+1}^{-\frac{3}{2}}(2\zeta) = -L_n^{-\frac{1}{2}}(2\zeta) + L_{n+1}^{-\frac{1}{2}}(2\zeta) \tag{D.3}$$

and take into account that

$$\gamma_n^H = \left(n+\frac{1}{2}\right)\gamma_n^E = \left(n+\frac{1}{2}\right)\frac{\Gamma\left(n+\dfrac{1}{2}\right)}{\Gamma(n+1)} = \frac{\Gamma\left(n+\dfrac{3}{2}\right)}{\Gamma(n+1)}. \tag{D.4}$$

then we obtain

$$\int_0^\infty \sqrt{\zeta'} e^{-\zeta'} K_0\left(\mid \zeta - \zeta' \mid\right) L_n^{\frac{1}{2}}\left(2\zeta'\right) d\zeta' = \frac{\pi e^{-\zeta}}{2\sqrt{2}} \gamma_n^H L_{n+1}^{-\frac{3}{2}}(2\zeta). \tag{D.5}$$

Now it is necessary to determine the second order derivative of the equation (D.5). We have

$$\frac{d^2}{d\zeta^2}\int_0^\infty \sqrt{\zeta'} e^{-\zeta'} K_0\left(\mid \zeta - \zeta' \mid\right) L_n^{\frac{1}{2}}\left(2\zeta'\right) d\zeta' = -\frac{\pi\gamma_n^H}{2\sqrt{2}}\frac{d^2}{d\zeta^2}\left[e^{-\zeta} L_{n+1}^{-\frac{3}{2}}(2\zeta)\right] \tag{D.6}$$

It can be shown that

$$\frac{d^2}{d\zeta^2}\left[e^{-\zeta}L_{n+1}^{-\frac{3}{2}}(2\zeta)\right]=e^{-\zeta}\left[L_{n+1}^{-\frac{3}{2}}(2\zeta)+4L_n^{\frac{1}{2}}(2\zeta)\right] \tag{D.7}$$

Substituting (D.7) into (D.6), we have

$$\frac{d^2}{d\zeta^2}\int_0^{\infty}\sqrt{\zeta'}e^{-\zeta'}K_0\left(\mid\zeta-\zeta'\mid\right)L_n^{\frac{1}{2}}\left(2\zeta'\right)d\zeta'$$
$$=-\frac{\pi e^{-\zeta}}{2\sqrt{2}}\gamma_n^H\left[L_{n+1}^{-\frac{3}{2}}(2\zeta)+4L_n^{\frac{1}{2}}(2\zeta)\right] \tag{D.8}$$

Now by using equation (D.5) and (D.8) it can be easily shown that the equation (35) is valid.

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