Anomalous radiation from a circular array of cylinders with longitudinal slits

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A quasistatic resonance has been found for a circular cylinder with a narrow longitudinal slit.[1] When this radiating structure is excited by an in-phase current element which produces a magnetic field, sharp changes occur in the directivity near the resonance frequencies. The effect is even more pronounced with two such cylinders with slits in mirror-image positions.[2]

In this note we study the electrodynamic characteristics of a new kind of resonance antenna system: This is a circular array consisting of N circular cylinders of radius a each with a longitudinal slit which subtends 2θ. The cylinders are arranged periodically along a circle of radius b (Fig. 1) and are excited by the current element of intensity I at the center of the circular array.

The longitudinal component of the total field outside the array consists of the source field and the field radiated by all the cylinders; this component must satisfy the Helmholtz equation, the Neumann boundary condition at the cylinder surfaces, the periodicity condition, the radiation condition, and the condition that the energy must be finite in any bounded volume. We use the method of Ref. 3 to find a rigorous solution in the form of an infinite system of linear algebraic equations of the second kind in the unknown Fourier coefficients of the surface current density on one of the cylinders. The scattered fields are expressed in terms of these Fourier coefficients.

When $a \ll 2b \sin \pi/N)$, and $ka < 1$ (the cylinders are small compared to the wavelength; $k = 2\pi/\lambda$, where λ is the wavelength), with narrow slits in the cylinders ($\theta \ll 1$) the problem can be solved approximately in analytic form.

The surface current density is

$$j(\varphi) = \frac{1}{D} i\pi (ka)^3 \frac{I}{4} k H_0^{(1)}(kb)\, u(\varphi) \ln \left| \frac{\sin \frac{\varphi}{2} + \sqrt{\frac{\cos\theta - \cos\varphi}{2}}}{\sin \frac{\theta}{2}} \right|, \quad (1)$$

where

$$D = 1 + 2(ka)^2 \left\{ 1 + i\pi \left(\frac{ka}{2}\right)^2 [1 + G_0(kb, N)] \right\} \ln \sin \frac{\theta}{2};$$

$$u(\varphi) = \begin{cases} 1, & \theta < |\varphi| \leqslant \pi, \\ 0, & |\varphi| < \theta. \end{cases}$$

The quantity

$$G_0(kb, N) = \sum_{m=1}^{N-1} H_0^{(1)}\left(2kb \sin \frac{\pi}{N} m\right)$$

determines the electrodynamic interaction of the cylinders of the circular array with each other.

The field in the far zone and the directional pattern can now be described by

$$H_z(R, \varphi) = \sqrt{\frac{2}{\pi k R}}\, e^{i\left(kR - \frac{\pi}{4}\right)} \frac{I}{4} k F(ka, kb, N, \theta, \varphi), \quad (2)$$

where F, the directivity of the array, is written

$$F = 1 + \frac{i\pi (ka)^2}{D} H_0^{(1)}(kb) \left[ka f_0(kb, N, \varphi) \ln \sin \frac{\theta}{2} - f_1(kb, ka, N, \theta, \varphi) \right],$$

$$f_0 = \sum_{m=0}^{N-1} e^{ikb \cos\left(\varphi + \frac{2\pi}{N} m\right)}; \quad f_1 = \cos^2 \frac{\theta}{2} \sum_{m=0}^{N-1} e^{ikb \cos\left(\varphi + \frac{2\pi}{N} m\right)}$$

$$\left(1 - e^{-i\frac{ka}{2} \cos\theta \cos\left(\varphi + \frac{2\pi}{N} m\right)}\right). \quad (3)$$

TABLE I

	N = 2	N = 3	N = 4	N = 5
$\varkappa$	4.775	5.5	6	6
$\frac{\varkappa}{W_0}$; ($\theta = 5°$)	1.91	2.2	2.4	2.4
$\sum_{m=1}^{N-1} J_0\left(\frac{2\varkappa}{W_0} \sin \frac{\pi}{N} m\right)$	−0.4027	−0.8053	−0.9678	−0.9977
$1 + \sum_{m=1}^{N-1} J_0\left(\frac{2\varkappa}{W_0} \sin \frac{\pi}{N} m\right)$	0.5973	0.1947	0.0322	0.0023
ka	0.36	0.38	0.332	0.387
$\lvert F \rvert$; ($\theta = 5°$)	0.7545	1.3067	16.444	85.585
$\lvert F \rvert$; ($\theta = 0°$)	0.0665	0.06957	0.086	0.0924

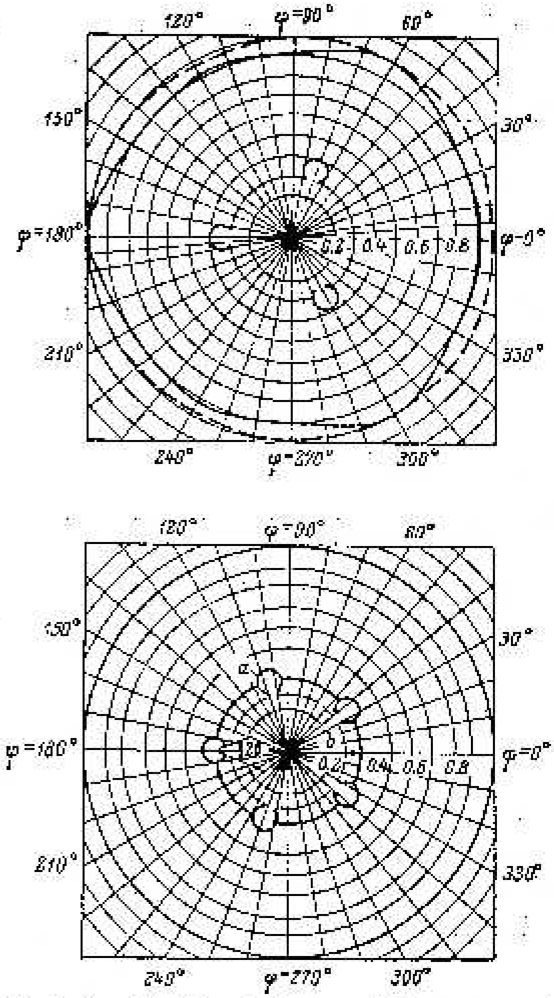

FIG. 1. Normalized directional pattern of the circular array for N = 3 and 5 (see Table I). Solid curves) $\theta = 0$; dashed) 5°.

The complex resonant frequencies of the structure are determined from the equation D = 0:

$$1 + 2(ka)^2\left\{1 + i\pi\left(\frac{ka}{2}\right)^2\left[1 + G_0(ka,\ N)\right]\right\}\ln\sin\frac{\theta}{2} = 0. \tag{4}$$

These are $ka = x_1 - ix_2$,

$$x_1 = \frac{1}{W_0}\left(1 + \frac{\pi\,\mathrm{Im}\,G_0}{8W_0^2}\right);\quad x_2 = -\frac{\pi}{8W_0^3}(1 + \mathrm{Re}\,G_0);\quad W_0 = \left(-2\ln\sin\frac{\theta}{2}\right)^{-1/2}.$$

The quality factor for this circular array, which is a composite open resonator, is given by

$$Q = -\frac{x_1}{2x_2} = \frac{Q_0}{1 + \sum_{m=1}^{N-1} J_0\left(\frac{2s}{W_0}\sin\frac{\pi}{N}m\right)}\left[1 + \frac{\pi}{8W_0^2}\sum_{m=1}^{N-1} N_0\left(\frac{2s}{W_0}\sin\frac{\pi}{N}m\right)\right], \tag{5}$$

where $s = b/a$, and $Q_0 = -(8/\pi)\ln\sin\theta/2$ is the quality factor for a single cylinder with a narrow slit when $ka \ll 1$. The quantity $1 + \sum_{m=1}^{N-1} J_0\left(\frac{2s}{W_0}\sin\frac{\pi}{N}m\right)$ has minima and these minima are listed in Table I for particular values of s and N. By choosing the parameters appropriately it is possible to vary the radiation Q of the circular array over a broad range. At resonance ($ka = x_1$) the surface current density at each cylinder increases to a much greater extent than for a single cylinder.[1] The amplitudes of the radiated waves, i.e., the field intensity in the far zone, also increase. These results are illustrated in Table I, which shows the values of F (the directivity) of a circular array of cylinders with longitudinal slits ($\theta = 5°$) as calculated from Eq. (3). Also shown are the corresponding results for a similar array of solid cylinders. This result is of considerable interest in antenna design.[4] At resonance the directivity pattern of the array is basically isotropic. Figure 1 shows normalized directivity patterns for three-element and five-element arrays.

According to Eqs. (4) and (5) the resonance properties of each element of the array (circular cylinder with the longitudinal slit) and their resonance interaction lead to the unique properties of this structure not only as it radiates, because of the in-phase excitation of the current elements, but also in other situations, in particular, in the efficient "trapping" of the field in the interior of the structure (the structure is equivalent to a "trap" resonator).

[1] A. I. Nosich and V. P. Shestopalov, Dokl. Akad. Nauk SSSR 234, 53 (1977) [Sov. Phys. Dokl. 22, 251 (1977)].

[2] V. N. Koshparenok, P. N. Melezhik, and V. P. Shestopalov, Pis'ma Zh. Tekh. Fiz. 4, 1145 (1978) [Sov. Tech. Phys. Lett. 4, 460 (1978)].

[3] E. I. Veliev and V. P. Shestopalov, ZhVMiMF 17, 1234 (1977).

[4] G. Z. Aizenberg, Microwave Antennas [in Russian], Svyaz'izdat, Moscow (1967).

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