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TRAPPING EFFECT FOR AN OPEN SCREEN ILLUMINATED BY A FIXED SOURCE

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The "trapping" effect arising with the excitation of an open circular cylindrical screen by a magnetic current filament is examined. It is shown that under certain conditions practically all of the energy from the source is captured by the open structure, while the emissivity of the antenna drops sharply. It is noted that this phenomenon is characteristic of the source + resonator pair, but not of the resonator itself.

At the present time, in antenna theory, there is great interest in problems concerning the radiation of a fixed source situated near thin metallic open surfaces. A number of papers have appeared that are concerned with developing quite general numerical methods [1-3], based on the solution of an integral equation of the first kind, and their realization for angular, parabolic, and circular cylindrical reflectors [2, 4, 5].

Unfortunately, in the papers mentioned, the physical characteristics of the wave fields arising as a result of scattering of the field from the source by the open screen were not adequately studied. This is related to the fact that the main emphasis was on calculating the directivity patterns (DP) at certain fixed frequencies. The values of the latter were usually chosen so that the wave dimensions of the screen constituted some integer.

Nevertheless, the radiation of a concentrated source in the presence of an open screen is so complicated that the DP, calculated at nearly equal frequencies, could be completely different from one another. In addition, the intensities of the radiation can differ by several orders of magnitude. Such phenomena are explained by the resonant properties of the open curvilinear screen, as well as by the interference of the primary and scattered fields.

In this paper, we investigate in detail the electrodynamic structure consisting of a source and a resonant screen, for the example of the emission by a filament with current situated near an open metallic circular cylinder.

1. Formulation of the Problem and Construction of Its Formal Solution. Let the synchronous filament of magnetic current be located at a distance l from the axis of an open circu-

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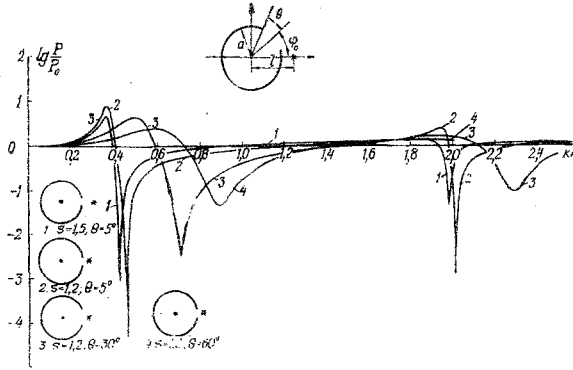


Fig. 1

lar cylinder parallel to it with radius α (Fig. 1). The angular size of the gap in the cylinder is 2θ , its angle of orientation is φ_0 , and the surface of the screen is assumed to be ideally conducting and infinitely thin.

As is well known, in free space, the current filament emits a cylindrical wave

$$H_z^0 = AH_0^{(1)}(kr'), \quad (1)$$

where $H_0^{(1)}(x)$ is a Hankel function, $k = 2\pi/\lambda$, $A = I_0 k/\pi$, $r' = \sqrt{r^2 + l^2 - 2rl \cos \varphi}$.

The investigation of the properties of an antenna system, consisting of a filament and a screen, reduces to determining the electromagnetic field H_z^S , scattered by the screen. The function $H_z^S(r, \varphi)$ must satisfy well-known conditions at infinity, on the surface of the screen, and on its sharp edges, and can be represented in the following form:

$$H_z^S = \sum_{n=-\infty}^{\infty} \mu_n \begin{cases} J_n(kr) H_n^{(1)'}(ka) \\ H_n^{(1)}(kr) J_n'(ka) \end{cases} e^{in\varphi}, \quad \begin{matrix} r < a \\ r > a \end{matrix}, \quad (2)$$

where the quantities μ_n are Fourier coefficients of the surface density function of the current (supplemented by zero in the gap), induced on the cylinder, and satisfy a system of equations of the first kind in paired series (with a trigonometric kernel) of the same type as in the problem of diffraction of a plane wave [6]. The permissible solution of these

equations belong to the class \tilde{l}_2 ; $\sum_{n=-\infty}^{\infty} |\mu_n|^2 |n| < \infty$, which follows from the condition that the

energy in the vicinity of the edge of the screen is finite. The paired equations are regularized by inverting the static part of the corresponding operator using the conjugate problem method [7], which leads to a system of algebraic equations of the second kind:

$$\mu_n + \sum_{m=-\infty}^{\infty} A_{nm} \mu_m = B_n, \quad n = 0, \pm 1, \pm 2, \dots, \quad (3)$$

where

$$A_{nm} = [|m| + i\pi(ka)^2 J_m'(ka) H_m^{(1)'}(ka)] T_{nm},$$

$$T_{nm} = (-1)^{m+n} e^{-i(n-m)\varphi_0} \begin{cases} V_{n-1}^{m-1}(-\cos \theta)/n, & n \neq 0 \\ V_{m-1}^{-1}(-\cos \theta)/m, & n = 0, m \neq 0 \\ -\ln [(1 - \cos \theta)/2], & n = 0, m = 0 \end{cases}$$

$$B_n = i\pi(ka)^2 \sum_{m=-\infty}^{\infty} \begin{cases} J_m'(ka) H_m^{(1)}(kl) \\ H_m^{(1)'}(ka) J_m(kl) \end{cases} T_{nm}, \quad \begin{matrix} l > a \\ l < a \end{matrix}$$

while the quantities V_{m-1}^{n-1} are defined in [7].

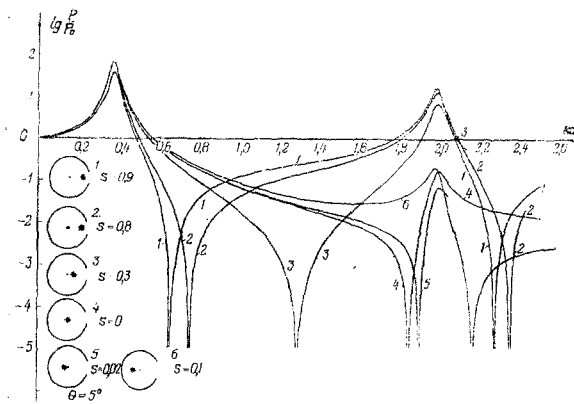


Fig. 2

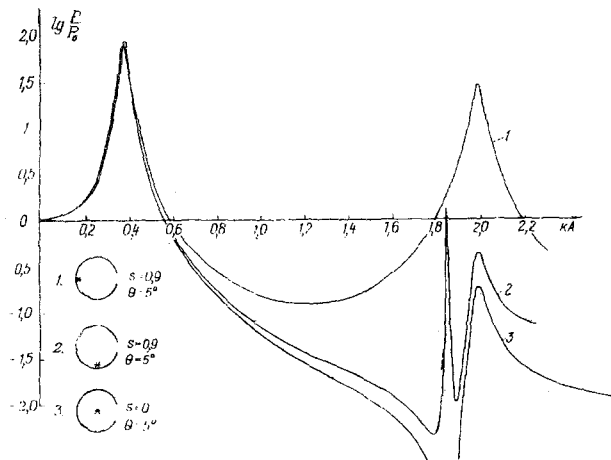


Fig. 3

We note that the solution obtained for the starting boundary-value problem is rigorous in the sense that Eq. (3) can be solved with any predetermined accuracy. This follows from the Fredholm system (3). The only exception is the case $l = \alpha$, for which the series $\sum_{n=-\infty}^{\infty} |B_n|^2$ diverges logarithmically, which makes it impossible to apply the method of reduction. In the long wavelength region, an estimate of the norm of the matrix operator leads to the inequality

$$q = \max_{m \neq 0} \{ |1 - A_{mm}|^{-1} \sum_{n=-\infty}^{\infty} |A_{mn}| \} < (ka)^2 f(\theta), \quad (4)$$

so that for sufficiently small ka Eq. (3) can be solved by iteration ($q < 1$). The numerical summation permits refining the estimate (4). It turns out that the iteration process converges up to $ka = 1.7$ for any θ and φ_0 . This permits, for small wave dimensions of the cylinder, determining the characteristics of the radiation in analytic form with an estimate of the error.

As far as the numerical realization is concerned, the much more rapid, compared to direct methods [2, 3], convergence of the reduction method applied to the solution of (3) permits studying effectively the characteristics of an antenna over a wide range of its parameters. For example, calculation of ninety values of the emitted power in the interval $ka = 0.2-2$ with a uniform step size takes only 20 min on a M-222 computer.

2. Radiation Characteristics with Excitation of Characteristic Regimes of the Antenna. Calculation of the frequency dependences of the integral characteristics of the antenna permits clarifying the resonance frequencies of the screen and the regions of extreme radiation. Such a characteristic is, for example, the total radiation power P normalized to the power of the radiation of the filament in free space P_0 (P/P_0 also coincides with the normalized value

of the radiation resistance of the antenna). Of course, calculation of the DP requires much less machine time than calculation of the frequency characteristics, but, on the other hand, the latter make it possible to choose, in a well-founded manner, the frequencies for calculating the DP. The results of the calculation of the frequency dependences of P/P₀ on a computer for different values of θ and s = a/l are shown in Figs. 1-3.

As for scattering of a plane wave by an open screen [6], excitation of high Q factor quasicharacteristic regimes of the screen is accompanied by a jumplike change in the DP and an increase in the total radiation power. In this case, it is as if a powerful linear source of secondary radiation appears on the surface of the cylinder at the location of the slit, since the slit is equivalent to a magnetic current filament. If the Q factor is high, then a field with high amplitude, whose structure is close to the structure of the characteristic oscillation of a closed circular cylinder, forms inside the screen (of the open resonator). For this reason, at the resonance frequency, the amplitude of the secondary radiation is determined by the field in the gap and exceeds the amplitude of the radiation of the filament in free space, in view of the fact that the radiation of the concentrated source is amplified.

The resonance frequencies are close to the zeros of the functions J_n¹(x) and exceed them by some amount depending on the gap. The increment turns out to be different for symmetrical (H_{mn}⁺) and antisymmetrical (H_{mn}⁻) oscillations, which removes the polarization degeneracy for m > 0:

$$k_{mn}^+ a = \nu_{mn} + \nu_{mn} [\varepsilon_m (\nu_{mn}^2 - m^2) \ln \sin(\theta/2)]^{-1} (1 - i \xi_m \ln^{-1} \sin(\theta/2)); \quad (5)$$

$$k_{mn}^- a = \nu_{mn} + \nu_{mn} m^2 (\nu_{mn}^2 - m^2)^{-1} \sin^2(\theta/2) (1 - 2 i x_m \sin^2(\theta/2)),$$

$$\varepsilon_m = \begin{cases} 1, & m = 0 \\ 2, & m \neq 0 \end{cases},$$

$$J'_m(\nu_{mn}) = 0, \quad \xi_m = \frac{1}{\pi \nu_{mn}^2} \sum_{k \neq 0, m} \frac{1}{|H_k^{(1)'}(\nu_{mn})|^2}, \quad (6)$$

$$x_m = \frac{1}{\pi \nu_{mn}^2} \sum_{k \neq m} \frac{k^2}{|H_k^{(1)'}(\nu_{mn})|^2}.$$

The quasistatic resonance regime of the gap type, which is usually denoted by H₀₀, occupies a special position [8]:

$$k_0 a = (-2 \ln \sin(\theta/2))^{-1/2} (1 + (i\pi/16) \ln^{-1} \sin(\theta/2)). \quad (7)$$

When the current filament lies in the plane of symmetry of the screen φ₀ = 0.180° (Fig. 1, 2), the maxima in the frequency dependence of P/P₀ correspond to the excitation of even-type characteristic regimes: H₀₀, H₁₁⁺, H₂₁⁺, etc. If, on the other hand, the screen is illuminated in an asymmetrical manner (Fig. 3), then the resonances correspond to the regimes H₀₀, H₁₁⁻, H₁₁⁺, etc.

In the long wavelength region, iteration leads to the following equations for the power and directivity pattern:

$$\frac{P}{P_0} = \frac{1}{|D_0|^2} \begin{cases} \left[1 - \left(\frac{x}{x_0} \right)^2 \left(1 + \frac{x^2}{2} \ln \frac{x}{2s} \right) - x^2 \ln(1-s) \right]^2, & a < l \\ \left[1 - \left(\frac{x}{x_0} \right)^2 \left(\frac{x}{2s} \right)^2 + x^2 \ln \left(1 - \frac{1}{s} \right) \right]^2, & a > l \end{cases}; \quad (8)$$

$$\Phi(\varphi) = \frac{A}{D_0} \left[1 - \left(\frac{x}{x_0} \right)^2 \left(1 + \frac{x^2}{2} \ln \frac{x}{2s} \right) + x^2 \ln(1-s) - \frac{\pi x^3}{2} \left(1 + \frac{2i}{\pi} \ln \frac{x}{2s(1-s)} \right) \cos \varphi \right], \quad a < l, \quad (9)$$

where x = ka, D₀ = 1 - (x/x₀)² + iπx⁴(4x₀)⁻², x₀ = [-2 ln sin(θ/2)]^{-1/2}, and s = a/l; these equations are valid for s ≠ 1.

The frequency $k = k_0 = x_0/a$ is a resonant frequency of the Helmholtz mode H_{00} . In this regime, the increase in the radiation power is described by the expression

$$\frac{P_{\text{res}}}{P_0} = \frac{16}{\pi^2} \begin{cases} \ln^2 [x_0(1-s)/(2s)], & a < l \\ [1 - x_0^2/(4s)^2 + x_0^2 \ln(1-1/s)]/x_0^4, & a > l \end{cases} \quad (10)$$

As the curves in Figs. 1-3 show, in the case of a H_{00} resonance, the increase in P/P_0 is not large (1-1.5 orders of magnitude), since this regime has a small Q factor $Q_{00} = -\text{Re}k_0/2 \text{Im} k_0 = (8/\pi) \ln \sin^{-1}(\theta/2)$. The same is true for oscillations of the type H_{mn}^+ . The odd-type oscillations H_{mn}^- have a higher Q factor and when they are excited, the increase in P/P_0 can be appreciable.

We note that the phenomenon of an increase in the radiation power of a concentrated source with the help of an open resonator, weakly coupled to external space, is well known in acoustics. Indeed, many musical instruments are nothing more than an acoustical antenna equipped with an open resonator, above whose coupling opening an active emitter, viz. the oscillating string, is located. The quality of the resonator and its shape and dimensions determine the spectrum of the characteristic regimes and their Q factors, on which the musical properties of the instrument depend. The theoretical investigation of the problem of interaction of a sound source and an open resonator is contained in papers by Karnovskii [9].

3. Antiresonance Phenomena and Extinction of Radiation. Aside from the resonance maxima, the frequency dependences P/P_0 have a number of deep troughs (Figs. 1-3). Thus, a characteristic of the system consisting of a source and an open screen placed next to one another is the presence of both resonance and antiresonance phenomena. If the former are explained by excitation of the characteristic oscillations of the screen, then the latter originate according to an entirely different mechanism. The characteristic, for an antiresonance, sharp drop in the radiation power occurs due to the compensation of the field from the source by the diffraction field.

Let us turn to Eq. (8), which describes the frequency dependence of the power in the long wavelength region. When the radiation frequency somewhat exceeds the resonant frequency, namely:

$$k_1 a = \begin{cases} x_0 [1 + x_0^2 \ln(1-s)]^{-1/2}, & a < l \\ [-\ln(1-1/s)]^{-1/2}, & a > l \end{cases} \quad (11)$$

the emission efficiency drops by several orders of magnitude: $P_\alpha/P_0 = O[x^6 |D_0|^{-2} \ln^2(x/s)]$. Relation (11) can be called the antiresonance condition. The cases of external ($l > a$) and internal ($l < a$) positioning of the source relative to the open screen must be distinguished.

If $l > a$, then at the frequency k_1 the filament with the current and the open resonator act as a pair of emitters of the same type, compensating one another in the far zone due to the near equality of the amplitudes and the opposite phases of the radiated fields. The effect is all the more clearly manifested, the closer the filament, and the open resonator are to one another and the narrower the gap in the screen (Fig. 1). In the limit, when the source is located at infinity and the screen is illuminated by a plane wave, nothing resembling compensation of radiation is observed [6]. Thus, we can conclude that antiresonance phenomena are characteristic of the pair source + open resonator, but not the resonator by itself. Compensation is always observed at frequencies higher than the frequency of the corresponding resonance ($k_1 > k_0$). This is related to the fact that at lower frequencies the scattered field is in phase with the incident field, while at the point of resonance, the phase of the field changes to the opposite value.

With internal excitation ($l < a$), this explanation is no longer obvious, so that it is more correct to speak of screening of the radiation source (Figs. 2, 3).

The attenuation of the radiation is of interest in itself in connection with the problem of extinction of electromagnetic waves. Extinction occurs most efficiently at antiresonance frequencies, but with an appropriate choice of parameters of the system, it is possible to achieve attenuation of radiation by one to two orders of magnitude in the octave range and higher (Fig. 2).

As the aperture of the gap increases, the open cylinder transforms into a cylindrical strip. It turns out that the power of the radiation of such an antenna oscillates near the

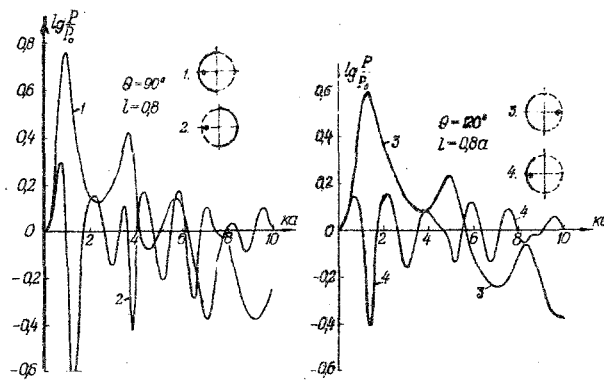


Fig. 4

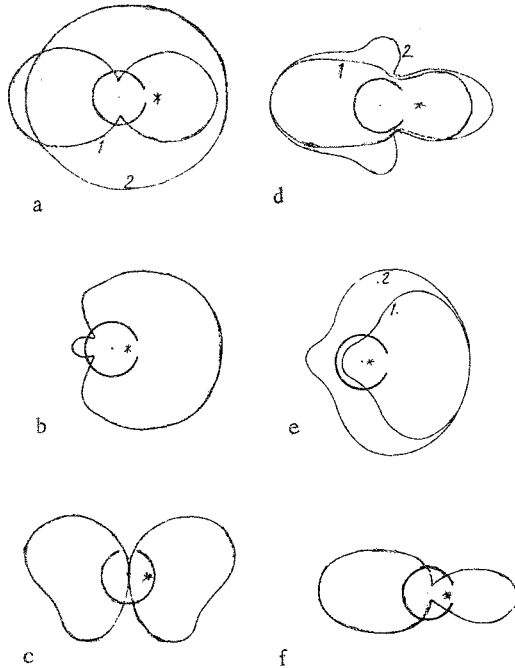


Fig. 5. Directivity patterns for different θ , ka , and s ; a) $\theta = 5^\circ$, $s = 1.2$, $ka_1 = 0.454$; b) $\theta = 5^\circ$, $s = 0.5$, $ka = 1.985$; c) $\theta = 5^\circ$, $s = 0.9$, $ka = 1.84$, $ka_2 = 0.375$; d) $\theta_1 = 30^\circ$, $\theta_2 = 60^\circ$, $s = 1.2$; e) $\theta = 5^\circ$, $s_1 = 0.5$, $s_2 = 0.3$; f) $\theta = 5^\circ$, $s = 0.8$, $ka = 0.7473$, $ka_1 = 1.04$, $ka_2 = 1.28$.

value $P/P_0 = 1$ with a small amplitude, and the role of the screen lies mainly in forming the directivity pattern (Fig. 4).

4. Directivity Patterns and the Field in the Near Zone. The directivity pattern of the antenna being examined is determined by the superposition of the incident cylindrical wave and the scattered field. The resonance and antiresonance frequencies, near which there is a sharp almost jumplike change in the DP, are of special interest.

As noted above, when high Q factor resonance regimes are excited, the amplitude of the secondary, scattered field exceeds the amplitude of the incident field. Naturally, in this case, the DP is formed mainly by the scattered field, which is close to the field of the radiation of the magnetic current filament (gap), lying on the surface of an ideally conducting circular cylinder.

In particular, when a quasistatic resonance regime H_{00} is excited, the DP becomes almost omnidirectional (Fig. 5a, curve 2): the cylinder with small wave dimensions distorts little the circular DP of the secondary source, namely, the emitting gap. For the resonance regime H_{11}^+ , the effect of the cylinder is now much stronger: the backward radiation ($\varphi = \pi$) is much weaker than the forward radiation ($\varphi = 0$) (Fig. 5b). The absence of radiation in the direction of orientation of the slit, as is evident from Fig. 5c, for regime H_{11}^- , is characteristic for odd-type resonance regimes, since here the emitting aperture (gap) is excited with opposite phase. In this case, the DP is close to the diagram of a pair of current filaments with opposite phase, lying on the surface of a closed cylinder. It has a characteristic double-lobe shape, while the radiation in space, opposite to the gap, is attenuated.

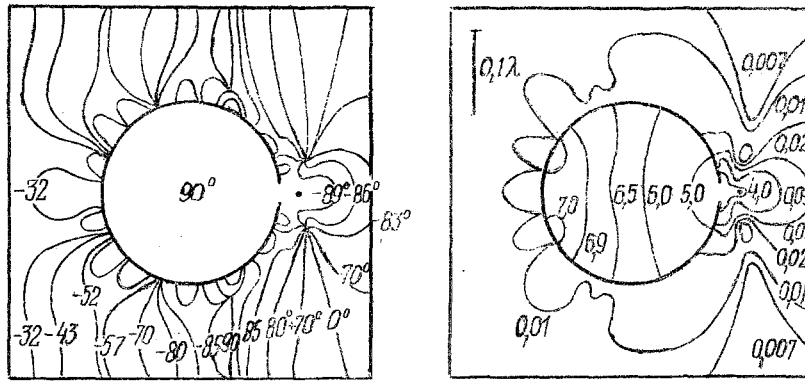


Fig. 6. The lines of equal phases and amplitudes ($H_z = \text{const}$).

At the antiresonance frequencies, with external excitation, the current filament and the gap in the open resonator act as a pair of linear emitters with nearly equal amplitudes and opposite phases, and in addition, one of them lies on the surface of the cylinder. For example, at low frequencies, the effect of the cylinder is small, and for $k = k_1$ the DP assumes the form of an almost regular figure 8 (Fig. 5a, curve 1). As the width of the gap increases, the figure 8 is distorted (Fig. 5e), since a wide gap can no longer completely compensate the emission of the filament with the current.

With internal excitation at the antiresonance frequency, the DP no longer has such a simple shape and strongly depends on the position of the source (Fig. 5e, f), which again indicates the difference between internal and external excitation.

As far as the field in the near zone is concerned and inside the cylindrical open resonator, at resonant frequencies it is close to the field of the corresponding characteristic oscillation of the closed cylinder. The amplitude of the field in the resonator is proportional to its Q factor. Figure 6 shows the phase and amplitude distribution of the field H_z near the screen at the antiresonance frequency. The pattern force lines does not follow any clear law, but its amplitude within the cylinder remains finite, while the phase is constant.

5. "Trapping" Effect. The preservation of a finite amplitude of the field inside the resonator, together with the sharp drop of the emissivity, suggested to us the idea that the "trapping" properties of the hollow resonator with a small coupling opening must be considered keeping in mind the antiresonance phenomena.

In the past, this term was used without indicating its meaning sufficiently clearly. For example, in [10, 11], the "trapping" action of the open resonator was discussed in connection with the increase in amplitude of the field inside the cavity at the resonance frequency, when the resonator is illuminated with a plane wave. But, in this case, the important circumstance that the amplitude of the scattered field increases simultaneously was neglected [6]. For this reason, we believe that here we cannot speak directly about "trapping" of the energy of the incident field. On the other hand, under antiresonance, conditions the emissivity of the system drops by several orders of magnitude, while the amplitude of the field inside the screen remains finite and drops rapidly with distance away from the screen (Fig. 6), which corresponds more closely to the term "trap."

In addition, speaking descriptively, the resonator is capable of "trapping" only the field of the source situated at a finite distance away from it, and the closer the source, the more successfully is the radiation trapped. For this, the geometry of the resonant screen must in some sense be related to the geometry of the source. The open cylinder can compensate the radiation of the filament with the current due to the fact that the longitudinal gap plays the role of such a filament, but with "specular" parameters. In order to compensate, for example, the radiation of a point source, the inhomogeneity in the screen, evidently, must have a similar pointlike character.

In conclusion, we emphasize once again that, in our opinion, the results presented above indicate the necessity of treating with care the calculation of directivity patterns at fixed frequencies. In studying an antenna consisting of a source and an open screen, priority must be given to calculating the frequency dependences of the integral characteristics of the radiation field.

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DIFFRACTION OF TWO-DIMENSIONAL GAUSSIAN BEAMS BY A
REFLECTION GRATING OF RECTANGULAR BARS

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UDC 538.574.6

An algorithm is synthesized for solving numerically the problem of diffraction of a two-dimensional wave beam by a reflection grating of rectangular bars with an arbitrary ratio between its period and the wavelength. The dependence of the shape of the directivity pattern and the transmission coefficient on the geometrical dimensions of the structure and the parameters of a Gaussian beam is investigated.

"Corrugation" reflection gratings of rectangular bars are used extensively in the design of various quasioptical and microwave devices.

The diffraction properties of a rectangular corrugation in the case of interaction with a plane electromagnetic wave have been thoroughly studied to date [1]. In real situations, however, a wave beam is incident on a corrugation grating as a constituent part of a device, and so for the successful application of corrugations in devices and instruments at millimeter and submillimeter wavelengths it is necessary to consider the diffraction characteristics of these structures when wave beams are incident on them. Below, for a rectangular corrugation grating in the case of H-polarization we give the fundamental physical results of a study of the diffraction of two-dimensional wave beams with a Gaussian field distribution as a function of the geometrical dimensions of the corrugation and the beam parameters.

If the field distribution

$$\psi_0(y, z_0) = \exp[-(y - z_0 \operatorname{tg} \alpha)^2 / (w \cos \alpha)^2]$$

is created in a plane that is parallel to the X axis, forms an angle α with the XY plane, and is situated at a distance $z_0 / \cos \alpha$ from the origin, a Gaussian wave beam will be incident at an angle α on a corrugation grating situated in the XY plane (Fig. 1). The field of the beam can be represented by a Fourier integral expansion with respect to plane waves:

$$\psi_0(y, z) = \int_{-\infty}^{\infty} q(\xi) \exp\{ik[y\xi - z\gamma(\xi)]\} d\xi,$$

where, as is readily shown,

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