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# PLANE WAVE DIFFRACTION BY A THIN MATERIAL STRIP: SOLUTION BY THE ANALYTICAL-NUMERICAL APPROACH

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*Abstract:* The plane wave diffraction by a thin material strip is analyzed for both $E$ and $H$ polarizations using a new analytical-numerical approach together with approximate boundary conditions. The problem is reduced to the solution of infinite systems of linear algebraic equations. The final results are valid provided that the thickness of the strip is small compared with the wavelength. The scattered field is evaluated asymptotically and a far field expression is derived. Illustrative numerical examples on the radar cross section are presented and the far field scattering characteristics are discussed. Some comparisons with the other existing method are also given.

## 1. INTRODUCTION

Analysis of the scattering by imperfectly conducting and absorbing strips is an important subject in electromagnetic theory, and it is relevant to many engineering applications such as antenna and radar cross section (RCS) studies. A resistive sheet can be regarded as a suitable model of thin material layers and there have been investigations on the scattering by resistive strips based on several analytical methods [1,2]. It is also known that a thin material layer with arbitrary permittivity and permeability can be modeled more accurately by a pair of modified resistive and conductive sheets each satisfying given boundary conditions [3-5]. In a previous paper [6], we have analyzed the plane wave diffraction by a thin dielectric strip using a new analytical-numerical approach [7] based on the orthogonal polynomial expansion in conjunction with the Fourier transform, where an efficient solution has been obtained for the strip thickness small compared with the wavelength. In this paper, we shall consider a thin strip consisting of a homogeneous material with arbitrary permittivity and permeability as a generalization to our previous geometry in [6], and analyze the plane wave diffraction for both $E$ and $H$ polarizations. The solution method is again based on the analytical-numerical approach.

Applying the approximate boundary conditions to an integral representation of the scattered field, the problem is formulated as two integral equations satisfied by the unknown current density functions. Expanding the current density functions in terms of the orthogonal polynomials, our problem is reduced to the solution of two infinite systems of linear algebraic equations (SLAE) satisfied by the unknown expansion coefficients. These coefficients are determined numerically with high accuracy via appropriate truncation of the SLAE. The scattered field is evaluated asymptotically and a far field expression is derived.

We shall present illustrative numerical examples on the RCS and discuss the far field scattering characteristics in detail. Some comparisons with Volakis [4] are also given to validate the present method. In the following, the analysis procedure is presented only for the $E$-polarized case, but numerical results are given for both polarizations.

The time factor is assumed to be $e^{-i\omega t}$ and suppressed throughout this paper.

## 2. SOLUTION BASED ON THE ANALYTICAL-NUMERICAL APPROACH

We consider the diffraction of an $E$-polarized plane wave by a thin material strip as shown in Fig. 1, where the relative permittivity and permeability of the strip are denoted by $\varepsilon_r$ and $\mu_r$, respectively. Let the total field be $E_z(x,y) = E_z^i(x,y) + E_z^s(x,y)$, where $E_z^i(x,y) = [e^{-ik\left(x\alpha_0 + y\sqrt{1-\alpha_0^2}\right)}]$ is the incident filed with $\alpha_0 = \cos\theta_0$ for $0 < \theta_0 < \pi$ with $k(=\omega\sqrt{\varepsilon_0\mu_0})$ being the free-space wavenumber. Here $E_z^s(x,y)$ is the unknown scattered filed. The material strip is replaced by a strip of zero thickness satisfying the boundary condition corresponding to modified resistive and conductive sheets [5] under the condition that the strip thickness is small compared with the wavelength. Then the total field satisfies the boundary condition on the strip given by

$$H_x(x,+0) + H_x(x,-0) + 2R_m\left[E_z(x,+0) - E_z(x,-0)\right] = 0, \tag{1a}$$

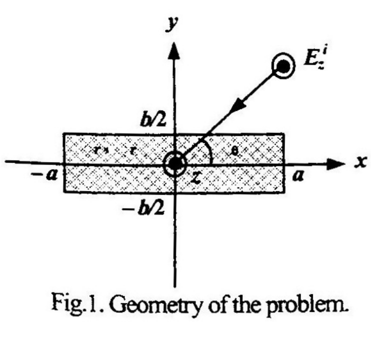



Fig.1. Geometry of the problem.

$$\left[\frac{1}{2R_e}+\frac{1}{2\tilde{R}_m}\left(1+\frac{1}{k^2}\frac{\partial^2}{\partial y^2}\right)\right][E_z(x,+0)+E_z(x,-0)]=H_x(x,+0)-H_x(x,-0) \tag{1b}$$

where $R_e = iZ/[kb(\varepsilon_r - 1)]$, $R_m = iY/[kb(\mu_r - 1)]$, $\tilde{R}_m = iZ\mu_r/[kb(\mu_r - 1)]$ with $Z$ and $Y$ being the intrinsic impedance and admittance, respectively. Using Green's formula, we can express the scattered as

$$E_z^s(x,y)=\frac{kZ}{4}\int_{-a}^{a}\left[J_z(x')-\frac{iY}{k}J_x^*(x')\frac{\partial}{\partial y}\right]H_0^{(1)}\left[k\sqrt{(x-x')^2+y^2}\right]dx', \tag{2}$$

where $H_0^{(1)}(\cdot)$ is the Hankel function of the first kind. In (2), $J_z(x)[\equiv H_x(x,+0)-H_x(x,-0)]$ and $J_x^*(x)[\equiv E_z(x,+0)-E_z(x,-0)]$ are the electric and magnetic current density functions, respectively.

Taking into account the boundary condition as given (1a,b), we obtain from (2) that

$$2ZR_mJ_x^*(x)=2\sin\theta_0 e^{-ikx\cos\theta_0}+\lim_{y\to 0}\frac{1}{2k}\frac{\partial^2}{\partial y^2}\int_{-a}^{a}J_x^*(x')H_0^{(1)}\left[k\sqrt{(x-x')^2+y^2}\right]dx', \tag{3a}$$

$$J_z(x)=-\left(\frac{1}{R_e}+\frac{\cos^2\theta_0}{\tilde{R}_m}\right)e^{-ikx\cos\theta_0}-\lim_{y\to 0}\frac{kZ}{2}\left[\frac{1}{2R_e}+\frac{1}{2\tilde{R}_m}\left(1+\frac{1}{k^2}\frac{\partial^2}{\partial y^2}\right)\right] \cdot\int_{-a}^{a}J_z(x')H_0^{(1)}\left[k\sqrt{(x-x')^2+y^2}\right]dx'. \tag{3b}$$

Taking the finite Fourier transform of (3a,b) over the interval $|x|<a$ and using the integral representation of the Hankel function, we are led to

$$\varsigma F_x(\beta)=4\sqrt{1-\alpha_0^2}\frac{\sin\kappa(\beta+\alpha_0)}{\kappa(\beta+\alpha_0)}-\frac{1}{\pi}\int_{-\infty}^{\infty}F_x(\alpha)\sqrt{1-\alpha^2}\frac{\sin\kappa(a-\beta)}{\alpha-\beta}d\alpha, \tag{4a}$$

$$F_z(\beta)=-2\left(\frac{1}{R_e}+\frac{\alpha_0^2}{\tilde{R}_m}\right)\frac{\sin\kappa(\beta+\alpha_0)}{\kappa(\beta+\alpha_0)}-\frac{Z}{2\pi}\left(\frac{1}{R_e}+\frac{1}{\tilde{R}_m}\right)\int_{-\infty}^{\infty}\frac{F_z(\alpha)}{\sqrt{1-\alpha^2}}\frac{\sin\kappa(\alpha-\beta)}{\alpha-\beta}d\alpha +\frac{Z}{2\pi\tilde{R}_m}\int_{-\infty}^{\infty}F_z(\alpha)\sqrt{1-\alpha^2}\frac{\sin\kappa(\alpha-\beta)}{\alpha-\beta}d\alpha, \tag{4b}$$

where $\kappa = ka$, $\varsigma = 2ZR_m$, and

$$F_x(\alpha)=\int_{-1}^{1}J_x^*(ax')e^{-i\kappa\alpha x'}dx',\quad F_z(\alpha)=\int_{-1}^{1}J_z(ax')e^{-i\kappa\alpha x'}dx'. \tag{5}$$

It can be shown that $J_x^*(x)$ and $J_z(x)$ are expanded in the form

$$J_x^*(x)=\sqrt{1-\left(\frac{x}{a}\right)^2}\sum_{n=0}^{\infty}J_n^xU_n\left(\frac{x}{a}\right),\quad J_z(x)=\frac{1}{a\sqrt{1-(x/a)^2}}\left[J_0^z+2\sum_{n=1}^{\infty}\frac{J_n^z}{n}T_n\left(\frac{x}{a}\right)\right], \tag{6}$$

where $T_n(\cdot)$ and $U_n(\cdot)$ denote the Chebyshev polynomial of the first and second kinds, respectively. In (6), $J_n^{x,z}$ for $n=0,1,2,\cdots$ are unknown coefficients to be determined. Substituting (6) into (4a,b), respectively and applying some properties of the Weber-Schafheitlin discontinuous integrals, we derive the two infinite systems of linear algebraic equations (SLAE) as in

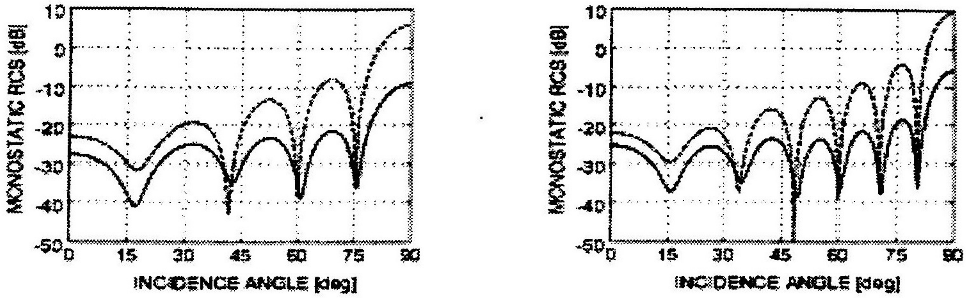
$$\sum_{n=0}^{\infty}(A_{mn}+\varsigma B_{mn})J_n^x=\gamma_m \tag{7a}$$

$$\sum_{n=0}^{\infty}X_n^z(-C_{mn}^v-D_{mn}^v+E_{mn}^v)=\gamma_m^v \tag{7b}$$

for $m=0,1,2,\cdots$, where $X_0^Z=J_0^z$, $X_n^Z=2(-i)^nJ_n^z/n$ for $n=1,2,3,\cdots$, and $v(>1)$ is a suitable parameter. In (7a,b), $A_{mn}, B_{mn}, C_{mn}^v, D_{mn}^v, E_{mn}^v, \gamma_m$, and $\gamma_m^v$ are known coefficients. The unknowns $J_n^{x,z}$ can be determined with high accuracy by solving (7a,b) numerically via truncation. In numerical computation, the value of $v$ has chosen as $v=2$.

## 3. NUMERICAL RESULTS AND DISCUSSION

We shall discuss the far field scattering characteristics of the strip based on numerical examples of the RCS. Figure 2 shows the monostatic RCS as a function of incidence angle, where the strip width is $2a=2\lambda$, $3\lambda$, and the strip thickness is




(a) $2a = 2\lambda$. (b) $2a = 3\lambda$.

Fig. 2. Monostatic RCS versus incidence angle for $b = 0.05\lambda$, $\varepsilon_r = 2.5 + i1.25$, $\mu_r = 1.6 + i0.8$.
: $E$ polarization; - - - - : $H$ polarization.

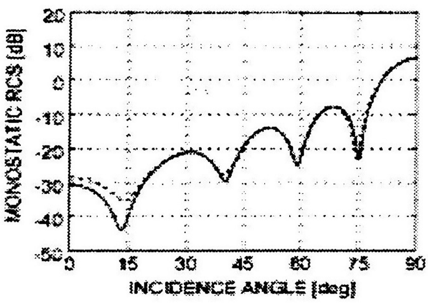



Fig. 3. Monostatic RCS versus incidence angle for $H$ polarization, $2a = 2\lambda$, $b = 0.05\lambda$, $\varepsilon_r = 4.0$, $\mu_r = 1.0$ and its comparison with Volakis [4].
: this paper; - - - - : Volakis.

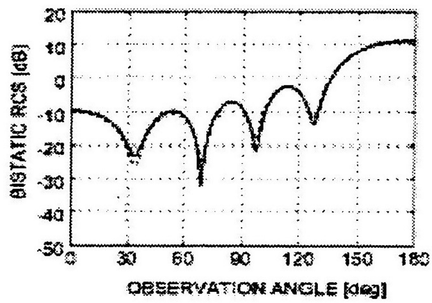



Fig. 4. Bistatic RCS versus observation angle for $H$ polarization, $\theta_0 = 1°$, $2a = 2\lambda$, $b = 0.05\lambda$, $\varepsilon_r = 1.5 + i0.1$, $\mu_r = 1.0$ and its comparison with Volakis [4].
: this paper; - - - - : Volakis.

$b = 0.05\lambda$. As an example of existing lossy materials, we have chosen the ferrite with $\varepsilon_r = 2.5 + i1.25$ and $\mu_r = 1.6 + i0.8$ in numerical computation. It is obvious that the peaks at $90°$ in the figure correspond to the specular reflection from the upper surface of the strip. We also notice that the RCS shows sharp oscillation with an increase of the strip width as can be expected. It is interesting to note that the RCS level for the $E$ polarization is lower than that for the $H$ polarization over the whole range of the incidence angle. This problem has previously been analyzed by Volakis [4] for the $H$-polarized case based on the plane wave spectrum method together with the extended spectral ray method. In the formulation, Volakis employed the same approximate boundary conditions as in this paper. Figures 3 and 4 show comparisons with the results obtained by Volakis, where the monostatic RCS as a function of incidence angle and the bistatic RCS as a function of observation angle are illustrated. It is seen from the figure that our results agree reasonably well with Volakis's results.

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