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Generalization of Green's Theorem with Fractional Differintegration

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Some of the potential mathematical applications of fractional calculus in electromagnetic theory have been recently explored by Engheta [see, e.g., N. Engheta, "Fractional Paradigm in Electromagnetic Theory" a chapter in *Frontiers in Electromagnetics*, D. H. Werner and R. Mittra (eds.), IEEE Press, New York, chapter 12, pp. 523-552, (2000)], and certain tools of fractional differintegration have been applied in some EM problems, resulting in interesting outcomes. Fractional calculus deals with mathematical operators involving fractional (non-integer, or even complex) order differentiation and integration.

In the present study, we consider possibility of generalizing the Green's theorem using the concept of fractional differentiation and/or fractional integration. It is well known that in the conventional Green's theorem for a scalar potential function $\phi(x, y, z)$ satisfying the Helmholtz equation, the value of the function ϕ at any given point inside a closed mathematical volume can be expressed in terms of the values of ϕ and $\partial\phi / \partial n$ over the boundary of this closed volume and in terms of the source inside this volume. This involves the standard Green function for the Helmholtz operator. However, if one wants to use fractional differintegration on this function, how can this relation be generalized? Starting from the basic principles, we develop mathematical steps that include fractional differintegration operators towards obtaining Green's theorem with these operators. Our analysis shows that using such generalization the ν^{th}-order differintegration of function ϕ at any point inside a close volume can be expressed in terms of μ^{th}-order differintegration of ϕ and $\partial\phi / \partial n$ over the boundary of this region, where ν and μ are two arbitrary non-integer constants. This also involves $\nu^{th} - \mu^{th}$ order differintegration of the Green function for the Helmholtz operator. This can lead to an interesting possibility for a novel alternative way of defining the fractional order differintegration of function in terms of the values of the function and its spatial normal derivative over a closed boundary.

In this talk, we will present our mathematical steps in developing the generalized Green's theorem using fractional differentiation and integration.