FRACTAL ELECTRICAL AND MAGNETICAL RADIATORS

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In the present paper we apply the concept of fractional calculus to one of the basic problems in electromagnetics, the problem of radiation of the fractal radiators and their corresponding potentials.

The concept of a radiating contour fractal geometrical structure is the physical basis of our consideration. In this connection, we assume the occurrence of fractal distribution of the contour in current. In space such current forms the appropriate electromagnetic field.

The directional pattern of the fractal analogue of electrical Hertz dipole we have investigated. The features of our fractal model and it differences from classical Hertz dipole are discussed.

We make comments on the model of the fractal magnetic radiator. The numerical account of the directional patterns of fractal electrical and magnetic radiators is compared.

Introduction

The Hertz dipole is known as a variant of the circuit, which ensures an intensive radiation at a rather small-connected part of energy. For mathematical simulation of such dipole usually the idealized elementary electrical vibrator is used, which is located in an unlimited homogeneous isotropic nonconducting medium. The vibrator is presented as wire, short in comparison with wave's length, the amplitude and phase of a current being constant along itslength. In inconsistency with this classical model there is a problem of practical creation of a vibrator with amplitude and phase of a current being constant along its length. Nor it is obviously possible to ensure the presence of a material medium with properties of a homogeneity, isotropy and non-conductivity.

The study of properties of an elementary vibrator is important for understanding the process of antennas' radiation even in such idealized statement of the problem, the antennas considered as consisting from a set of vibrators. The principle of superposition allows studying an emitted field as a sum of fields of elementary vibrators.

Hereby we try, proceeding from fractal conceptions of the current's structure in a conductor and the field in a semiconducting medium, to enter α-features of an electromagnetic field ([1], [2]) and to develop a differintegral model of an electrical vibrator.

Construction of differintegral model

As well as in a classical case, we shall assume, that an indirect current creats the monochromatic field

$$I_m^{in} e^{i\omega t}. \tag{1}$$

We shall place a vibrator so that l the beginning of a spherical frame r, θ, φ is in middle of its length l. The polar axis coincides with axis of the vibrator $\theta = 0, \theta = \pi$.

Instead of a classical vectorial potential, used in such case,

$$\vec{z}_o A_z(r,\theta,t) = \vec{z}_o \frac{1}{4\pi} I_m^{in} \int_{-l/2}^{l/2} \mu \frac{e^{i(\omega t - kR)}}{R(r,\theta,\xi)} d\xi, \tag{2}$$

where μ - magnetic permeability of a medium, $k = 2\pi / \lambda$, λ - length of a wave,

$$R(r,\theta,\xi) = \sqrt{r^2 + \xi^2 - 2r\xi\cos\theta}, \quad -l/2 \le \xi \le l/2,$$

we consider α-features (differintegral, $0 < \alpha < 1$) till a θ (see [1], [2]):

$$A_z^{\alpha}(r,\theta,t) = \theta_0^{\alpha}\ {}_aD_{\theta}^{\alpha} A_z(r,\theta,t), \tag{3}$$

where
$${}_aD_{\theta}^{\alpha} f(x) \equiv \frac{1}{\Gamma(1-\alpha)} \frac{d}{d\theta} \int_a^{\theta} \frac{f(u)}{(\theta-u)^{\alpha}} du$$

is fractional derivative of the α-order (definition see in [3]), Γ(x) – Gamma-function.

Thus,

$$A_z^{\alpha}(r,\theta,t) = \theta_0^{\alpha} \frac{1}{\Gamma(1-\alpha)} \frac{\partial}{\partial\theta} \int_a^{\theta} \frac{A_z(r,\theta,t)}{(\theta-\theta')^{\alpha}} d\theta' = = \frac{\theta_0^{\alpha}}{4\pi} \frac{\partial}{\partial\theta} \int_a^{\theta} \frac{1}{\Gamma(1-\alpha)} \frac{\mu I_m^{in}}{(\theta-\theta')^{\alpha}} \left[\int_{-l/2}^{l/2} \frac{e^{i(\omega t - kR)}}{R(r,\theta',\xi)} d\xi \right] d\theta' \quad . \tag{4}$$

Potential (4) we call α-feature of potential $A_z(r,\theta,t)$. Obviously, at $\alpha = 0$ of (4) we obtain a classical potential.

The introduction of α-features can be proved through reviewing the two new physical and mathematical models corresponding with two statements of the problem of radiation of the Hertz dipole:

1) Electrical dipole has a geometric fractal structure. It implies modification of magnitude of current (1) in the direction of the angle θ under the law

$${}_aD_{\theta}^{\alpha}(const_{\theta}) = \theta_0^{\alpha} \frac{const_{\theta}}{\Gamma(1-\alpha)} \frac{1}{(\theta-a)^{\alpha}}, \tag{5}$$

where $const_{\theta} = I_m^{in} e^{i\omega t}$, magnetic permeability μ - constant.

Here we use the formula of α-order fractional derivative for a constant. It is obvious, that fractality of the current distributions a on φ and r is necessary to take into account in models of a vibrator with a defined thickness ΔS.

2) Electrical dipole - ideal (thickness ΔS = 0, current vary on (1)) and is in a medium with a permeability μ, which has a fractal structure of type (5) on cone to θ=const.

α-features diagrams of a vibrator field

For determinancy, we shall consider the field of the vibrator in a distant (wave) zone $(r >> l, l << \lambda, 2\pi r >> \lambda)$.

In this case
$$\vec{z}_o A_z = \vec{z}_o \frac{\mu}{4\pi r} I_m^{in} e^{i(\omega t - kr)} . \tag{6}$$

In view of that
$$\vec{E} = -\frac{i}{\omega\varepsilon} rot\,\vec{H}, \quad \vec{H} = \frac{1}{\mu} rot\,\vec{A} \quad , \tag{7}$$

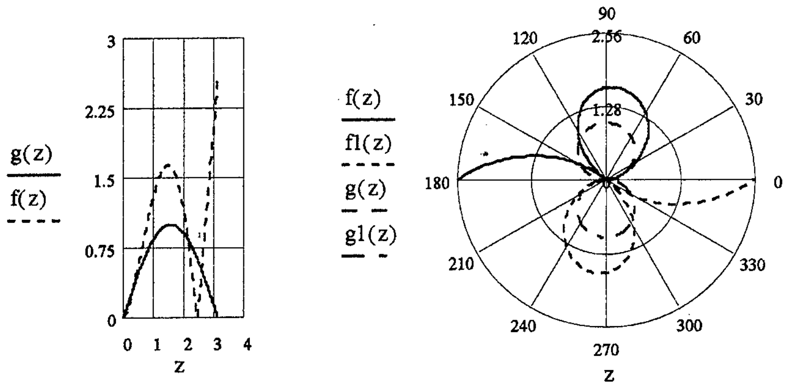
Having disregarded the addends $(1/kr)^2$ and $(1/kr)^3$, it becomes possible to obtain 0- and α-feature of θ-component of strength electrical vector [4]:

$$E_{\theta} = \frac{i I_m^{in} l}{2\pi r} \sqrt{\frac{\mu}{\varepsilon}} \sin\theta\ e^{i(\omega t - kr)} \tag{8}$$

$$E_{\theta}^{\alpha} = \frac{iI_m^{in} l}{2\lambda r}\theta_0^{\alpha}\sqrt{\frac{\mu}{\varepsilon}}\left[\frac{\theta^{\alpha}}{\Gamma(\alpha+1)}\left({}_1F_1(1;\alpha+1;i\theta) - {}_1F_1(1;\alpha+1;-i\theta)\right)\right] \qquad (9)$$

(${}_1F_1(a;c;z)$ - hypergeometric function (Kummer)).

We have obtained the graphs of amplitudes of strength of a field, normalized to assotiations' maximum



value, from direction to the viewpoint (at r=const in distant zone $2\pi r$>>λ).

Figure 1. The θ-component of wave is shown as function of the z=θ in Cartesians and polar coordinates

In Fig., curves g(z)= E_{θ}^{0}, g1(z)=-g(z) correspond to 0-feature; curves f(z)= E_{θ}^{α}, f1(z)=-f(z) correspond to 0.5 –feature. The graphs $\left|E_{\theta}^{0}\right|$ and $\left|E_{\theta}^{\alpha}\right|$ completely coincide when $\alpha = 0$. The stream of waves is noticed in direction $\theta = 0$ and $\theta = \pi$, when $\alpha \neq 0$ (this radiation is absent in a classical model).

Conclusion

For deriving differintegral models of the elementary magnetic vibrator, in the formula for E_{θ}^{α} we substitute I_m^{in} with I_m^{mg}, $\mu \leftrightarrow \varepsilon$. Therefore, we obtain α-feature for a θ-component of a magnetic field.

Obtained in paper α-features can be used for development of differintegral model of Huygens element and for solution of such a diffraction problem, as [5].

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