ELECTROMAGNETIC THEORY RADIATION OF ELECTRICAL AND MAGNETIC FRACTAL SURFACE CURRENTS

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1.Introduction

The using possibility of scaled - invariant fractals models for the nature objects description was marked still B.Mandelbrot [1]. Successes of mathematical development of problems on a fractals structure and dimensionality [2] promoted application of differintegrals [3] - [5] for a research of electromagnetic waves interaction with fractal surfaces [6]. In paper [7], using the concept and tools of fractional calculus, N.Engheta introduce a definition for "fractional-order" multipoles of electric-charge densities.

In the present work, we apply the concept of fractional calculus to one of the basic problems in electromagnetics, namely, the problem of electric and magnetic current radiation. We use fractional differintegrals to define α-features [8] of electromagnetic field radiation of fractal currents.

2. On fractal surface simulation

The fractal outlines and surfaces in an electromagnetism are used as a model for actual rough, strongly cut up objects. The prefractal surface simulation is connected to covering by compact sets (segments, quadrates, circles, cubes, and spheres) [9] and with a construction Hausdorff measure [10].

For covering problem solving, we select an universal covering unit as α-measure of a sphere with size V_α (Hausdorff α-measure)

$$V_\alpha = \gamma(\alpha) r^\alpha ,$$
$$\gamma(\alpha) = \frac{2\pi^{\alpha/2}}{\alpha\Gamma(\alpha/2)},$$

where $\Gamma(\alpha/2)$ - gamma-function.

This measure will be derivated after an ordinary integration $(\alpha-1)$-variate of a sphere cuts (surface area)

$$S_\alpha = \frac{\alpha\,\gamma(\alpha)}{r^{1-\alpha}}. \tag{1}$$

Subject to (1), we define Hausdorff measure as

$$V_\alpha = \frac{1}{\Gamma(\alpha)}\int_0^r \frac{\mu(\alpha)}{(r-t)^{1-\alpha}}\,dt, \qquad V_\alpha = \frac{1}{\Gamma(\alpha)}\mu(\alpha)\, r^\alpha = ({}_0I_r^\alpha \mu)(\alpha), \tag{2}$$

where $({}_0I_r^\alpha\mu)(\alpha)$- integral fractal order α [4]; $\mu(\alpha) = \Gamma(\alpha)\gamma(\alpha)$. Obviously, the integer values scaling metric would correspond to covering as spheres, circles, segments.

Electrical and magnetic ñomponents behavior close by prefractal surface we shall determine with their differintegrals (2) (as the α-features field [6]).

It will be agreed physical assumptions (e.g. [11]) about deformation of linear currents by a structure of a fractal surface in Millimeter and Sub-Millimeter wave band, when the discontinuities sizes of an actual rough surface become comparable with a wavelength.

So, in the equations of the Maxwell integrals $\int_L (\vec{H}, d\vec{l})$ and $\int_L (\vec{E}, d\vec{l})$ on a fractal contour L is considered as differintegrals (i.e. as the α-features):

$$
\begin{aligned}
{}_a D_r^{\alpha}(\vec{H}(l)) &= {}_a I_r^{-\alpha}(\vec{H}(l)) = \vec{H}^{(\alpha)}(l), \\
{}_a D_r^{\alpha}(\vec{E}(l)) &= {}_a I_r^{-\alpha}(\vec{E}(l)) = \vec{E}^{(\alpha)}(l),
\end{aligned} \tag{3}
$$

generated by currents $\vec{J}^{\alpha}$.

3. Waves radiation by fractal currents on a metal surface

Subject to (3), we have received expressions for components of a radiation field of the fractal Hertz dipole and Huygens fractal surface element [12-13].

α - features of electrical $\vec{E}^{(\alpha)}$ and magnetic $\vec{H}^{(\alpha)}$ vectors of Hertz 2^{α}-pole in a free space are determined by the formulas

$$
\begin{aligned}
\vec{E}^{(\alpha)} &= \frac{1}{i\omega\varepsilon_0} rot\ rot(\vec{P}^{(\alpha)} G(r)), \\
\vec{H}^{(\alpha)} &= rot(\vec{P}^{(\alpha)} G(r)),
\end{aligned} \tag{4}
$$

where $\vec{P}^{(\alpha)} = \vec{J}\, l^{(\alpha)} = \vec{J}^{(\alpha)}\, l$ - moment of a fractal 2^{α}-pole current; $G(r) = e^{-ikr}/r$; ω - frequency; ε_0 - dielectric constant; r – distance from a 2^{α}-pole up to a point view.

Is separable some fractal surface on elementary two-dimensional $\Delta S^{(\alpha)}$ sites . The site with a current $\vec{J}_S^{e}$ is considered as the fractal Hertz 2^{α}-pole. A moment of a current are defined by

$$
\Delta\vec{P}^{(\alpha)} = \vec{J}_S^{e} \Delta S^{(\alpha)} = J_S^{e(\alpha)} \Delta S .
$$

The radiation of all fractal surface is equal to a sum of 2^{α}-pole radiation. After a passage to the limit $\Delta S \to 0$, we obtain

$$
\begin{aligned}
\vec{E}^{e(\alpha)} &= \frac{1}{i\omega\varepsilon_0} \int_S rot\ rot(J_S^{e(\alpha)} G(r)) dS, \\
\vec{H}^{e(\alpha)} &= \int_S rot(J_S^{e(\alpha)} G(r)) dS.
\end{aligned} \tag{5}
$$

We have decided a interaction problem of an electromagnetic wave with a fractal metal surface [14]. On boundary of such surface there are surface electric currents $\vec{J}_S^{e(\alpha)}$ allocations and surface magnetic currents $\vec{J}_S^{m(\alpha)}$ allocation:

$$
\begin{aligned}
\left[\vec{n}\vec{H}_S^{(\alpha)}\right] &= -\vec{J}_S^{e(\alpha)}, \\
\left[\vec{n}\vec{E}_S^{(\alpha)}\right] &= \vec{J}_S^{m(\alpha)} .
\end{aligned} \tag{6}
$$

Radiation fields of surface magnetic fractal currents are defined by

$$\vec{E}^{m(\alpha)} = -\int_S rot(J_s^{m(\alpha)} G(r))dS,$$

$$\vec{H}^{m(\alpha)} = \frac{1}{i\omega\mu_0} \int_S rot\ rot(J_s^{m(\alpha)} G(r))dS.$$

As a result, fields of fractal surface currents can by written as

$$\begin{aligned} \vec{E}^{(\alpha)} &= \vec{E}^{e(\alpha)} + \vec{E}^{m(\alpha)} = \frac{1}{i\omega\varepsilon_0} \int_S rot\ rot(J_s^{e(\alpha)} G(r))dS - \int_S rot(J_s^{m(\alpha)} G(r))dS, \\ \vec{H}^{(\alpha)} &= \vec{H}^{e(\alpha)} + \vec{H}^{m(\alpha)} = \int_S rot(J_s^{e(\alpha)} G(r))dS + \frac{1}{i\omega\mu_0} \int_S rot\ rot(J_s^{m(\alpha)} G(r))dS. \end{aligned} \tag{7}$$

Using a principle of equivalence S.A. Schelkunov we uppose, that the tangents electromagnetic fields, α-features component, near to a fractal surface are equivalent to surface fractal electrical and magnetic currents. Subject to (6), we obtain from (7):

$$\vec{E}^{(\alpha)} = \frac{1}{i\omega\varepsilon_0} \int_S rot\ rot([\ \vec{n}\vec{H}_s^{(\alpha)}\ ]G(r))dS - \int_S rot([\ \vec{n}\vec{E}_s^{(\alpha)}\ ]G(r))dS,$$

$$\vec{H}^{(\alpha)} = -\int_S rot([\ \vec{n}\vec{H}_s^{(\alpha)}\ ]G(r))dS - \frac{1}{i\omega\mu_0} \int_S rot\ rot([\ \vec{n}\vec{E}_s^{(\alpha)}\ ]G(r))dS.$$

Having taken advantage the equation of a continuity, we enter concept of equivalent fractal surface densenesses of charges: electrical $\sigma^{e(\alpha)}$ and magnetic $\sigma^{m(\alpha)}$ under the formulas

$$\sigma^{e(\alpha)} = -\varepsilon_0 (\vec{n}\,\vec{E}),$$

$$\sigma^{m(\alpha)} = -\mu_0 (\vec{n}\,\vec{H}).$$

4. Waves radiation by fractal surfaces slots

At reviewing of the theory of ideal slot antennas emerges, that on a part of a metal surface the tangent magnetic fields component turns to zero. The field is determined on a tangent electrical fields component. It corresponds to reviewing only of equivalent magnetic currents. In case of an ideal fractal slot (fractal outline or the surface) is necessary to take into account in the equations both fractal electrical influence, and fractal magnetic currents.

A.A.Pistolkors duality principle, we shall as suppose, that to an ideal fractal slot antenna in a free space there corresponds an ideal fractal vibrator. The boundary conditions are identical to α-features of a component field, if them to express through tangents component both for a fractal slot and through tangents component and for a fractal vibrator.

The identity of electromagnetic fields of an ideal fractal slot antenna and ideal fractal lamellar vibrator concerns not only to zone Fraunhofer, but also and to zone immediately contiguous to a slot and according to a vibrator, together with to other zones.

5. Conclusion

In this paper, we analysed initiation of fractal electrical and magnetic currents in an electrodynamic model of waves radiation. We use fractional differintegrals to define α-features of electromagnetic field radiation of fractal currents. The coincidence of the polar pattern of a thin fractal 2^{α}-pole with the diagram for a classical surface unit of the Huygens is marked. The further development of the theory is important for solution of problems of near link.

In recent years chiral media have received considerable attentions due to its potential applications in the fields of electromagnetic and microwave. S.Bassiri, C.H.Papas and N.Engheta [15] analysed reflection and transmission through a semi-infinite chiral medium by obtaining the Fresnel equations in terms of parallel and perpendicular polarized modes. We mark that suggested approach in this paper submits possibility to study the radiation and passing problems of electromagnetic waves in chiral mediums.

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