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SOLUTION OF THE PLANE WAVE DIFFRACTION PROBLEM BY AN IMPEDANCE STRIP USING A NUMERICAL-ANALYTICAL METHOD: *E*-POLARIZED CASE

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Abstract–In this study, the diffraction of a plane wave by an infinitely long strip, having the same impedance on both faces with a width of $2a$ is investigated. The diffracted field is expressed by an integral in terms of the induced electric and magnetic current densities. Applying the boundary condition to the integral representation of the scattered field, the problem is formulated as simultaneous integral equations satisfied by the electric and magnetic current density functions. By obtaining the Fourier transform of the integral equations the unknown current density functions can be expanded into the infinite series containing the Chebyshev polynomials. This leads to two infinite systems of linear

algebraic equations satisfied by the expansion coefficients. These coefficients are determined numerically with high accuracy via appropriate truncation of the systems of linear algebraic equations. Evaluating the scattered field asymptotically, a far field expression is derived. Some illustrative numerical examples on the monostatic and bistatic radar cross section (RCS) are presented and the far field scattering characteristics are discussed.

1. INTRODUCTION

The solution of canonical problems such as half-plane, cylinder or sphere are important in the sense of diffraction theory and strip is one of the most important canonical structures. First of all, due to its geometry, strips are frequently used to investigate the multiple diffraction phenomenon. Furthermore, especially in remote sensing, a large number of practical problems can be simulated by conducting, resistive or impedance strips. On the other hand, diffraction by a slit in an infinite conducting plane can be reduced to a perfectly conducting strip problem by using the duality principle. Scattering from gaps or cracks that may exist on the surface of an obstacle, which is entirely or partially filled with some material can provide a significant contribution to the overall scattering pattern. In such problems the gaps or cracks may be simulated by strips and/or slits. Therefore, due to its conformity

to many practical problems, strips have been extensively investigated by many authors by using different analytical and numerical methods [1–9].

The development in numerical techniques for the solution of the scattering problems has always been parallel to the developments in computer technology. Although numerical methods may be considered as more straightforward compared to analytical methods, due to the matrix inversion procedure for the analysis, computer capacity restricts the size of the problem that can be handled. Generally, for the obstacles which have a maximum dimension of a few wavelengths, numerical methods can provide accurate solutions. Although the integral equations are usually solved by numerical methods, they can also be converted to a set of algebraic equations by using some analytical methods. Then the obtained matrix equation can be solved by standard matrix inversion algorithms. The time required for the solution of this matrix equation is proportional to the size of the resultant matrix. So for large bodies, especially for RCS estimation the time required can be enormously large; therefore, the size of the matrix must be kept as small as possible.

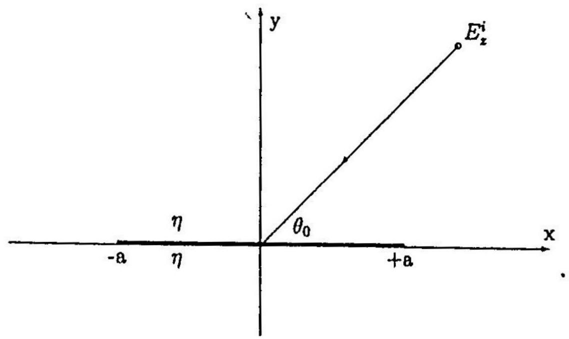
It is a well known fact that the electrical size of the body limits the tractability of numerical methods while the geometrical complexity of the object restricts the applicability of the analytical methods. For the asymptotic solution of a problem in the high-frequency region, hybrid methods are used besides techniques based upon the extension of classical optics. Hybrid methods incorporating both numerical and high frequency asymptotic techniques may have the potential to enlarge the class of electromagnetic scattering problems that can be treated.

The hybrid approach can be formulated as a field-based analysis where the GTD (geometrical theory of diffraction) solution for the field associated with edge or surface diffraction are used as the starting point. These solutions serve as the ansatz to the MM (moment method) formulation and represent the parts of a scatterer not conforming to a canonical geometry which is not amenable to a GTD solution itself [10, 11]. Alternatively current-based formulation is possible where the analysis proceeds from ansatz solutions for the currents obtained from physical optics and PTD (physical theory of diffraction) [12, 13].

Although there are many powerful analytical techniques, the main advantage of numerical techniques is that they may be applied to a scatterer of arbitrary shape and are generally only limited by the size of the scatterer. But this limitation is a practical problem. Theoretically, a set of linear equations which describe the scattering problem can be generated but the obtained set may be too large to be solved. Fortunately, the developments in computer technology make possible the solution of many electromagnetics problem for a desired degree of accuracy. In contrast, the asymptotic techniques work best when the scatterer size is large compared to the wavelength. Unfortunately the difficulty of the problem increases when the complex shaped bodies are of interest. Use of analytical and numerical methods together may overcome these restrictions. The numerical methods are limited to the bodies having a maximum dimension of less than few wavelengths, whereas the analytical methods yield accurate results for the scatterer much larger than those of one wavelength. So, these methods may be combined to solve the scattering problems involving scatterers of intermediate size and size in the resonance region. Additionally by using analytical-numerical methods the computation time may be reduced to a reasonable level.

An alternative method was developed by Veliev et al. [14], where the solution comprises any preassigned accuracy. The scattered field was represented using the Fourier transform of the corresponding surface current density which offers a number of advantages for constructing the solution of the problem. A hybrid technique based on the semi-inversion procedure for equation operators and the method of moments was used to obtain the desired solution. The essentials of the solution and its application to the wave scattering by polygonal cylinders and flat conducting strip structures are given by Veliev and Veremey [15]. This analytical-numerical method, using a spectral approach, reduces the problem to a system of linear algebraic equations for the unknown Fourier coefficients of the current density function. Appropriate truncation of the infinite system of equations can yield the solution with any desired accuracy. It should be noted that the applicability of the truncation method cannot always be justified, and the matrix elements associated with the system of linear algebraic equations usually decay slowly with an increase of their index.

The aims of the present study are to obtain a solution which may work in a wide frequency range and to reduce the computation time to a reasonable level. In this study, diffraction by an impedance strip is investigated by using the analytical-numerical technique proposed by



Veliev and Veremey [15]. In Section 2, the formulation of the problem for E-polarized case is given. By expressing the electric and magnetic current as infinite series in terms of Gegenbauer polynomials, two integral equations in spectral domain for electric and magnetic currents are derived. In Section 3, the integral equations are reduced to a system of linear algebraic equations for both currents with some unknown coefficients. In Section 4 some physical quantities are represented in terms of the unknown coefficients which will be determined by solving the system of linear algebraic equations. In Section 5 the results of branch-cut integrals are presented and the curves for both far field and RCS are represented. The results are compared with some previously obtained results.

2. FORMULATION OF THE PROBLEM

The scatterer is a strip of width $2a$ where same impedance is assumed to be imposed on both sides. The geometry of the problem is illustrated in Fig. 1, where η denotes the normalized impedance of the strip.

Since the strip is uniform along the z-axis, the problem can be reduced to a two dimensional problem. The time dependence of the fields is assumed as $\exp(-i\omega t)$ and suppressed throughout the analysis. The incident field is given as a linearly polarized plane wave

$$E_z^i(x,y) = \exp\left[-ik\left(x\alpha_0 + y\sqrt{1-\alpha_0^2}\right)\right]. \qquad (1)$$

where $\alpha_0 = \cos\theta_0$ with θ_0 denoting the incidence angle. The total field will be expressed as the sum of the incident and scattered fields for all y, such as,

$$E_z(x,y) = E_z^i(x,y) + E_z^s(x,y). \tag{2}$$

On the strip, the total field must satisfy the Leontovich boundary condition which is frequently called as impedance boundary condition, given by,

$$\left\{\frac{\partial E_z(x,y)}{\partial y} \pm \frac{ik}{\eta} E_z(x,y)\right\} |_{y=\pm 0}= 0 \quad \text{for} \quad | x |< a. \tag{3}$$

where k is the propagation constant.

By considering the magnetic and electric currents which are denoted by $f_e(x)$ and $f_m(x)$ respectively, the integral representation of the total electric field can be obtained as [16]:

$$E_z(x,y) = E_z^i(x,y) - \frac{i}{4}\int_{-a}^{a}\left\{f_e(x') + f_m(x')\frac{\partial}{\partial y}\right\}H_0^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right)dx' \tag{4}$$

where

$$f_e(x) = H_x(x,+0) - H_x(x,-0), \tag{5}$$

and,

$$f_m(x) = E_z(x,+0) - E_z(x,-0). \tag{6}$$

2.1. Application of Boundary Conditions

If we rewrite the Leontovich boundary condition for $y = +0$ and $y = -0$, and if we subtract and add these two equations we can obtain the following expressions:

$$f_e(x) + \frac{ik}{\eta}\{E_z(x,+0) + E_z(x,-0)\} = 0, \tag{7}$$

and

$$f_m(x) + \frac{\eta}{ik}\left\{\frac{\partial E_z(x,y)}{\partial y}|_{y=+0} + \frac{\partial E_z(x,y)}{\partial y}|_{y=-0}\right\} = 0. \tag{8}$$

The expressions of the total electric field for $y = +0$ and for $y = -0$ can be obtained from (4) and if the resultant equations are substituted into (7) and by considering the following equation [16]

$$\left(\lim_{y\to+0} + \lim_{y\to-0}\right)\int_{-a}^{a} f_m(x')\frac{\partial}{\partial y}H_0^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right)dx' = 0 \tag{9}$$

it yields that,

$$-\frac{\eta}{ik}f_e(x) = 2E_z^i(x,0) - \frac{i}{2}\int_{-a}^{a} f_e(x')H_0^{(1)}(k\mid x-x'\mid)dx'. \tag{10}$$

In a similar way, an integral equation for $f_m(x)$, is obtained from (8) and (4) as follows:

$$f_m(x) = 2\eta\sqrt{1-\alpha_0^2}e^{-ikx\alpha_0} + \frac{\eta}{4k}\left(\lim_{y\to+0} + \lim_{y\to-0}\right)\frac{\partial}{\partial y} \cdot \int_{-a}^{a} f_m(x')\frac{\partial}{\partial y}H_0^{(1)}\left(k\sqrt{(x-x')^2+y^2}\right)dx'. \tag{11}$$

2.2. Fourier Transformation of the Integral Equations

Substituting the integral representation of Hankel function

$$H_0^{(1)}\left(k\sqrt{(x-x')^2+(y-y')^2}\right) = \frac{1}{\pi}\int_{\infty}^{\infty} e^{ik[(x-x')\alpha+|y-y'|\sqrt{1-\alpha^2}]}\frac{d\alpha}{\sqrt{1-\alpha^2}} \tag{12}$$

into (10) with the following variable changes

$$x = a\zeta, \quad x' = a\zeta', \quad \text{and} \quad \xi = ka \tag{13}$$

yields that,

$$-\frac{\eta}{\xi}F_e(\beta) = 4i\frac{\sin\xi(\beta+\alpha_0)}{\xi(\beta+\alpha_0)} + \frac{1}{\xi\pi}\int_{-\infty}^{\infty}\frac{F_e(t)}{\sqrt{1-t^2}}\frac{\sin\xi(t-\beta)}{(t-\beta)}dt \tag{14}$$

where

$$F_e(\beta) = \int_{-1}^{+1}\tilde{f}_e(\zeta)e^{-i\xi\beta\zeta}d\zeta, \quad \text{with} \quad \tilde{f}_e(\zeta) = af_e(a\zeta). \tag{15}$$

This is the integral equation of $f_e(x)$ in spectral domain for the E-polarized case.

In a similar way, by using the after-mentioned variable changes given in (13) and integral representation of Hankel function in (12), in (11) the integral equation of $f_m(x)$ in spectral domain for the E-polarized case is obtained as follows

$$\frac{1}{\eta}F_m(\beta) = 4\sqrt{1-\alpha_0^2}\frac{\sin\xi(\beta+\alpha_0)}{\xi(\beta+\alpha_0)} - \frac{1}{\pi}\int_{-\infty}^{\infty} F_m(t)\sqrt{1-t^2}\frac{\sin\xi(t-\beta)}{t-\beta}dt \tag{16}$$

where

$$F_m(\beta) = \int_{-1}^{+1} \tilde{f}_m(\zeta)e^{-i\xi\beta\zeta}d\zeta, \quad \text{with} \quad \tilde{f}_m(\zeta) = f_m(a\zeta). \tag{17}$$

3. REDUCTION OF INTEGRAL EQUATIONS TO SYSTEMS OF LINEAR ALGEBRAIC EQUATIONS

The solution of the integral equations for electric (F_e) and magnetic (F_m) current densities will be reduced to the solution of two uncoupled systems of linear algebraic equations. The first step of the reduction process is to express the current densities in Fourier transform domain and obtain a general spectral expression for currents. Then by using the constraints implied by edge conditions, electric and magnetic current density expressions will be obtained in the transform domain. Finally, they will be written in the form of infinite system of linear algebraic equations involving Gegenbauer polynomial coefficients as unknowns.

3.1. General Expressions for Current Density Functions in Transform Domain

Since it is necessary to express the current functions in spectral domain, the Fourier transform $F(\beta)$ of the current density function $\tilde{f}(\zeta)$ must be found as

$$F(\beta) = \int_{-1}^{+1} \tilde{f}(\zeta)e^{-i\xi\beta\zeta}d\zeta. \tag{18}$$

The current density function $\tilde{f}(\zeta)$ is defined for $|\zeta| \leq 1$ and it is zero elsewhere. Let $\tilde{f}(\zeta)$ be represented by a uniformly convergent

series, such as,

$$\tilde{f}(\zeta) = (1-\zeta^2)^{\nu}\sum_{n=0}^{\infty} f_n C_n^{\nu+\frac{1}{2}}(\zeta) \tag{19}$$

where $C_n^{\nu+\frac{1}{2}}(\zeta)$ denote the Gegenbauer polynomials and ν is a constant related to the edge condition. The value of ν in (19) will be determined by enforcing the functions such as to satisfy the edge conditions for electric and magnetic current densities separately. For the electric current density function $\tilde{f}_e(\zeta)$, and the magnetic current density function $\tilde{f}_m(\zeta)$, from Meixner's edge conditions [17], ν can be determined for $\zeta \to 0$ as,

$$\tilde{f}_e(\zeta) = O\left(\zeta^{-1/2}\right) \quad \text{and} \quad \tilde{f}_m(\zeta) = O\left(\zeta^{1/2}\right). \tag{20}$$

The order relations given above for electric and magnetic current densities can be obtained respectively as $\nu = -1/2$ and $\nu = 1/2$ by considering the asymptotic behavior of Gegenbauer polynomials together with (19). The Fourier transform of the current density function can be derived as follows:

$$F(\beta) = \frac{2\pi}{\Gamma\left(\nu+\frac{1}{2}\right)}\sum_{n=0}^{\infty}(-i)^n f_n \frac{\Gamma(n+2\nu+1)}{\Gamma(n+1)}\frac{J_{n+\nu+\frac{1}{2}}(\xi\beta)}{(2\xi\beta)^{\nu+\frac{1}{2}}}. \tag{21}$$

This completes the calculation for the Fourier transform of a current element represented in terms of Gegenbauer polynomials given by (19). Since no restriction is imposed on the current series expression during the derivation, it is obvious that this representation is valid for both electric and magnetic currents. Now considering the edge conditions separately for electric and magnetic current components, the corresponding spectral expressions in the Fourier domain can easily be obtained. First, the magnetic current density will be obtained simply from (21) by substituting $\nu = 1/2$, which yields

$$F_m(\beta) = \pi\sum_{n=0}^{\infty}(-i)^n(n+1)f_n^m\frac{J_{n+1}}{\xi\beta} \tag{22}$$

and from (19)

$$\tilde{f}_m = (1-\zeta^2)^{\frac{1}{2}}\sum_{n=0}^{\infty} f_n^m C_n^1(\zeta) \tag{23}$$

or

$$\tilde{f}_m(\zeta) = \left(1-\zeta^2\right)^{\frac{1}{2}} \sum_{n=0}^{\infty} f_n^m U_n(\zeta) \tag{24}$$

where $U_n(\zeta)$ is the Chebyshev polynomial of second type and $U_n(\zeta) = C_n^1(\zeta)$.

Similarly, by inserting $\nu = -1/2$ in (21) the Fourier transform of the electric current density function can be derived as

$$F_e(\beta) = \pi f_0^e J_0(\xi\beta) + 2\pi \sum_{n=1}^{\infty} \frac{f_n^e}{n}(-i)^n J_n(\xi\beta) \tag{25}$$

and from (19)

$$\tilde{f}_e(\zeta) = \left(1-\zeta^2\right)^{-\frac{1}{2}} \sum_{n=0}^{\infty} f_n^e C_n^0(\zeta) \tag{26}$$

or

$$\tilde{f}_e = (1-\zeta^2)^{-\frac{1}{2}} \left\{ f_0^e + 2\sum_{n=1}^{\infty} \frac{f_n^e}{n} T_n(\zeta) \right\} \tag{27}$$

where $T_n(\zeta)$ is the Chebyshev polynomial of first type, and $2T_n(\zeta) = nC_n^0(\zeta)$. As seen from these equations the unknown coefficients appear in both current density functions and Fourier transform of them are identical.

3.2. System of Linear Algebraic Equations for f_n^e

In a convenient form, the Fourier transform of $\tilde{f}_e(\zeta)$ for $\nu = -\frac{1}{2}$ was obtained as

$$F_e(\beta) = \pi \sum_{n=0}^{\infty} X_n J_n(\xi\beta) \tag{28}$$

where

$$X_n = f_0^{(e)} \quad \text{for} \quad n = 0 \tag{29}$$

and

$$X_n = 2(-i)^n \frac{f_n^e}{n} \quad \text{for} \quad n \neq 0. \tag{30}$$

If (28) is substituted into (14), the problem is reduced to that of finding

the unknowns f_n^e with $n = 0, 1, 2, \cdots$ as follows:

$$-\frac{\eta}{\xi}\pi \sum_{n=0}^{\infty} X_n J_n(\xi\beta) = 4i \frac{\sin\xi(\beta+\alpha_0)}{\xi(\beta+\alpha_0)} + \frac{1}{\xi}\sum_{n=0}^{\infty} X_n \int_{-\infty}^{\infty} \frac{\sin\xi(t-\beta)}{t-\beta} \frac{1}{\sqrt{1-t^2}} J_n(\xi t)dt. \tag{31}$$

In order to be able to express (31) in a more convenient form for numerical calculations, both sides of the equations will be multiplied by

$$\frac{J_{l+\tau}(\xi\beta)}{\beta^\tau} \quad \text{for} \quad l = 0, 1, 2\cdots$$

and by integrating each term with respect to β from $-\infty$ to ∞

$$-\eta\pi \sum_{n=0}^{\infty} X_n \int_{-\infty}^{\infty} \frac{J_{l+\tau}(\xi\beta)J_n(\xi\beta)}{\beta^\tau} d\beta = 4i\pi(-1)^l \frac{J_{l+\tau}(\xi\alpha_0)}{\alpha_0^\tau} + \sum_{n=0}^{\infty} X_n \int_{-\infty}^{\infty} \pi \frac{J_{l+\tau}(\xi t)}{t^\tau} \frac{J_n(\xi t)}{\sqrt{1-t^2}} dt \tag{32}$$

is obtained. The integral on the left-hand side of (32) is denoted as

$$d_{ln}^{E1} = \int_{-\infty}^{\infty} \frac{J_{l+\tau}(\xi\beta)J_n(\xi\beta)}{\beta^\tau} d\beta \tag{33}$$

and the integral on the right hand side is named as

$$D_{ln}^{E1} = \int_{-\infty}^{\infty} \frac{J_{l+\tau}(\xi t)J_n(\xi t)}{t^\tau} \frac{1}{\sqrt{1-t^2}} dt. \tag{34}$$

So,

$$-\eta \sum_{n=0}^{\infty} X_n d_{ln}^{E1} = \gamma_l^{E1} + \sum_{n=0}^{\infty} X_n D_{ln}^{E1} \tag{35}$$

is derived with

$$\gamma_l^{E1} = 4i(-1)^l \frac{J_{l+\tau}(\xi\cos\theta_0)}{\cos^\tau\theta_0}. \tag{36}$$

Now (35) can be rearranged simply as

$$-\gamma_l^{E1} = \sum_{n=0}^{\infty} X_n \left(\eta d_{ln}^{E1} + D_{ln}^{E1}\right) \tag{37}$$

which gives an infinite system of linear algebraic equations for f_n^e.

3.3. System of Linear Algebraic Equations for f_n^m

The Fourier transform of $\tilde{f}_m(\zeta)$ for $\nu = \frac{1}{2}$ was obtained as,

$$F_m(\beta) = \pi \sum_{n=0}^{\infty} Y_n \frac{J_{n+1}(\xi\beta)}{\xi\beta} \tag{38}$$

where

$$Y_n = (-i)^n (n+1) f_n^m. \tag{39}$$

If this equation is substituted into (16)

$$\frac{\pi}{\eta} \sum_{n=0}^{\infty} Y_n \frac{J_{n+1}(\xi\beta)}{\xi\beta} = 4\sqrt{1-\alpha_0^2} \frac{\sin\xi(\beta+\alpha_0)}{\xi(\beta+\alpha_0)} - \sum_{n=0}^{\infty} Y_n \left\{ \int_{-\infty}^{\infty} \frac{\sin\xi(t-\beta)}{t-\beta} \frac{J_{n+1}(\xi t)}{\xi t} \sqrt{1-t^2}dt \right\} \tag{40}$$

is derived. By multiplying both sides of (40) by $\beta^{-1}J_{l+1}(\xi\beta)$ and integrating each term with respect to β from $-\infty$ to ∞, it yields that

$$\gamma_l^{E2} = \sum_{n=0}^{\infty} Y_n \left(D_{ln}^{E2} + \frac{1}{\eta} d_{ln}^{E2} \right), \quad l = 0, 1, 2 \cdots \tag{41}$$

which is the system of linear algebraic equations for f_n^m, where

$$d_{ln}^{E2} = \int_{-\infty}^{\infty} \frac{J_{n+1}(\xi\beta) J_{l+1}(\xi\beta)}{\beta^2} d\beta \tag{42}$$

$$\gamma_l^{E2} = -(-1)^l 4 \frac{\sqrt{1-\alpha_0^2}}{\alpha_0} J_{l+1}(\xi\alpha_0), \tag{43}$$

and

$$D_{ln}^{E2} = \int_{-\infty}^{\infty} J_{n+1}(\xi t) J_{l+1}(\xi t) \frac{\sqrt{1-t^2}}{t^2} dt. \tag{44}$$

4. FIELD ANALYSIS

The total field expression in (4), involving the electric and magnetic current densities as unknowns is the fundamental formula for field analysis. As stated in (4), the scattered field expression was given as

$$E_z^s(x,y) = -\frac{i}{4} \int_{-a}^{a} \left\{ f_e(x') + f_m(x') \frac{\partial}{\partial y} \right\} H_0^1 \left(k\sqrt{(x-x')^2 + y^2} \right) dx'.$$

The asymptotic expression of Hankel function for large argument is used to get the scattered far field as follows:

$$E_z^s(r,\theta) = -\frac{i}{4} \sqrt{\frac{2}{\pi i k r}} e^{ikr} \int_{-1}^{1} \left\{ \tilde{f}(\zeta') + \frac{\xi \sin\theta}{4} \tilde{f}_m(\zeta') \right\} e^{-i\xi\zeta' \cos\theta} d\zeta'. \tag{45}$$

Then, let the scattered field be expressed as

$$E_z^s(r,\theta) = A(kr)\phi(\theta) \tag{46}$$

where,

$$A(kr) = \sqrt{\frac{2}{\pi k r}} e^{i(kr \frac{\pi}{4})} \tag{47}$$

and,

$$\phi(\theta) = \phi_e(\theta) + \phi_m(\theta). \tag{48}$$

In this case $\phi(\theta)$ represents the far field total radiation pattern. The integrals in (45) are the Fourier transforms of the current density functions and by using (28) and (38) the $\phi_e(\theta)$ and $\phi_m(\theta)$ can be obtained as follows:

$$\phi_e(\theta) = \frac{\pi}{4} \sum_{n=0}^{\infty} X_n J_n(\epsilon \cos\theta) \tag{49}$$

and,

$$\phi_m(\theta) = \frac{\xi \sin\theta}{4} \pi \sum_{n=0}^{\infty} Y_n \frac{J_{n+1}(\xi\cos\theta)}{\xi\cos\theta}. \tag{50}$$

The total scattering cross section can be calculated as

$$\frac{\sigma_s}{4a} = -\frac{1}{\xi} Re\{\phi(\theta)\} \tag{51}$$

where θ is the incident angle. It is obvious that the calculation of the RCS requires to know the values of X_n and Y_n. It should be clear that the determination of X_n and Y_n is reduced to numerical evaluation of D_{ln}^{E1}, D_{ln}^{E2}, d_{ln}^{E1} and d_{ln}^{E2}.

5. NUMERICAL ANALYSIS

As shown in the previous section the analysis of the scattered field is reduced to the numerical evaluation of the functions $d_{ln}^{E1,2}$ and $D_{ln}^{E1,2}$.

The integral expressions of these terms given by equations (33), (42), (34) and (44) are not convenient for numerical calculations. Therefore, these integrals must be evaluated in terms of some well known functions which are convenient for numerical calculations. Therefore by using some analytical methods these integrals are evaluated as follows:

$$d_{ln}^{E1} = \frac{1}{\xi}\left(\frac{\xi}{2}\right)^{\tau} \frac{[1+(-1)^{l+n}]\Gamma(\tau)\Gamma\left(\frac{n+l+1}{2}\right)}{\Gamma\left(\frac{n-l+1}{2}\right)\Gamma\left(\frac{l-n+1+2\tau}{2}\right)\Gamma\left(\frac{n+l+2\tau+1}{2}\right)} \tag{52}$$

$$d_{ln}^{E2} = \frac{8\xi(-1)^{\frac{n-l}{2}}}{\pi(n+l+3)(n+l+1)\{1-(n-l)^2\}} \tag{53}$$

$$\begin{aligned}
D_{ln}^{E1} = &\left\{1+(-1)^{l+n}\right\}\Bigg\{\frac{\xi^{l+n+\lambda+\tau}}{2}\sum_{k=0}^{\infty} h_{kln}^{\lambda,\tau}\xi^{2k}\frac{\Gamma\left(k+\frac{l+n}{2}+\frac{1}{2}\right)}{\Gamma\left(k+\frac{l+n}{2}+1\right)} \\
&- i\Bigg\{\sum_{k=0}^{\frac{l+n}{2}-1}\frac{1}{2\pi}\frac{\Gamma\left(-k+\frac{l+n}{2}\right)\Gamma\left(k+\frac{1}{2}\right)\Gamma\left(k+\frac{\lambda+\tau+1}{2}\right)}{\Gamma(k+1)\Gamma\left(-k+\frac{l+n}{2}+l+\lambda+1\right)} \\
&\times\frac{\Gamma\left(k+\frac{\lambda+\tau}{2}+1\right)}{\Gamma\left(k+\frac{l+n}{2}+\lambda+\tau+1\right)\Gamma\left(-k+\frac{l+n}{2}+n+\tau+1\right)} \\
&\cdot\xi^{2k+\lambda+\tau}W_n - \sum_{k=0}^{\infty}\frac{1}{2\pi}\frac{\Gamma\left(k+\frac{l+n+1}{2}\right)}{\Gamma\left(k+\frac{l+n}{2}+1\right)}h_{kln}^{\lambda,\tau}\xi^{2k+l+n+\lambda+\tau} \\
&\times\Bigg\{2\ln\xi+\Psi\left(k+\frac{l+n+1}{2}\right)+\Psi\left(k+\frac{l+n+\lambda+\tau+1}{2}\right) \\
&+\Psi\left(k+\frac{l+n+\lambda+\tau}{2}+1\right)-\Psi(k+1)-\Psi\left(k+\frac{l+n}{2}+1\right) \\
&-\Psi(k+l+\tau+1) \\
&-\Psi(k+n+\lambda+1)-\Psi(k+l+n+\lambda+\tau+1)\Bigg\}\Bigg\}\Bigg\}
\end{aligned} \tag{54}$$

where $\tau = 1$, $\lambda = 0$ and $W_n = 0$, for $l+n=0$, $W_n = 1$, for $l+n \neq 0$. Similarly,

$$\begin{aligned}
D_{ln}^{E2} = &\left\{1+(-1)^{l+n}\right\}\Bigg\{\frac{\xi^{l+n+\lambda+\tau}}{4}\sum_{k=0}^{\infty} h_{kln}^{\lambda,\tau}\xi^{2k}\frac{\Gamma\left(k+\frac{l+n}{2}+\frac{1}{2}\right)}{\Gamma\left(k+\frac{l+n}{2}+2\right)} \\
&+ i\Bigg\{\sum_{k=-1}^{\frac{l+n}{2}-1}\frac{1}{4\pi}\frac{\Gamma\left(-k+\frac{l+n}{2}\right)\Gamma\left(k+\frac{1}{2}\right)\Gamma\left(k+\frac{\lambda+\tau+1}{2}\right)}{\Gamma(k+2)\Gamma\left(-k+\frac{m+n}{2}+m+\tau+1\right)} \\
&\times\frac{\Gamma\left(k+\frac{\lambda+\tau}{2}+1\right)}{\Gamma\left(k+\frac{l+n}{2}+\lambda+\tau+1\right)\Gamma\left(-k+\frac{l+n}{2}+n+\lambda+1\right)} \\
&\cdot\xi^{2k+\lambda+\tau} - \sum_{k=0}^{\infty}\frac{1}{4\pi}\frac{\Gamma\left(k+\frac{l+n+1}{2}\right)}{\Gamma\left(k+\frac{l+n}{2}+2\right)}h_{kln}^{\lambda,\tau}\xi^{2k+l+n+\lambda+\tau} \\
&\times\Bigg\{2\ln\xi+\Psi\left(k+\frac{m+n+1}{2}\right)+\Psi\left(k+\frac{l+n+\lambda+\tau+1}{2}\right) \\
&+\Psi\left(k+\frac{l+n+\lambda+\tau}{2}+1\right)-\Psi(k+1) \\
&-\Psi\left(k+\frac{l+n}{2}+2\right)-\Psi(k+l+\tau+1) \\
&-\Psi(k+n+\lambda+1)-\Psi(k+l+n+\lambda+\tau+1)\Bigg\}\Bigg\}\Bigg\}
\end{aligned} \tag{55}$$

is obtained, where $\tau = 1$, $\lambda = 1$ and

$$\Psi(z) = \frac{\Gamma'(z)}{\Gamma(z)}, \tag{56}$$

$\Gamma(.)$ denotes the Gamma function and,

$$h_{kln}^{\lambda,\tau} = (-1)^k \frac{\Gamma\left(k+\frac{l+n+\lambda+\tau+1}{2}\right)\Gamma\left(k+\frac{l+n+\lambda+\tau+2}{2}\right)}{\Gamma(k+1)\Gamma(k+l+n+\lambda+\tau+1)\Gamma(k+n+\lambda+1)\Gamma(k+l+\tau+1)}. \tag{57}$$

5.1. Asymptotic Expressions

The asymptotic expressions of the far field radiation pattern can be easily derived by using the following assumptions

$$\lim_{\xi\to 0}\frac{J_1(\xi\alpha_0)}{\alpha_0}=\frac{\xi}{2}+O(\xi^2) \tag{58}$$

and,

$$\lim_{\xi\to\infty}\frac{\sin\xi(t-\beta)}{t-\beta}=\pi\delta(t-\beta). \tag{59}$$

For high frequency asymptotic expression, substitute (60) into (14) and (16) to get,

$$F_e(\beta)=4i\frac{\sqrt{1-\beta^2}}{1+\eta\sqrt{1-\beta^2}}\frac{\sin[\xi(\beta+\alpha_0)]}{\beta+\alpha_0} \tag{60}$$

and,

$$F_m(\beta)=4\eta\frac{\sqrt{1-\alpha_0^2}}{1+\eta\sqrt{1-\beta^2}}\frac{\sin[\xi(\beta+\alpha_0)]}{\xi(\beta+\alpha_0)}. \tag{61}$$

So the far field radiation pattern due to electric and magnetic currents as $\xi\to\infty$ is derived from (49) and (50) as

$$\phi(\theta)=i\frac{\sqrt{1-\beta^2}}{1+\eta\sqrt{1-\beta^2}}\frac{\sin[\xi(\beta+\alpha_0)]}{\beta+\alpha_0}+\eta\frac{\sqrt{1-\beta^2}\sqrt{1-\alpha_0^2}}{1+\eta\sqrt{1-\beta^2}}\frac{\sin[\xi(\beta+\alpha_0)]}{\xi(\beta+\alpha_0)}, \tag{62}$$

where $\beta=\cos(\theta)$. For the low frequency asymptotic expressions, (37) can be rewritten as

$$-\gamma_0^{E1}\approx X_0\left(\eta d_{00}^{E1}+D_{00}^{E1}\right)+O(\xi^2) \tag{63}$$

and by using (59) in (36)

$$\gamma_0^{E1}\approx 2i\xi+O\left(\xi^2\right). \tag{64}$$

If the expressions of d_{00}^{E1} and D_{00}^{E1} are derived from (52) and (54) and the resultant expressions are substituted into (37) one can get that,

$$X_0=f_e^0=-\frac{2i\xi}{\frac{4}{\pi}\eta+\frac{\xi}{2}\left[\pi+i\left(\gamma+\ln\frac{\xi}{4}\right)\right]}+O\left(\xi^2\right) \tag{65}$$

where $\gamma=0.5772$. By the same way, it is easily derived from (41) that,

$$Y_0=-\frac{6\xi\pi\eta\sqrt{1-\alpha_0^2}}{8\xi-\frac{i}{8}3\pi\eta}. \tag{66}$$

Finally by substituting (66) and (67) into (49) and (50) respectively, the low frequency asymptotic expressions of far field radiation patterns are obtained as:

$$\phi_m=O\left(\xi^2\right) \tag{67}$$

and,

$$\phi(\theta)=\phi_e(\theta)=-\frac{1}{2}\frac{i\pi\xi}{\frac{4}{\pi}\eta+\frac{\xi}{2}\left[\pi+i\left(\gamma+\ln\frac{\xi}{4}\right)\right]}+O\left(\xi^2\right). \tag{68}$$

6. CONCLUSION

The method used in this approach is a hybrid method named as analytical-numerical method. As in general, the aim of using hybrid methods is to eliminate the disadvantages of the analytical methods which operate well at high frequencies and of the numerical methods which operate well at low frequencies. In other words its aim is, to obtain an accurate solution for a wide frequency range. By using this method some physical quantities such as magnetic current density Fig. 2a and Fig. 2b, electric current density Fig. 3a and Fig. 3b along the strip and scattered far field Fig. 4, etc.., may be obtained without any modification on the computer programs.

By comparing the figure obtained by Herman and Volakis [5] and the Fig. 4a obtained by using this method it may be concluded that our results are much close to the results obtained by using the method of moment. For $\eta=0.1-i0.27$ and $\eta=0.1+i0.27$ the results are almost same in both figures. But for $\eta=1.1$ Herman's result is different than the result obtained by using method of moment. Probably this may be explained by the accuracy of this method for different values of the strip impedance and for different values of the incidence angle.

It is well known that the electromagnetic dual to a resistive strip is a magnetically conductive strip supporting only a magnetic current. For an impedance strip both electric and magnetic currents are represented. It is obvious that these currents are uncoupled. Therefore,

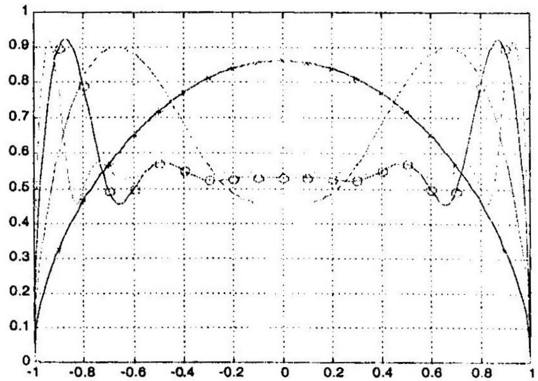


Figure 2a. Magnetic current density versus x/a for $\eta = 0.1 + i0.27$, $\theta = 90°$. (*) $ka = 0.1\pi$, (+) $ka = \pi$, (o) $ka = 2.5\pi$, (-) $ka = 5\pi$.

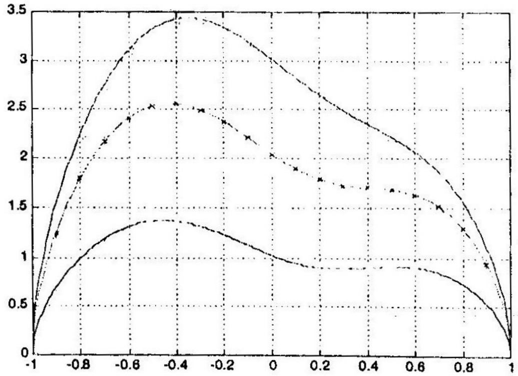


Figure 2b. Magnetic current density versus x/a for $\eta = 0.0 + i1.1$, $ka = \pi$. (o) $\theta = 45°$. (*) $\theta = 30°$, (+) $\theta = 15°$.

from the solution of the impedance strip we can easily obtain the results for a resistive strip by considering only the electric current and for a conductive strip by considering only the magnetic current. Echowidth versus incidence angle graphics are represented for both resistive strip Fig. 4b and conductive strip Fig. 4c to compare with Volakis results [5].

As shown in Section 5 both low and high frequency asymptotics can

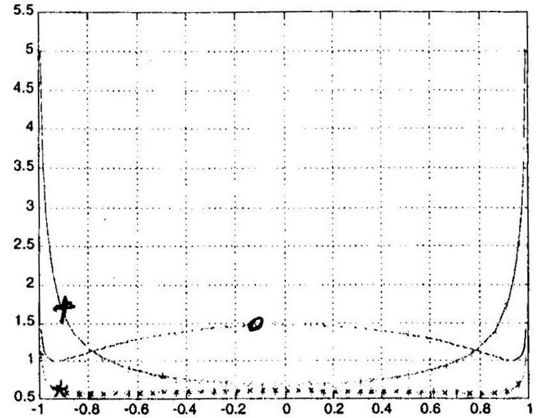


Figure 3a. Electric current density versus x/a for $\theta = 90°$, $ka = 0.2\pi$. (+) $\eta = 0.1 + i0.27$, (*) $\eta = 0.1 - i0.27$, (o) $\eta = 0.0$.

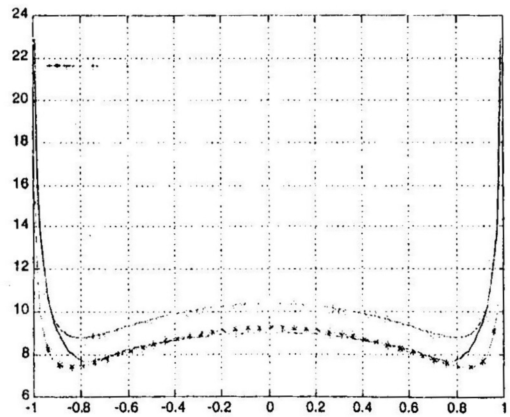


Figure 3b. Electric current density versus x/a for $\theta = 90°$, $ka = 5\pi$. (+) $\eta = 0.0 + i1.1$, (*) $\eta = 0.1 - i0.27$, (o) $\eta = 0.0$.

be derived easily. For $ka = 5\pi$, high frequency asymptotics are given for both $\eta = 0.1 + i0.27$ and $\eta = 0.1 - i0.27$ Fig. 6a and Fig. 6b.

The configuration which was considered in this study was the simple impedance strip illuminated normally by a plane wave. The reason

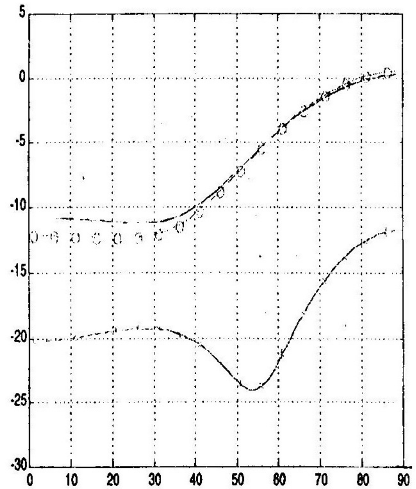


Figure 4a. Backscattered echowidth from a 0.5λ wide impedance strip. (-) $\eta = 0.1 - i0.27$, (*) $\eta = 1.1$, (o) $\eta = 0.1 + i0.27$.

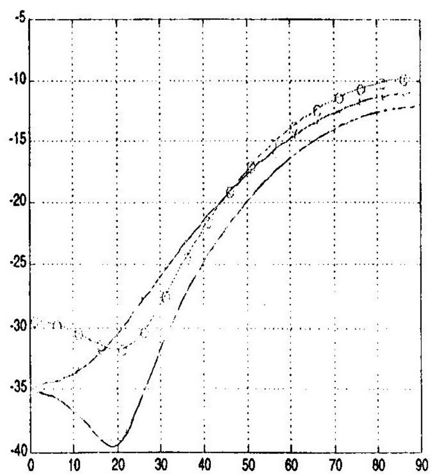


Figure 4b. Backscattered echowidth from a 0.5λ wide resistive strip. (+) $\eta = i4$, (*) $\eta = -i4$, (o) $\eta = 4$.

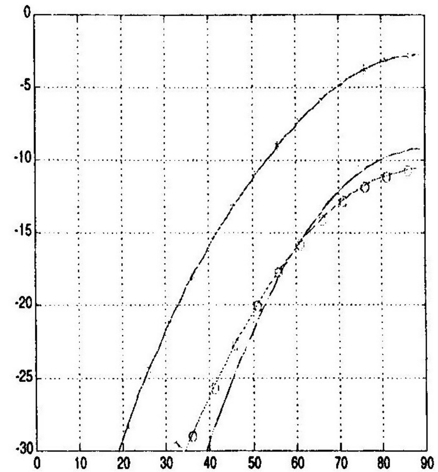


Figure 4c. Backscattered echowidth from a 0.5λ wide conductive strip. (*) $\eta = 0.1 + i0.27$, (*) $\eta = 0.1 - i0.27$, (+) $\eta = 1.1$.

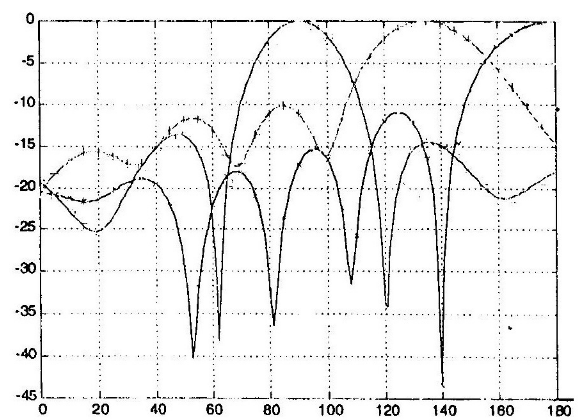


Figure 5. Scattered far field versus incidence angle for $\eta = i1.1$, $ka = 2\pi$. (o) $\theta = 90°$, (+) $\theta = 45°$, (*) $\theta = 0°$.

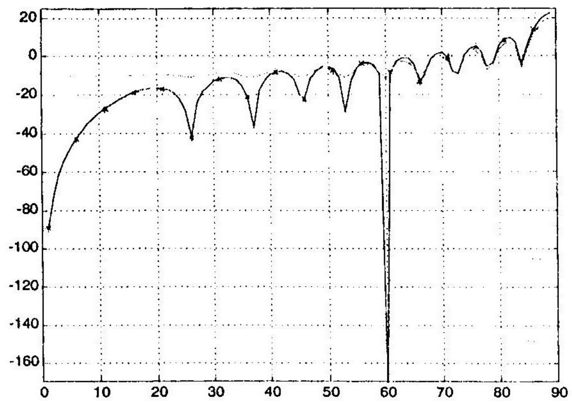


Figure 6a. Comparison of the results for an impedance strip. $\eta = 0.1 + i0.27$ and $ka = 5\pi$. (*) high frequency asymptotics and (-) real values.

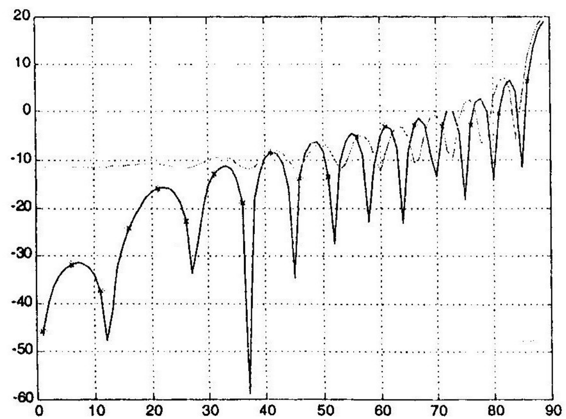


Figure 6b. Comparison of the results for an impedance strip. $\eta = 0.1 - i0.27$ and $ka = 5\pi$. (*) high frequency asymptotics and (-) real values.

for choosing the canonical strip structure was its conformity to many practical problems. Although the strip geometry is frequently used to investigate the multiple diffraction phenomena, the method which is being applied here yields the total field and does not gives the opportunity of such analysis. This may be considered as a disadvantages of the method used in this approach compared with the analytical methods. On the other hand the superiority of the used method with respect to both numerical and analytical methods in various aspects.

The analytical steps applied in the method give null elements in the matrix which is required to be calculated for the analysis. Naturally this results a considerable reduction in the calculation time. Therefore it is much easier to make investigations with respect to the different values of the physical parameters. Especially in the resonant regions the analysis can be easily accomplished by choosing very small frequency intervals. It is also important to note that the diagonal elements of the matrices reveal the characteristics of the physical parameters.

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