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NEW GENERALIZED ELECTROMAGNETIC BOUNDARIES: FRACTIONAL OPERATORS APPROACH

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Abstract – **This paper is devoted to the description of new boundaries using the fractional field, which is constructed by applying the fractional curl operator. Fractional field allows to describe the solution of problem of reflection from specific boundaries of "impedance type", which generalize canonical boundaries (perfect electric conductor (PEC), perfect magnetic conductor (PMC), isotropic impedance boundary). Fractional field approach also gives a possibility to obtain boundaries with new features.**

I. INTRODUCTION

In this paper, new boundaries obtained by the fractional operators approach are considered. Fractional operators are defined as fractionalization of some commonly used operators. In this article we consider fractional curl operator $curl^{\alpha}$ with fractional order α ($0<\alpha<1$) as fractionalization of the usual curl operator [1]. A new electromagnetic field $(\vec{E}^{\alpha},\vec{H}^{\alpha})$ can be defined by applying $curl^{\alpha}$ to certain known original field $(\vec{E}^{0},\vec{H}^{0})$ – solution of electromagnetic problem with certain values of input parameters:

$$(\vec{E}^{\alpha},\vec{H}^{\alpha})=(ik_0)^{-\alpha}\operatorname{curl}^{\alpha}(\vec{E}^{0},\vec{H}^{0}) \quad (1)$$

where $k_0=\omega\sqrt{\varepsilon_0\mu_0}$ is a propagation constant in the medium with permittivity ε_0 and permeability μ_0. Time dependence is assumed to be $e^{-i\omega t}$. We call the field $(\vec{E}^{\alpha},\vec{H}^{\alpha})$ a "fractional field".

We have applied the concept of the fractional field to several specific two-dimensional problems of reflection from infinite boundaries of impedance type. This approach can be useful in description of solutions to reflection problems for some known materials of special kind and also new boundaries with new features. Due to specific properties of surface currents correspond to fractional field, we call such boundary as electromagnetic boundary described via certain boundary conditions (BC) with parameters depending on α. In some cases, such boundaries can be described by anisotropic or bi-anisotropic impedance BC, and some of them need further analysis to obtain adequate model.

Using the operator $curl^{\alpha}$, N.Engheta [1] obtained new boundary with impedance $\eta_{\alpha}=itg(\pi\alpha/2)$ as intermediate case between PEC and PMC in the problem of normal incidence of a plane wave on an impedance boundary. Lindell and Sihvola [2] considered new boundary called Perfectly Electromagnetic Conducting Boundary (PEMC) as generalization of PEC, PMC defined by BC like $\vec{E}+M\vec{H}=0$, where M is an admittance. Another type of fractional boundaries was considered in [3].

II. FRACTIONAL FIELD

Fractional field $(\vec{E}^{\alpha},\vec{H}^{\alpha})$ (1) acts as an intermediate solution between the original and dual solutions [1].

If original field is a field radiated by the „original" line currents with the densities $(\vec{j}_{e0},\vec{j}_{m0})$ then the fractional field represents a field radiated by new sources $(\vec{j}_{e,\alpha},\vec{j}_{m,\alpha})$ expressed via the original currents and the order α [4]:

$$\vec{j}_{e,\alpha}=\cos(\frac{\pi\alpha}{2})\vec{j}_{e0}+\sin(\frac{\pi\alpha}{2})\vec{j}_{m0},\quad \vec{j}_{m,\alpha}=-\sin(\frac{\pi\alpha}{2})\vec{j}_{e0}+\cos(\frac{\pi\alpha}{2})\vec{j}_{m0} \quad (2)$$

We call the currents $(\vec{j}_{e,\alpha},\vec{j}_{m,\alpha})$ "fractional currents" keeping in mind the currents, which correspond to the fractional field. Note that fractional currents are distributed in the same volume as original currents are.

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Duality principle in the reflection problems states that the dual solution $(\vec{E}^1,\vec{H}^1)$, obtained as $\vec{E}^1=\eta_0\vec{H},\eta_0\vec{H}^1=-\vec{E}$, corresponds to:

(i) PMC boundary when the original field is a solution for the PEC one;

(ii) Impedance surface with η^{-1} when the original solution is a solution for the impedance η ;

(iii) Conductive surface when the original is a solution for the resistive surface [5].

The fractional field can be a solution of the problem of reflection from certain boundaries which are intermediate cases between the original boundary and the "dual boundary" [1].

Having presentation for the fractional field (expressed via the original field components and the order α), we want to describe the boundary corresponding to this solution. Our aim is to find the BC, which is satisfied by the constructed fractional field.

Impedance boundary supports surface electric and magnetic currents. The original solution corresponds to the surface currents (j_{e0},j_{m0}) and the fractional solution corresponds to the "fractional" surface currents expressed from the original ones as (2). Equation (2) can be treated as follows: fractional currents represent a mixture of the original electric and magnetic currents. We call the boundaries, which support such kind of surface currents as "fractional" boundaries or "electromagnetic" boundaries. Fractional electric field represents a mixture of the original electric and magnetic fields.

III. FRACTIONAL FIELD APPROACH IN REFLECTION PROBLEM

Consider a classic problem of the TE polarized plane wave $\vec{E}^i(0,0,e^{ik(x\cos\varphi+y\sin\varphi)})$ incidence at the angle φ on the plane boundary $y=0$. Normal $\vec{n}$ to the boundary has the same direction as the axis y. Total field should satisfy isotropic impedance BC:

$$\vec{n}\times\vec{E}=\eta\vec{n}\times(\vec{n}\times\vec{H}),\ \text{for}\ y\to+0 \tag{3}$$

where $\vec{E}^0=\vec{E}^i+\vec{E}^r$ ($y>0$) is the total field, which is the sum of the incident $(\vec{E}^i,\vec{H}^i)$ and reflected $(\vec{E}^r,\vec{H}^r)$ waves, η is an isotropic impedance. As a special case the BC (3) describes the PEC boundary if $\eta=0$, and the PMC boundary if $\eta=i\infty$.

Having the field $(\vec{E}^0,\vec{H}^0)$ as a known solution for a certain value of impedance η, we can build the fractional field as $curl^\alpha$ to the original total field (1). Taking into account the property (2), we can assume that the fractional field represents the solution of the reflection problem for some boundary, which can be characterized by the following BC [5]

$$\vec{n}\times\vec{E}^\alpha=\hat{\eta}_\alpha\vec{n}\times(\vec{n}\times\vec{H}^\alpha), \tag{4}$$

where the impedance $\hat{\eta}_\alpha$, in general, can be written as a tensor

$$\hat{\eta}_\alpha=\begin{pmatrix} t_{11} & t_{12} \\ t_{21} & t_{22}\end{pmatrix} \tag{5}$$

and can be expressed via the original impedance η and the order α.

Using the expressions for the $curl^\alpha$ for the function $\vec{F}(x,y)=\vec{z}e^{iax+iby}$ [1, 4, 6],

$$curl^\alpha(\vec{z}e^{iax+iby})=e^{iax+iby}[\vec{x}i^\alpha\sin(\frac{\pi\alpha}{2})\frac{b}{(a^2+b^2)^{\frac{1-\alpha}{2}}}-\vec{y}i^\alpha\sin(\frac{\pi\alpha}{2})\frac{a}{(a^2+b^2)^{\frac{1-\alpha}{2}}}+\vec{z}\cos(\frac{\pi\alpha}{2})(a^2+b^2)^{\frac{\alpha}{2}}],$$

the components of the fractional field $\vec{E}^\alpha(E^\alpha_x,E^\alpha_y,E^\alpha_z)$, $\vec{H}^\alpha(H^\alpha_x,H^\alpha_y,H^\alpha_z)$ are expressed via the components of the original field:

$$\vec{E}^\alpha(\sin(\frac{\pi\alpha}{2})H_x,\sin(\frac{\pi\alpha}{2})H_y,\cos(\frac{\pi\alpha}{2})E_z),\quad \vec{H}^\alpha(\cos(\frac{\pi\alpha}{2})H_x,\cos(\frac{\pi\alpha}{2})H_y,-\sin(\frac{\pi\alpha}{2})E_z) \tag{6}$$

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Consider two important special cases of the tensor (5):

(a) anisotropic BC with $\hat{\eta}_\alpha = \begin{pmatrix} t_{11} & 0 \\ 0 & t_{22} \end{pmatrix}$, *where* $t_{11} = -\frac{E_z^\alpha}{H_x^\alpha}$, $t_{22} = \frac{E_x^\alpha}{H_z^\alpha}$.

(b) bi-anisotropic BC with $\hat{\eta}_\alpha = \begin{pmatrix} 0 & t_{12} \\ t_{21} & 0 \end{pmatrix}$, where $t_{12} = -\frac{E_z^\alpha}{H_z^\alpha}$, $t_{21} = \frac{E_x^\alpha}{H_x^\alpha}$.

Case (a).

$$t_{11} = i\frac{1}{\sin\varphi}\bullet tg(\pi\alpha/2)\frac{1-i\eta\sin\varphi\bullet ctg(\pi\alpha/2)}{1+i\eta\sin\varphi\bullet tg(\pi\alpha/2)},\quad t_{22} = i\sin\varphi\bullet tg(\pi\alpha/2)\frac{1-i\eta\sin\varphi\bullet ctg(\pi\alpha/2)}{1+i\eta\sin\varphi\bullet tg(\pi\alpha/2)} \tag{7}$$

If the original field is a solution for a PEC ($\eta = 0$) boundary, then we have

$$t_{11} = i\frac{1}{\sin\varphi}\bullet tg(\pi\alpha/2),\quad t_{22} = i\sin\varphi\bullet tg(\pi\alpha/2) \tag{8}$$

For normal incidence ($\varphi = \frac{\pi}{2}$) from (8): $t_{11} = t_{22} = i\bullet tg(\pi\alpha/2)$. These expressions are agreed with the results obtained in [1].

Case (b).

$$t_{12} = -i\frac{1-i\eta\sin\varphi\bullet ctg(\pi\alpha/2)}{1+i\eta\sin\varphi\bullet tg(\pi\alpha/2)},\quad t_{21} = -i\bullet tg^2(\pi\alpha/2)\bullet\frac{1-i\eta\sin\varphi\bullet ctg(\pi\alpha/2)}{1+i\eta\sin\varphi\bullet tg(\pi\alpha/2)} \tag{9}$$

In case of a PEC boundary ($\eta = 0$) $t_{12} = -i$, $t_{21} = -itg^2(\frac{\pi\alpha}{2})$.

For $0 < \alpha < 1$ these equations describe a bi-anisotropic boundary [3], which supports surface currents given by the equation

$$\vec{j}_e^\alpha = -\hat{\eta}_\alpha \vec{j}_m^\alpha \tag{10}$$

Unlike for an isotropic impedance boundary, the surface electric and magnetic currents $\vec{j}_e(0,0,j_{ez}^\alpha)$, $\vec{j}_m(0,0,j_{mz}^\alpha)$ for this kind of boundary are parallel to each other.

Note that equation similar to (10) was used in [2] for the description of the PEMC boundary.

IV. Conclusion

The fractional curl operator allows to obtain intermediate solutions in electromagnetic problems. In the reflection problems for the boundaries of impedance type, fractional solution is a useful tool to describe solutions for anisotropic and bi-anisotropic boundaries of special kind. New boundaries, which generalize PEC and PMC and also isotropic impedance boundaries, are introduced as the boundaries corresponding to the fractional field. The fractional operators can be used as a mathematical tool to obtain intermediate situations in electromagnetics.

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