Total reflection by a small slotted capacitive cylinder

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A solid capacitive cylinder of small dimensions inside a rectangular waveguide only perturbs an incident H_{10} wave to a small extent.[1] For this reason these are not used in practice.[2] However, if a cylinder has a narrow longitudinal slot, total reflection of the H_{10} wave can be achieved under certain conditions.

Consider an H_{10} wave incident from $z < 0$ on a cylinder with a longitudinal slot in a rectangular waveguide (Fig. 2b). Our problem is to determine the diffracted field; this field, together with the incident wave, must satisfy the Helmholtz equation, the boundary conditions at the waveguide walls, the Neumann boundary condition at the surface of the cylinder, the condition that the field be continuous at the slot, the condition that the energy remain finite in a bounded volume, and the radiation condition at infinity. To find the scattered field we use the mirror-reflection method,[4] writing this field as the sum of double-layer potentials distributed on "mirror" sources with an unknown density $\mu(y_1, z_1)$. To determine the Fourier amplitude of the function $\mu(y_1, z_1)$, we use the method of Ref. 3 to derive a system of linear algebraic equations of the second kind, which determine the solution of our problem.

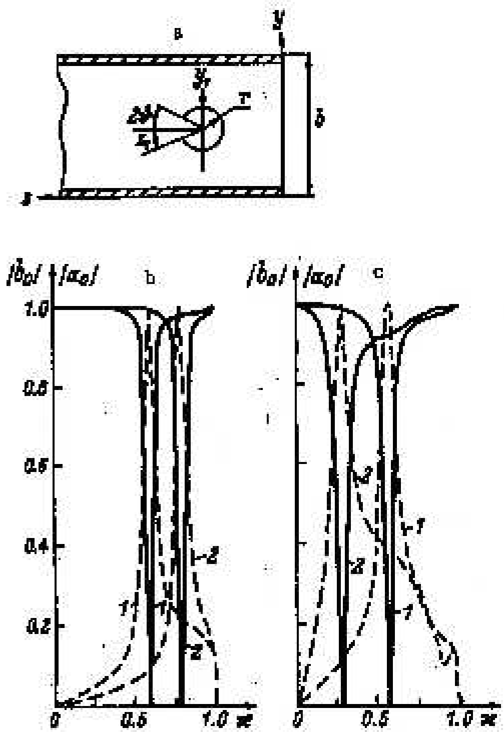

FIG. 1. a: The structure under consideration. b, c) The transmission coefficient (solid curves) and the reflection coefficient (dashed curves) as functions of the frequency parameter ϰ. b: ϑ = 5°. 1) s = 0.02; 2) 0.15. c: ϑ = 15°. 1) s = 0.25; 2) s = 0.5.

For the single-mode case, with a frequency parameter $\varkappa = \frac{a}{2\pi}\beta = \frac{a}{2\pi}k\sqrt{1-\left(\frac{\pi}{ka}\right)^2}$ ($k = \frac{2\pi}{\lambda}$, λ is the wavelength wall), the H_{10} wave is the only undamped harmonic in the diffraction spectrum. The total field far from the inhomo geneity can then be written as the field of the fundamental

$$H_x = \begin{cases} i\frac{\beta a}{\pi}\sin\frac{\pi}{a}x\left(e^{i\beta z} + a_0 e^{-i\beta z}\right)e^{-i\omega t}, & z > \frac{r}{2}, \\ i\frac{\beta a}{\pi}\sin\frac{\pi}{a}x\, b_0 e^{i\beta z - i\omega t}, & z < -\frac{r}{2}. \end{cases} \quad (1)$$

In this case the reflection coefficient $|a_0|$ and the trar mission coefficient $|b_0|$ of the fundamental are completely governed by the properties of the inhomogeneity. For the parameter values $s = 2r/b < 0.5$ and $\vartheta \to 0$, i.e., cylinder with a narrow longitudinal slot and small dimensions (βr $\varkappa s < 1$) we find the following equation for b_0:

$$b_0 = 1 - \frac{1}{\varkappa}\cdot\frac{(\beta r)^4 \ln\sin\frac{\vartheta}{2}}{1 + 2(\beta r)^2\left[1 + i\pi\left(\frac{\beta r}{2}\right)^2(1+Q)\right]\ln\sin\frac{\vartheta}{2}} + O\left[\vartheta^2 + \varkappa^4\left(\frac{r}{b}\right)^4\right], \quad (2)$$

where

$$Q = \frac{1}{\pi\varkappa} - 1 - \frac{2i}{\pi}\left(\ln\frac{\varkappa}{2} + C + 0.601\cdot\frac{\varkappa^2}{\pi}\right).$$

and C is the Euler constant.

The quantity is determined from the condition $|a_0|^2 + |b_0|^2 = 1$, which holds for $0 \le \varkappa < 1$. Setting the denominat equal to zero in the equation for b_0 in (2), we find a dispersion relation for the waveguide with this obstacle. Thi dispersion relation is

$$\frac{1}{2\ln\sin\frac{\vartheta}{2}} + (\beta r)^2\left[1 + i\pi\left(\frac{\beta r}{2}\right)^2(1+Q)\right] = 0. \quad (3)$$

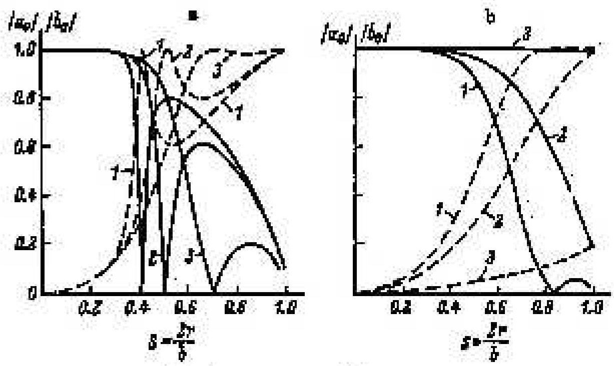

FIG. 2. Curves of $|a_0|$ (dashed) and $|b_0|$ (solid) as functions of the parameter $s = 2r/b$. a: ϰ = 0.2525. 1) ϑ = 1°; 2) ϑ = 5°; 3) ϑ = 20°; b: ϰ = 0.2525. 1) ϑ = 30°; 2) ϑ = 90°; 3) ϑ = 150°.

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The roots of this equation, which are determined by the Newton method, are written $\beta r = q_1 + q_2$, where

$$q_1 = \frac{1}{\sqrt{-2\ln\sin\frac{\vartheta}{2}}}\left(1 + \frac{\pi \operatorname{Im} Q}{8\ln\sin\frac{\vartheta}{2}}\right) + O\left(\frac{1}{\ln^2\sin\frac{\vartheta}{2}}\right);$$

$$q_2 = \frac{\pi}{16\ln\sin\frac{\vartheta}{2}\sqrt{-2\ln\sin\frac{\vartheta}{2}}}(1 + \operatorname{Re} Q) + O\left(\frac{1}{\ln^2\sin\frac{\vartheta}{2}}\right). \quad (4)$$

For the parameter values $\beta r = q_1$ we find from (2) that $b_0 \to 0$ as $O[\vartheta^2 + \varkappa^2(\frac{\vartheta}{2})^3]$; in other words, almost total reflection of the H_{10} wave occurs from the obstacle. At $\vartheta = 0$, on the other hand, (in which case the inhomogeneity is a closed cylinder), we find from (2) that $\xi_0 = 1 + O[\varkappa^2(\frac{\vartheta}{2})^2]$ This result corresponds to nearly total transmission of the H_{10} wave. In this case the incident wave induces weak currents at the inhomogeneity, and the primary field is only slightly perturbed. The appearance of a narrow slot in the wall of a cylinder with a small wave dimension thus leads to a qualitatively new resonant effect: total reflection of an incident wave.

Figures 1 and 2 show that when $\varkappa < 1$ with small slot width ϑ we obtain total reflection of the incident wave for various values of s in correspondance with $\beta r = q_1$. As ϑ is increased, the Q of the resonances of $|a_0|$ and $|b_0|$ decreases, and these values shift towards larger values of s (Fig. 2a). With large slots in the cylinder, the resonances in $|a_0|$ and $|b_0|$ vanish, and the scattered field becomes similar to the field scattered by a capacitive iris[5] (Fig. 2b).

In conclusion we note that the curves in Figs. 1 and 2 are found from a computer solution of this system of linear algebraic equations of the second kind.

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