Diffraction of Waves by a Rectangular Cylinder*

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A rigorous method for solving problems of the diffraction of waves by an ideally conducting rectangular cylinder, regarded as a composite body formed by "gluing together" flat ribbons, is proposed. The method is based on the idea of the method of partial inversion of an operator and the method of moments. As a result the problem is reduced to the solution of infinite systems of linear algebraic equations (SLAE). It is shown that the method of reduction is applicable to the solution of these systems. The unknowns are the coefficients in the expansion of the density functions of the surface currents on the faces of the cylinder in a complete system of Gegenbauer polynomials with a weighting factor taking into account the form of the behavior of the currents at the edges. The proposed method incorporates, as a special case, a new rigorous solution of the problem of diffraction of waves by a collection of flat ribbons with parallel generatrices. The characteristics of the distribution of the currents on the faces of a rectangular cylinder, the total scattering cross section, and the radiation pattern of the scattered field are studied based on a numerical calculations.

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1. The main approach to the solution of the problems of diffraction of waves by ideally conducting polynomial cylinders (PC) is to reduce them to integral equations of the first kind in the current density induced by the incident wave on the surface of the cylinder. These integral equations are solved by numerical methods, in which the integral equations are reduced to systems of linear algebraic equations (SLAE) [1-5]. The presence of corner points (edges) on the contour of integration, however, appreciably reduces the accuracy with which the functions of the current are determined from these integral equations. To improve the situation, approaches taking into account, by different methods, the conditions on the edge in explicit form have been proposed [3-5]. In these papers the characteristics of the current distribution on the faces of the cylinders and the behavior of the fields in the far zone have been studied in part. In [6, 7], in which a solution is obtained based on the geometric theory of diffraction (GTD), the asymptotic solution of the problem in question in the short-wave region is considered.

Below we propose a rigorous method for solving the problem; the method enables us to determine the quantity required with any predetermined accuracy. The idea of the method is as follows: the ideally conducting PC is regarded as a composite body, formed by key elements, such as a flat ribbon, "glued together" (see Fig. 1b). In this formulation the problems of diffraction

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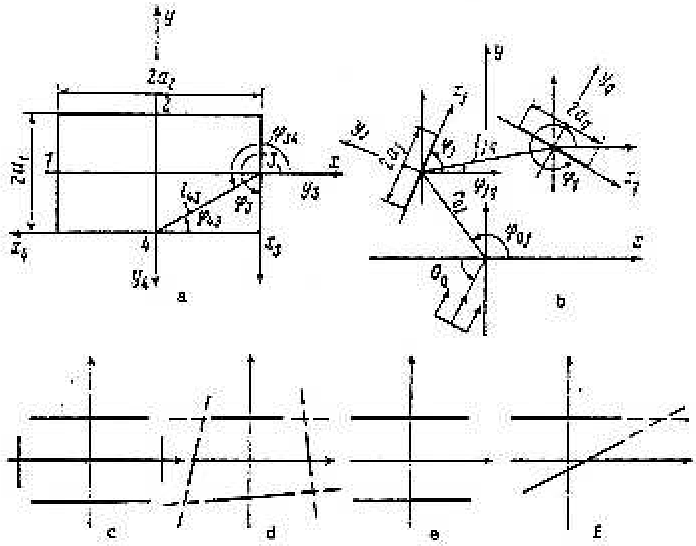

Fig. 1. The structures considered.

of waves by PC are closely related to problems of diffraction of waves by N bodies. In particular, the field scattered by the PC can be regarded as a superposition of fields scattered by each fact of the cylinder (i.e., in the form of the superposition of fields generated by the corresponding currents on the faces); the scattered fields are expressed in terms of the Fourier transformation of the density functions of the surface currents on the faces of the cylinder. Such representations for the scattered fields give a number of important advantages in the construction of the solution of the problem. The boundary conditions and the representation employed for the scattered fields lead to a system of paired integral equations (with a kernel in the form of trigonometric functions) relative to the unknown Fourier transforms. Then a hybrid (mixed) approach [8] employing the idea of the method of partial inversion of an operator [9] and the method of moments [7, 9], is used to solve these paired integral equations.

The proposed method also differs considerably from the well-known methods employed to solve the problems of diffraction of waves by PC in that it includes as a special case the solution of the problem of diffraction of waves by a collection of "key" elements (for example, a system of $N = 4$ ribbons (see Figs. 1c and d)).

In what follows our main attention will be devoted to the problem of the diffraction of a plane H-polarized electromagnetic wave by a PC, since it is precisely this case that is the most difficult to study.* Since the main ideas of the proposed method of solution, as applied to regular PC, have been published in [10, 11], in this paper an infinite SLAE, to which the problem reduces, is studied in detail using the example of a rectangular cylinder. The existence and uniqueness of the solutions and the applicability of the method of reduction to the solution of the problem are also considered. The computational results from the study of the current distribution on the faces of the cylinder, the total scattering cross section, and the radiation pattern of the field are also analyzed in detail.

2. Let a plane H-polarized electromagnetic wave $H_z^i = \exp[ik(\alpha_0 x + \sqrt{1-\alpha_0^2}\, y)]$ (where $k = 2\pi/\lambda$, λ is the wavelength and $\alpha_0 = \cos\vartheta_0$), varying in time as $e^{-i\omega t}$, be incident on an ideally conducting cylinder at an angle ϑ_0 (see Figs. 1a and b). The cylinder is unbounded in the $0z$ direction and has a rectangular cross section in the $x0y$ plane. We introduce the following notation (see Figs. 1a and b): $\{x_j 0_j y_j\}_{j=1}^4$ are the local coordinate systems connected to the centers of the faces of the cylinder; $\{2a_j\}_{j=1}^4$ are the widths of the faces; and, $\{\varphi_j\}_{j=1}^4$ are the angles fixing the orientation of the faces.

The problem consists of determining the component H_z of the field scattered by the cylinder. The function sought must satisfy Helmholtz's equation outside the surface of the cylinder, the Sommerfeld radiation condition at infinity, the condition that the energy is finite in any

*The case of E polarization can be studied in an analogous manner.

bounded volume of space (Miexner's condition on the edges), and Neumann's boundary condition on the surface of the cylinder.

Starting from this model of the PC the total field can be represented in the form $H_z = H_z^0 + \sum_{j=1}^{4} H_z^{(j)}$, where $H_z^{(j)}$ describes the scattered field, generated by the surface current on the j-th face. These fields in local coordinate systems, as in the problem of diffraction by a flat ribbon in the case of H-polarized excitation, can be represented in the form [12]

$$H_z^{(j)} = -\frac{\varepsilon_j}{2\pi}\frac{|y_j|}{y_j}\int_{-\infty}^{\infty} h_j(\alpha)\exp\{ik[\alpha x_j + \sqrt{1-\alpha^2}\,|y_j|]\}\,d\alpha, \tag{1}$$

where $\varepsilon_j = ka_j$, while $\{h_j(\alpha)\}_{j=1}^{4}$ is the Fourier transform of the functions

$$\{\mu_j(x_j)\}_{j=1}^{4} = \frac{1}{2\pi}\int_{-\infty}^{\infty} h_j(\alpha)\exp(ik\alpha x_j)\,d\alpha,$$

describing the surface-current density on the faces of the PC; in addition, $\mu_j(x_j) = 0$ for $|x_j| > a_j$, i.e., these functions are continued by zero outside the interval $x_j \in [-a_j, a_j]$. In (1) the branch of the function $\sqrt{1-\alpha^2}$ for which $\operatorname{Im}\sqrt{1-\alpha^2} > 0$ as $|\alpha| \to \infty$ is chosen. We note that for infinitely thin screens the current functions $\mu_j(x_j)$ are determined as jumps of the H_z components of the total magnetic field, whereas for PC the function $\mu_j(x_j)$ is the limiting value of the H_z component of the field on the faces of the PC, i.e., $\mu_j(x_j) = H_z(x_j, +0)$, $x_j \in [-a_j, a_j]$.

3. To determine the unknowns $\{h_j(\alpha)\}_{j=1}^{4}$ we make the total field obey Neumann's boundary condition on the surface of the cylinder

$$\frac{\partial}{\partial n}\Big(H_z^0 + \sum_{j=1}^{4} H_z^{(j)}\Big)\Big|_L = 0, \quad L = \bigcup_{j=1}^{4} L_j. \tag{2}$$

Here L_j is the contour of the face in the $x_j o_j y_j$ plane, while the normal n is oriented along the axis $o_j y_j > 0$.

To satisfy Eq. (2) the field H_z^0 and $H_z^{(q)}$ must be written in a coordinate system fixed on the j-th face (see Fig. 1b). Using the relationship between the coordinate systems, we obtain the following representations for these fields in the coordinate system $x_j o_j y_j$:

$$H_z^0 = \exp\{i\varepsilon_j[\eta_j B_j(\alpha_0) + \zeta_j A_j(\alpha_0)] + ikr_{0j}D_j(\alpha_0)\},$$

$$H_z^{(q)} = \frac{\varepsilon_q}{2\pi}\int_{-\infty}^{\infty} h_q(\alpha)\exp\{i\varepsilon_j[B_{jq}(\alpha)\eta_j + \zeta_j A_{jq}(\alpha)] + ikl_{jq}D_{jq}(\alpha)\}\,d\alpha, \tag{3}$$

$$\zeta_j < 0.$$

Here we have introduced the following notation:

$$\begin{aligned}
A_{jq}(\alpha) &= -\alpha\sin(\varphi_j-\varphi_q) - \sqrt{1-\alpha^2}\cos(\varphi_j-\varphi_q);\\
A_j(\alpha_0) &= -\alpha_0\sin\varphi_j + \sqrt{1-\alpha_0^2}\cos\varphi_j,\\
B_{jq}(\alpha) &= \alpha\cos(\varphi_j-\varphi_q) - \sqrt{1-\alpha^2}\sin(\varphi_j-\varphi_q);\\
B_j(\alpha_0) &= \alpha_0\cos\varphi_j + \sqrt{1-\alpha_0^2}\sin\varphi_j,\\
D_{jq}(\alpha) &= -\alpha\cos(\varphi_q-\varphi_{jq}) - \sqrt{1-\alpha^2}\sin(\varphi_q-\varphi_{jq});\\
D_j(\alpha_0) &= \alpha_0\cos\varphi_{0j} + \sqrt{1-\alpha_0^2}\sin\varphi_{0j},
\end{aligned} \tag{4}$$

$\eta_j = x_j/a_j$, $\zeta_j = y_j/a_j$ are the dimensionless coordinates, while the parameters l_{jq}, r_{0j} and φ_{jq} are determined in Figs. 1a and b.

Using now the representations (3) for the fields and keeping in mind the fact that the functions $\mu_j(\eta_j)$ are continued by zero outside the interval $[-a_j, a_j]$, from (2) we obtain the following system of paired integral equations with a kernel in the form of trigonometric functions in order to determine the unknowns $\{h_j(\alpha)\}_{j=1}^{4}$:

$$\int_{-\infty}^{\infty} h_j(\alpha)\sqrt{1-\alpha^2}\exp(i\varepsilon_j\alpha\eta_j)\,d\alpha=\frac{2\pi}{\varepsilon_j}A_j(\alpha_0)\times$$
$$\times\exp[i\varepsilon_jB_j(\alpha_0)\eta_j+ikr_{0j}D_j(\alpha_0)]+\frac{1}{\varepsilon_j}\sum_{q=1,q\neq j}^{4}\varepsilon_q\int_{-\infty}^{\infty}h_q(\alpha)A_{jq}(\alpha)\times \tag{5}$$
$$\times\exp[i\varepsilon_jB_{jq}(\alpha)\eta_j+ikl_{jq}D_{jq}(\alpha)]\,d\alpha,\qquad |\eta_j|<1,$$
$$\int_{-\infty}^{\infty} h_j(\alpha)\exp(i\varepsilon_j\alpha\eta_j)\,d\alpha=0,\qquad |\eta_j|>1.$$

The functions $\{h_j(\alpha)\}_{j=1}^{4}$ must satisfy, in addition to the system of equations (5), the relations

$$\int_{-\infty}^{\infty}(|\alpha|+1)\,|h_j(\alpha)|^2\,d\alpha<\infty,\qquad j=1,\ldots,4, \tag{6}$$

which follow from the condition that the energy of the scattered waves must be finite in any bounded region of space. It can be shown that this condition in Miexner's form of the current functions near the edges (at the vertices of the polygon) lead to the following restrictions:

$$\mu_j(\eta)\underset{\eta\to\pm1}{\approx}C_j(1-\eta)+C_{j+1}(1+\eta)+(1-\eta^2)^{\nu}, \tag{7}$$

where $\{C_j\}_{j=1}^{4}$ are constants to be determined.

In addition, the functions $\{\mu_j(\eta)\}_{j=1}^{4}$ must satisfy the condition of continuity:

$$\mu_j(+1)=\mu_{j+1}(-1),\ \ldots,\ \mu_{N+1}(+1)=\mu_1(-1);\quad C_{N+1}=C_1;\quad N=4. \tag{8}$$

The condition on the edge (7) when transforming from PC to a collection of flat ribbons by means of "separation" of the faces of the PC is written in the form $\mu_j(\eta)_{\eta\to\pm1}\sim(1-\eta^2)^{\nu}$, i.e., the coefficients $\{C_j\}_{j=1}^{4}$ are assumed to equal zero, while the parameter $\nu = 1/2$.

To satisfy condition (7) we represent the functions $\{\mu_j(\eta)\}_{j=1}^{4}$ in the form of uniformly converging series in the complete and orthogonal system of Gegenbauer polynomials $\{C_m^{\nu+1/2}(\eta)\}_{m=0}^{\infty}$ [13] with the weighting factor $(1-\eta^2)^{\nu}$:

$$\mu_j(\eta)=C_j(1-\eta)+C_{j+1}(1+\eta)+(1-\eta^2)^{\nu}\sum_{m=0}^{\infty}\mu_m^{(j)}C_m^{\nu+1/2}(\eta), \tag{9}$$

where $\{\mu_m^{(j)}\}_{m=0}^{\infty}$ are new unknown coefficients.

Relations (9) lead to new representations for the Fourier transformations of the functions $\mu_j(\eta)$. It can be verified that they will have the following form:

$$h_j(\alpha)=\frac{2i}{\varepsilon_j\alpha}[C_{j+1}K_j(-\alpha)-C_jK_j(+\alpha)]+$$
$$+\frac{2\pi}{\Gamma\left(\nu+\frac{1}{2}\right)}\sum_{m=0}^{\infty}(-i)^m\beta_m^{(\nu)}\mu_m^{(j)}\frac{J_{m+\nu+1/2}(\varepsilon_j\alpha)}{(2\varepsilon_j\alpha)^{\nu+1/2}}. \tag{10}$$

Here

$$K_j(\pm\alpha)=\exp(\pm i\varepsilon_j\alpha)-\frac{\sin(\varepsilon_j\alpha)}{\varepsilon_j\alpha},$$

$$\beta_m^{(\nu)}=\frac{\Gamma(m+2\nu+1)}{\Gamma(m+1)},$$

where $\Gamma(x)$ is the gamma function and $J_\nu(x)$ is a Bessel function.

4. To solve the system of paired integral equations (5) we shall use the method of partial inversion of an operator [8]. To this end the operators generated by the left-hand sides of the inhomogenous equations in system (4) must be divided into principal* and completely continuous parts. This procedure is realized by introducing the quantity γ according to the formula

$$\sqrt{1-\alpha^2}=i|\alpha|[1-\gamma(\alpha)];\quad \gamma(\alpha)\sim\underset{|\alpha|\to\infty}{O}\left(\frac{1}{\alpha^2}\right). \tag{11}$$

Substituting (10) and (11) into (5) and making use of the completeness and orthogonality (in the interval $\eta\in[-1, 1]$) of the Gegenbauer polynomials and using the discontinuous Weber-Shafkheytlin integrals [13], we obtain the following coupled infinite SLAE for finding the coefficients $\{\mu_m^{(j)}\}_{m=0}^{\infty}$:

$$\begin{aligned}&C_{j+1}[C_k^{(-j)}-d_k^{(-j)}]-C_j[C_k^{(+j)}-d_k^{(+j)}]+\\&+i\sum_{m=0}^{\infty}[1+(-1)^{k+m}]x_m^{(j)}[C_{km}^{(\nu)}-Q_{km}^{(\nu)}]=\\&=f_k^{(j)}-\sum_{q=1,\,q\neq j}^{4}\left[C_{q+1}P_k^{-jq}-C_qP_k^{+jq}+\sum_{m=0}^{\infty}x_m^{(q)}P_{km}^{jq}\right],\quad j=1,\ldots,4.\end{aligned} \tag{12}$$

Here the following notation has been introduced:

$$x_m^{(j)}=(-i)^m\mu_m^{(j)}\beta_m^{(\nu)};$$

$$C_k^{(\pm j)}=\frac{i}{\pi}K_\nu^{j}\int_{-\infty}^{\infty}\frac{|\alpha|}{\alpha}K_j(\pm\alpha)J_{k+\nu+1/2}(\varepsilon_j\alpha)\frac{d\alpha}{\alpha^{\nu+1/2}},$$

$$d_k^{(\pm j)}=\frac{i}{\pi}K_\nu^{j}\int_{-\infty}^{\infty}\frac{|\alpha|}{\alpha}\gamma(\alpha)K_j(\pm\alpha)J_{k+\nu+1/2}(\varepsilon_j\alpha)\frac{d\alpha}{\alpha^{\nu+1/2}}$$

$$C_{km}^{(\nu)}=\frac{\Gamma^2\left(\nu+\frac{1}{2}\right)\Gamma\left(\frac{k+m}{2}+1\right)}{\Gamma\left(\nu+\frac{1+k-m}{2}\right)\Gamma\left(\nu+\frac{1+m-k}{2}\right)\Gamma\left(\frac{k+m}{2}+2\nu+1\right)}$$

$$Q_{km}=K_\nu(\varepsilon_j)\int_0^{\infty}\gamma(\alpha)J_{k+\nu+1/2}(\varepsilon_j\alpha)J_{m+\nu+1/2}(\varepsilon_j\alpha)\frac{d\alpha}{\alpha^{2\nu}},$$

$$\begin{aligned}P_k^{\pm jq}=&\frac{i}{\pi}K_\nu^{jq}\left(\frac{\varepsilon_j}{\varepsilon_q}\right)^{\nu-1/2}\int_{-\infty}^{\infty}A_{jq}(\alpha)K_q(\pm\alpha)\times\\&\times\frac{\exp[ikl_{jq}D_{jq}(\alpha)]}{[B_{jq}(\alpha)]^{\nu+1/2}}J_{k+\nu+1/2}(\varepsilon_jB_{jq}(\alpha))\frac{d\alpha}{\alpha},\end{aligned}$$

$$\begin{aligned}P_{km}^{jq}=&K_\nu^{jq}\left(\frac{\varepsilon}{\varepsilon}\right)^{\nu-1/2}\int_{-\infty}^{\infty}A_{jq}(\alpha)J_{m+\nu+1/2}(\varepsilon_q\alpha)\times\\&\times J_{k+\nu+1/2}(\varepsilon_jB_{jq}(\alpha))\frac{\exp[ikl_{jq}D_{jq}(\alpha)]}{[\alpha B_{jq}(\alpha)]^{\nu+1/2}}d\alpha,\end{aligned}$$

$$\begin{aligned}f_k^{(j)}=&K_\nu(\varepsilon_j)2^{\nu+1/2}\Gamma\left(\nu+\frac{1}{2}\right)\exp[ikr_{0j}D_j(\alpha_0)]A_j(\alpha_0)\times\\&\times\frac{J_{k+\nu+1/2}(\varepsilon_jB_j(\alpha_0))}{[B_j(\alpha_0)]^{\nu+1/2}},\quad K_\nu(\varepsilon_j)=\left(\frac{2}{\varepsilon_j}\right)^{2\nu-1}\frac{2\Gamma^2(\nu+1/2)}{\Gamma(2\nu)}.\end{aligned}$$

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*The principal part of the integral operator corresponds to the case when the wave number $k=0$, i.e., it corresponds to the static situation.

$$K_\nu^{(j)}=2(2\varepsilon_j)^{\nu-1/2}\Gamma(\nu+1/2)K_\nu(\varepsilon_j).$$

The coefficients $\{C_j\}_{j=1}^4$ in the SLAE (12) are given by the relations

$$\exp[-i\varepsilon_j B_j(\alpha_0)+ikr_{1j}D_j(\alpha_0)]+\sum_{q=1,\,q\neq j,\,j-1}^{4}\left\{\sum_{m=0}^{\infty}x_m^{(q)}e_m^{jq}+\right.$$
$$\left.+i[C_{q+1}e_{jq}^{(-)}-C_q e_{jq}^{(+)}]\right\}=-C_j;\quad j=1,\ldots,4, \tag{13}$$

which follow from the definition of the current functions on the faces of the PC and from the conditions on the edge (7) and (8). In (13) the quantities $\{e_m^{jq}\}_{m=0}^{\infty}$, $\{e_{jq}^{(\pm)}\}_{j\neq q=1}^{4}$ are defined as follows:

$$e_{jq}^{(\pm)}=\frac{1}{\pi}\int_{-\infty}^{\infty}K_q(\pm\alpha)\exp[-i\varepsilon_j B_{jq}(\alpha)+ikl_{jq}D_{jq}(\alpha)]\frac{d\alpha}{\alpha},$$

$$e_m^{jq}=\frac{\varepsilon_q}{(2\varepsilon_q)^{\nu+1/2}\Gamma(\nu+1/2)}\int_{-\infty}^{\infty}J_{m+\nu+1/2}(\varepsilon_q\alpha)\exp[-i\varepsilon_j B_{jq}(\alpha)+$$
$$+ikl_{jq}D_{jq}(\alpha)]\frac{d\alpha}{\alpha^{\nu+1/2}}.$$

It has been proved that the Fredholm alternative is valid for the infinite SLAE (12). The proof is based on the fact that, first, the finiteness of the norm in the Hilbert space l_2 of the matrix operators $\{C^{(j)}\}_{j=1}^4$, $\{Q_j\}_{j=1}^4$, $\{P_{jq}\}_{j\neq q=1}^4$, to which the matrices $\{C_{km}^{(\nu)}\}_{k,m=0}^{\infty}$, $\{Q_{km}^{(\nu)}\}_{k,m=0}^{\infty}$ and $\{P_{km}^{jq}\}_{k,m=0}^{\infty}$, respectively, correspond, has been established and, second, the positive-definiteness of the operators $\{C^{(j)}\}_{j=1}^4$ has been proved. In other words, in the space l_2 the matrix operators $\{Q_j\}_{j=1}^4$, $\{P_{jq}\}_{j\neq q=1}^4$ are completely continuous operators, while the operators $\{C^{(j)}\}_{j=1}^4$ have a bilateral continuous inverse operator. It has also been shown that the numerical sequences $\{f_k^{(j)}\}_{k=0}^{\infty}$, $\{C_k^{(\pm j)}\}_{k=0}^{\infty}$, $\{d_k^{(\pm j)}\}_{k=0}^{\infty}$, $\{P_k^{\pm jq}\}_{k=0}^{\infty}$ belong to the space l_2. It can therefore be asserted that the infinite SLAE (12) belongs to the class of operator equations (see [14]) for which Fredholm's alternative is valid.

As an example we shall make the following estimates for the norms of the operators $\{P_{jq}\}_{j\neq q=1}^4$, which describe the interaction of the faces of the PC:

$$\|P_{12}\|=\|P_{21}\|\leqslant\frac{(\varepsilon_1\varepsilon_2)^{\nu-1/2}\Gamma(\nu)}{2\sqrt{\pi}\,\Gamma(\nu+1/2)}\left[\frac{1}{\Gamma(1+2\nu)}+\zeta(2\nu)-\right.$$
$$\left.-\nu(2\nu+1)\zeta(2\nu+1)+\ldots\right]<\infty,$$
$$\frac{1}{2}<\nu<1, \tag{14}$$
$$\|P_{31}\|^2=\|P_{13}\|^2\leqslant\frac{2^{1+\nu}}{\pi}\varepsilon_1{}^4\alpha_\nu^{(1)}\left\{\frac{d_\nu}{\varepsilon^{1/2}}K_{1/2}(2\varepsilon_1)+\alpha_\nu^{(2)}(\varepsilon_2)-\right.$$
$$-2^{1/2}\sum_{m=1}^{\infty}\sum_{n=1}^{\infty}\varepsilon_1{}^{2m}\frac{(-1)^n\Gamma(n+m+\nu+1)}{\Gamma(n+1)\Gamma(n+m+\nu+3/2)\Gamma(2m+n+2\nu+2)}\times$$
$$\left.\times\frac{\partial^{2n+2m-1}}{\partial(2\varepsilon_1)^{2n+2m-1}}\left[\frac{K_{1/2}(2\varepsilon_1)}{(2\varepsilon_1)}\right]\right\}<\infty.$$

Here $\zeta(x)$ is the Riemann zeta function, $K_{n/2}(x)$ are spherical MacDonald functions, and $\alpha_\nu^{(1)}$, d_ν, $\alpha_\nu^{(2)}(\varepsilon_2)$ are quantities that depend on the parameters ν and ε_2, respectively.

The approximate solutions of the infinite SLAE (12) can be found by the method of reduction. Indeed, since the positive-definite operators $\{C^{(j)}\}_{j=1}^4$ can be represented in the form (see [5]) $C^{(j)}=T_j+\alpha I$, where T_j is a positive operator, I is the unit operator, and α is a real number, taking into account the fact that the operators $\{Q_j\}_{j=1}^4$ and $\{P_{jq}\}_{j\neq q=1}^4$ are completely continous we arrive at the conclusion (see [16]) that the method of reduction is applicable to the solution of the infinite SLAE (12).

5. The specific value of the parameter ν, which is determined by the condition on the edge (in the case under study $\nu = 2/3$), was not substituted in all of the foregoing formulas. This is because one must also be able to obtain, in a special case, the new rigorous solution of the problem of diffraction of waves by a collection of key elements. It is clear that if $\nu = 1/2$ is substituted and it is assumed that the dimensions of the flat ribbons $\{a_j\}_{j=1}^{4}$ are such that the ribbons do not touch one another (the faces of the rectangle are separated), then with the help of the infinite SLAE from (12) it is possible to obtain the exact solution of the problem of diffraction of waves by a system of four ribbons. (In so doing, as pointed out in Sec. 3, the coefficients $\{a_j\}_{j=1}^{4}$ in the SLAE (12) must be set equal to zero.) In addition, for such values of the parameters ν and $\{C_j\}_{j=1}^{4}$ the system of equations (12) transforms into a Fredholm SLAE of the second kind with respect to the coefficients $\{\mu_m^{(j)}\}_{m=0}^{\infty}$. This follows from the fact that the matrix $[1+(-1)^{k+m}]C_{km}^{(\nu)}\beta_{km}{}^{\nu}-\delta_{km}$ becomes diagonal (δ_{km} is the Kronecker delta). Thus the solution obtained is of general form for structures consisting of a finite number of flat ribbons with parallel generatrices (ribbon resonators (Figs. 1c–e) and reflectors close to corner reflectors (Fig. 1f), etc.). The set of such structures is limited only by the fact that the surfaces of the ribbons as well as their continuation should not cross the surface of neighboring ribbons.

6. The most important part of the numerical implementation of this approach is the calculation of the matrix elements of the SLAE (12), which are not characteristic integrals. It has been established that as the variable of integration increases the integrand in the quantities $Q_{km}^{(\nu)}$ and P_{km}^{ij} decreases as $O\left(\frac{1}{\alpha^{3+2\nu}}\right)$ and $O\left(\frac{1}{\alpha^{2\nu+1}}\right)$, respectively. This decrease of the functions in the integrand leads to the fact that as the indices increase the quantities $Q_{km}^{(\nu)}$ and P_{km}^{12} decreases as

$$Q_{km}^{(\nu)}\underset{k,m\to\infty}{\sim}O\left[\frac{1}{(km)^{1+\nu}}\right];\quad P_{km}^{12}\underset{k,m\to\infty}{\sim}O\left[\frac{1}{(km)^{\nu}}\right];\quad \nu>\frac{1}{2}. \tag{15}$$

From here it may be concluded that the integrals in the quantities $Q_{km}^{(\nu)}$ can be evaluated, for example, by Simpson's method. As regards the integrals in P_{km}^{12}, to obtain reliable numerical results the convergence of the integrand must be improved. The simplest way to do this is to subtract and add the asymptotic form of the integrand.

We note here that the asymptotic behavior (15) of the matrix elements P_{km}^{12} and the estimate of the norm of the operators generated by these matrices (see (14)) enabled us to draw two very important conclusions. First, this approach does not enable one to pass continuously from the polygon to a flat ribbon, i.e., for example, the height of the rectangle a_1 cannot be made to approach zero, since the norm of the operator $\|P_{11}\|$ increases in an unbounded fashion (as follows from (14)). The transfer to a ribbon and to other ribbon structures can be made as indicated in Sec. 5. Second, it is impossible to join infinitely thin ribbons without changing the conditions on the edge (at the joining point), since if we set $\nu = 1/2$ in the SLAE (12), the operator P_2 will not be completely continuous.

A package of ALGOL-GDR programs has been developed for the BESM-6 computer. The results presented below were obtained using these programs.

Based on the coefficients $\{x_m^{(j)}\}_{m=1}^{\infty}$ found from the SLAE (12) the surface current densities were calculated using Eq. (9), while the total scattering cross section and the DP of the scattered field were determined from the formulas

$$\sigma_s^E=-\frac{4}{k}\operatorname{Re}\Phi(\theta_0),$$

$$\Phi(\varphi)=-\frac{1}{2}\sum_{j=1}^{4}\varepsilon_j\sin(\varphi-\varphi_j)\exp[ikr_{0j}\cos(\varphi-\varphi_{0j})]\times$$

$$\times\left\{\frac{2i}{\varepsilon_j\cos(\varphi-\varphi_j)}\Big[C_{j+1}K_j(-\cos(\varphi-\varphi_j))-C_jK_j(\cos(\varphi-\varphi_j))+\right.$$

$$\left.+\frac{2\pi}{\Gamma(\nu+1/2)}\sum_{n=0}^{\infty}x_n^{(j)}\frac{J_{n+\nu+1/2}(\varepsilon_j\cos(\varphi-\varphi_j))}{[2\varepsilon_j\cos(\varphi-\varphi_j)]^{1+\nu}}\Big]\right\},$$

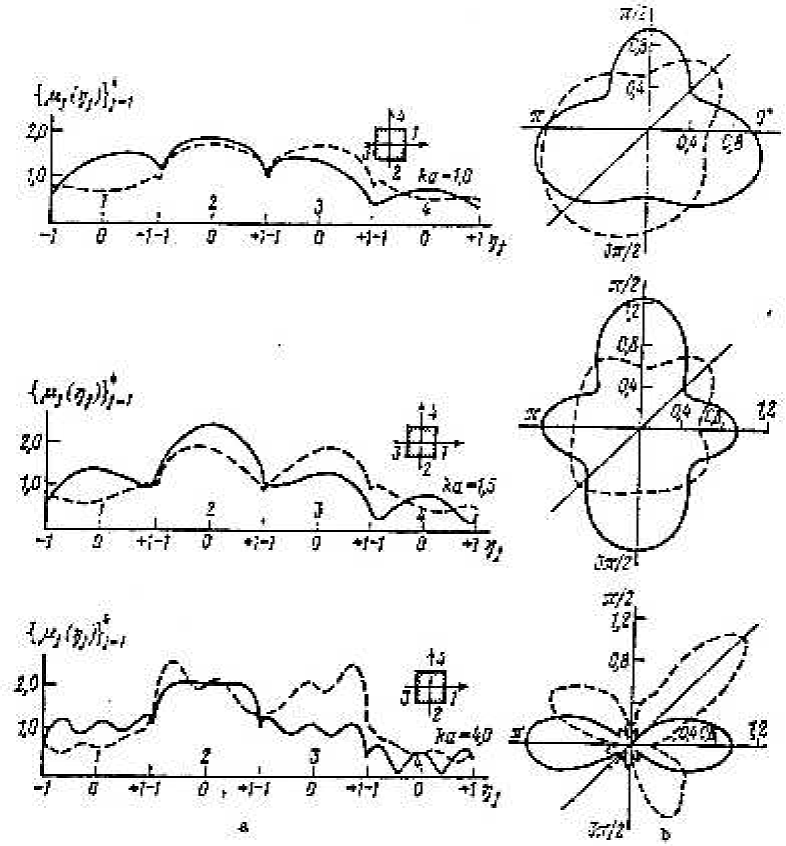

Fig. 2. Distribution of the current density function $|\mu_j(\eta)|_{j=1}^{4}$ on the faces of a square cylinder for different values of $\varepsilon = ka$; the radiation pattern of the field for $\vartheta_0 = 90°$ (the solid line) and 45° (the dashed line).

which can be derived using the expression for the scattered fields in the far zone and the optical theorem [12]. Figure 2 shows the distribution of the surface current densities on the faces of the cylinder with a square cross section for different values of the frequency parameter $\varepsilon = ka$ and angle of incidence ϑ_0. One can see from these figures that for a normally incident wave ($\vartheta_0 = 90°$) the amplitude of the current on the faces in the region of the shade is much smaller than the illuminated region, and as ε increases in the shaded region the number of oscillations of the current increases appreciably. For an angle of incidence $\vartheta_0 = 45$ the two lateral boundaries of the cylinder are illuminated uniformly, so that the amplitudes of the currents on them are appreciably higher than those of the currents on the unilluminated faces. This current distribution leads to the appearance in the radiation pattern of the sidelobe fields (see Fig. 2), comparable with the main and shade-forming lobes. The radiation pattern for $\vartheta_0 = 90°$ and $ka > 1$ is reminiscent of the radiation pattern of a flat ribbon (the irregularity of the radiation pattern near zero is associated with the effect of the side boundaries), while for $ka < 1$ it is reminiscent of the radiation pattern of a circular cylinder.

Figure 3 shows the current distribution and the radiation pattern of the field for a cylinder with a square cross section with $ka = 2\pi$. The DP of the field for cylinders with a rectangular cross section for different values of ka_1 and ϑ_0 are presented in Fig. 4. Note that the radiation pattern of the fields in Fig. 3 is identical, to within graphical accuracy, with the radiation pattern presented in [7].

Table 1 gives the values of σ_s^H for different values of ka and ϑ_0. Figure 5 shows, as an example, the frequency dependences of σ_s^H for a flat ribbon resonator for $\delta = h/a = 0.5$ and 1 as well as the radiation pattern and the current distributions on ribbons. These results agree

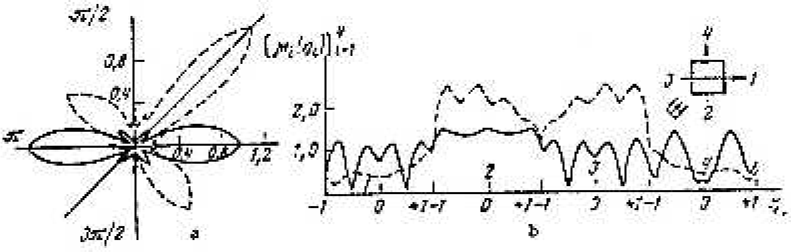

Fig. 3. Square cylinder with the wave size $ka = 2\pi$; a) radiation pattern of the field; b) distribution of the current density function for $\theta_0 = 90°$ (the solid line) and 45° (the dashed line).

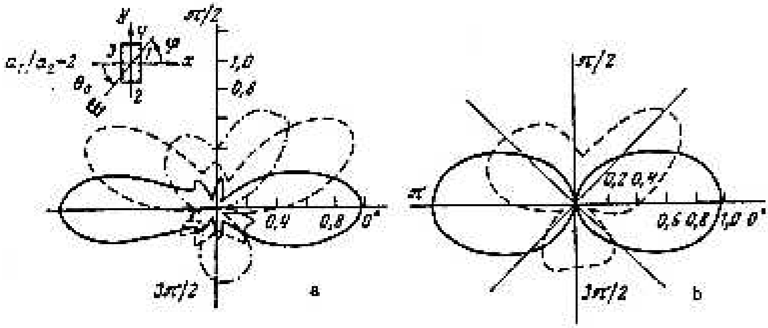

Fig. 4. Radiation pattern of the field for a rectangular cylinder for different values of ka and θ_0: a) $ka_1 = 4$ for $\theta_0 = 0$ (the solid line), 30° (the dashed line), and 60° (the dot-dash line); b) $ka_1 = 3$ for $\theta_0 = 0$ (the solid line) and 45° (the dashed line).

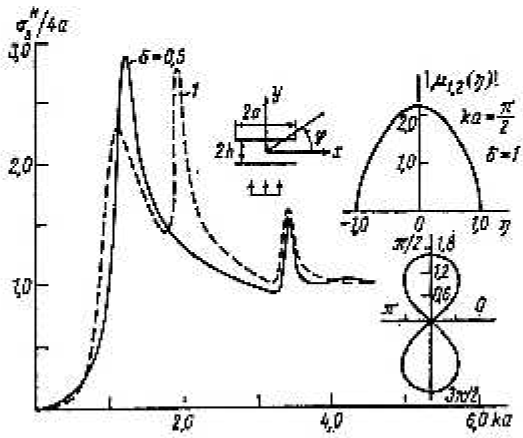

Fig. 5. Frequency dependences of the total transverse cross section $\sigma_s^H/4a$ for $\delta = h/a = 0.5$ and 1, and also the distribution function of the current density $|\mu_i(\eta)|_{i=1}^{2}$ on the surface of the resonator ribbons and the radiation pattern of the field for $ka = \pi/2$, $\delta = 1$, $\theta_0 = 90°$.

σ_s^H			σ_s^H		
ka	θ, deg		ka	θ, deg	
	90	45		90	45
0,4	0,13935	0,14117	3	0,97557	1,12862
0,8	0,60643	0,63168	4	0,96065	1,35306
1,2	0,89496	0,69736	5	1,03665	1,47678
1,5	1,14448	0,71475	2π	0,98557	1,5233

completely with the data of [17]. As follows from Fig. 5, the dependences of c_s^H on ka have a resonance form. The resonance are associated with the excitation of quasicharacteristic oscillations of the ribbon resonator [17].

In conclusion we note that the proposed method has also been employed to solve the problems of diffraction by polygonal cylinders with a more complicated transverse cross section [18, 19].

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