

Floquet states and persistent currents transitions in a mesoscopic ring

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We consider the effect of an oscillating potential on the single-particle spectrum and the time-averaged persistent current of a one-dimensional phase-coherent mesoscopic ring with a magnetic flux. We show that in a ring with an even number of spinless electrons the oscillating potential has a strong effect on the persistent current when the excited side bands are close to the eigen levels of a pure ring. Resonant enhancement of side bands of the Floquet state generates a sign change of the persistent current.

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Mesoscopic systems subject to a periodic in time, external driving force are now of considerable interest. This is illustrated by work concerning low frequency ac-transport^{1,2}, photon-assisted tunneling³⁻⁶, and quantum pumping^{7,8}. This list could easily be extended. Electrons interacting with a time-dependent potential⁹ can gain or loss energy and thus the electron system has no stationary states and, in particular, there is no stationary ground state. However if the external potential is periodic in time we can describe the state of a system using the Floquet function¹⁰⁻¹² which is a superposition of wave functions with energies shifted by $n\hbar\omega$ (here n is an integer; ω is the frequency of the driving potential). The existence of many components (side bands) of a wave function has a strong effect on the properties of a mesoscopic system. For instance, side bands open up additional channels for transmission through the mesoscopic system - photon assisted transmission (see e.g., Refs. 11,12). The existence of side bands is also a necessary condition for pumping charge through an unbiased mesoscopic sample¹³.

The aim of the present paper is to investigate the properties of a phase-coherent mesoscopic ring in the Floquet state. We are interested in the coherent properties of the Floquet state of a ring structures with an oscillating potential. In a ring structure the wave functions must not only be periodic in time but also periodic in space. As a consequence a ring, in the presence of an Aharonov-Bohm flux¹⁴ Φ exhibits a persistent current¹⁵⁻¹⁹. The persistent current is a signature of the coherence of the ground state of a mesoscopic ring²⁰. Therefore, it is interesting to investigate how the persistent current is affected by a time-dependent potential. We find that under certain conditions the system exhibits transitions between the different components of the Floquet state. Our analysis shows that the absolute value of the amplitudes of the side bands of the Floquet states are strong functions of frequency and of flux. As a consequence the persistent current displays transitions as a function of frequency

and flux.

Let us consider the time-dependent Schrödinger equation for an electron wave function $\Psi(x, t)$ on a circle of circumference L threaded by an Aharonov-Bohm magnetic flux Φ with an oscillating delta-function potential

$$i\hbar \frac{\partial \Psi(x, t)}{\partial t} = \hat{H}(x, t) \Psi(x, t),$$

$$\hat{H}(x, t) = -\frac{\hbar^2}{2m} \left(\frac{\partial}{\partial x} - 2\pi i \frac{\Phi}{\Phi_0} \right)^2 + V(t) \delta(x), \quad (1)$$

$$V(t) = 2LV \cos(\omega t).$$

Here $\Phi_0 = h/e$ is the magnetic flux quantum. To solve this equation we will use the method of Floquet functions^{11,12}. Because the Hamiltonian \hat{H} depends on time the system has no stationary eigenstates. Since the Hamiltonian is periodic in time, the states of Eq.(1) can, according to the Floquet theorem, be represented as a superposition of wave functions with energies shifted by $n\hbar\omega$

$$\Psi_E(x, t) = e^{-iEt/\hbar} \sum_{n=-\infty}^{\infty} \psi_n(x) e^{-in\omega t}. \quad (2)$$

By analogy with a pure ring problem we choose the functions $\psi_n(x)$ in the following form

$$\psi_n(x) = e^{2\pi i \frac{\Phi}{\Phi_0} \frac{x}{L}} (a_n e^{ik_n x} + b_n e^{-ik_n x}). \quad (3)$$

Here $k_n = \sqrt{2mE_n/\hbar^2}$ and $E_n = E + n\hbar\omega$. The coordinate x is directed along the ring $0 \leq x < L$. Note, that for the evanescent modes ($E_n < 0$) we put $k_n = i\kappa_n$ with $\kappa_n = \sqrt{2m|E_n|/\hbar^2}$.

On a ring the Floquet eigenfunction Ψ_E must be periodic in x . In addition its derivative is discontinuous at the delta function barrier. Thus Ψ_E is subject to the following boundary conditions

$$\begin{aligned}\Psi_E(x, t) &= \Psi_E(x + L, t), \\ \frac{\partial \Psi_E(x, t)}{\partial x} \Big|_{x=+0} - \frac{\partial \Psi_E(x, t)}{\partial x} \Big|_{x=L-0} &= \frac{2m}{\hbar^2} V(t) \Psi_E(0, t).\end{aligned}\quad (4)$$

These boundary conditions define the discrete set of Floquet eigenenergies $E^{(l)}$ (where l is an integer) and corresponding Floquet eigenfunctions $\Psi_{E^{(l)}}$ which are characteristic for the ring problem. The quantization of the Floquet energy in a finite-size system is quite analogous to the quantization of an energy in the time-independent problem. In addition each Floquet state can be occupied by only one electron (because of the Pauli principle) and thus the wave function Ψ_E must be normalized

$$\frac{1}{T} \int_0^T dt \int_0^L dx |\Psi_E|^2 \equiv \sum_n \int_0^L dx |\psi_n|^2 = 1. \quad (5)$$

Here $T = 2\pi/\omega$. Furthermore, note that usually the Floquet energy E is determined within the interval $0 \leq E < \hbar\omega$. However in our problem it is convenient not to reduce the discrete set of $E^{(l)}$ to this interval.

Substituting Eq.(2) and Eq.(3) into the Eqs.(4) we obtain the following relations between coefficients a_n and b_n of the Floquet function Ψ_E corresponding to the Floquet energy E

$$\begin{aligned}a_n A_n + b_n B_n &= 0, \\ a_n A_n - b_n B_n &= \\ -i \frac{2v}{k_n} (a_{n-1} + b_{n-1} + a_{n+1} + b_{n+1}).\end{aligned}\quad (6)$$

Here we have introduced

$$\begin{aligned}A_n &= e^{-2\pi i \frac{\Phi}{\Phi_0}} - e^{ik_n L}, \\ B_n &= e^{-2\pi i \frac{\Phi}{\Phi_0}} - e^{-ik_n L}, \\ v &= \frac{mLV}{\hbar^2}.\end{aligned}\quad (7)$$

Eqs.(6) couples amplitudes of different index n . As a consequence we obtain an infinite system of uniform linear equations for the coefficients a_n and b_n ($n = 0, \pm 1, \pm 2, \dots$). To have a nontrivial solution the corresponding (infinite range) determinant must be equal to zero. This condition defines the allowed values of the Floquet energy and the corresponding set of coefficients a_n and b_n .

Using the method of continued fractions²¹ the calculation of an infinite range determinant can be greatly simplified. To this end we rewrite the first equation of Eqs.(6) as follows

$$b_n = -a_n \frac{A_n}{B_n}, \quad (8)$$

Substituting the above relation into the second equation of Eqs.(6) we obtain a recursive equation for the coefficients a_n . It is convenient to introduce new quantities x_n ($n \neq 0$)

$$x_n = \frac{1}{v} \frac{a_n}{a_{n\mp 1}} \frac{\sin(k_n L)}{\sin(k_{n\mp 1} L)} \frac{B_{n\mp 1}}{B_n}. \quad (9)$$

Here and hereafter the upper (lower) sign is for $n \geq 1$ ($n \leq -1$). In terms of the x_n the recursive equation reads

$$x_n = \frac{\sin(k_n L)}{k_n D_n - v^2 \sin(k_n L) x_{n\pm 1}}, \quad (10)$$

where

$$D_n = \cos\left(2\pi \frac{\Phi}{\Phi_0}\right) - \cos(k_n L). \quad (11)$$

We can write the solution of Eq.(10) in the form of a continued fraction

$$x_n = \frac{\sin(k_n L)}{k_n D_n - \frac{\sin(k_{n\pm 1} L)}{k_{n\pm 1} D_{n\pm 1} - \frac{\sin(k_{n\pm 2} L)}{k_{n\pm 2} D_{n\pm 2} - \dots}}. \quad (12)$$

Here $h_{n\pm 1} = \sin(k_n L) \sin(k_{n\pm 1} L)$, Using the quantities x_n and Eq.(9) we can express any a_n through the a_0

$$a_n = a_0 v^{|n|} \frac{B_n \sin(k_0 L)}{B_0 \sin(k_n L)} \prod_{j=\pm 1}^n x_j, \quad n \neq 0. \quad (13)$$

The corresponding relation between b_n and b_0 can be easily obtained from the above equation and Eq.(8).

Now we can write down the equations containing only a_0 and b_0 . Using Eqs.(6) for $n = 0$ and expressing $a_{\pm 1}$ and $b_{\pm 1}$ in terms of a_0 and b_0 , respectively, we get

$$\begin{aligned}a_0 A_0 + b_0 B_0 &= 0, \\ a_0 A_0 - b_0 B_0 &= \\ -i \frac{2v^2}{k_0} (x_{-1} + x_{+1})(a_0 + b_0).\end{aligned}\quad (14)$$

This system of equations has a nontrivial solution if its determinant equals to zero

$$\begin{aligned}k_0 \left[\cos\left(2\pi \frac{\Phi}{\Phi_0}\right) - \cos(k_0 L) \right] \\ - v^2 (x_{-1} + x_{+1}) \sin(k_0 L) &= 0.\end{aligned}\quad (15)$$

The solutions $k_0^{(l)}$ ($l = 0, \pm 1, \pm 2, \dots$) of this (dispersion) equation give us a set of allowed Floquet eigenenergies $E^{(l)} = (\hbar k_0^{(l)})^2 / (2m)$ and corresponding side bands $E_n^{(l)} = E^{(l)} + n\hbar\omega$.

Note, that in the absence of an oscillating barrier ($v = 0$) we obtain the well known spectrum of a perfect ring with a magnetic flux

$$E^{(l)}(\Phi) = \frac{\hbar^2}{2mL^2} \left(l + \frac{\Phi}{\Phi_0} \right)^2. \quad (16)$$

For a weak potential ($v \rightarrow 0$) the Floquet energies are close to those given by Eq.(16).

The main component (corresponding to the energy $E^{(l)}$) of the Floquet wave function has a large amplitude: $a_0^{(l)}$ and/or $b_0^{(l)} \sim 1$. This is due to constructive interference in the ring. In the general case, the amplitudes of side bands (corresponding to energies $E_n^{(l)}$, $n \neq 0$) are small due to destructive interference. They are $a_n^{(l)}, b_n^{(l)} \sim v^{|n|}$ for a weak potential and $a_n^{(l)}, b_n^{(l)} \sim v^{-|n|}$ for a strong potential. Mathematically the effect of interference in a ring is described by Eq.(15) (for the main component) and by the denominator in Eq.(10) (for the side bands). However there is a special (resonant) case when the amplitude of a particular side band with an energy $E_{n_r}^{(l)}$ is comparable with the amplitudes of the main component (corresponding to the energy $E^{(l)}$). This is the case when the energy $E_{n_r}^{(l)}$ of the side band is close to another Floquet eigenenergy $E^{(l')}$. Note that at the same time the corresponding side band $E_{-n_r}^{(l)}$ is close to $E^{(l)}$.

To determine the Floquet wave function $\Psi_{E^{(l)}}$ we need to know $a_0^{(l)}$ (then the coefficients $b_0^{(l)}$, $a_n^{(l)}$ and $b_n^{(l)}$ can be found from Eq.(8) and Eq.(13)). We find this coefficient from the normalization condition Eq.(5). Substituting Eqs.(2), (3), (8), and Eq.(13) into Eq.(5) we obtain

$$|a_0^{(l)}|^2 = \frac{1}{L} \frac{\sin^2(k_0^{(l)}L/2 - \pi\Phi/\Phi_0)}{Z^{(l)} \sin^2(k_0^{(l)}L)}, \quad (17)$$

where

$$Z^{(l)} = \frac{\xi_0^{(l)}}{\sin^2(k_0^{(l)}L)} + \sum_{n \neq 0} v^{2|n|} \frac{\xi_n^{(l)}}{\sin^2(k_n^{(l)}L)} \prod_{j=\pm 1}^n |x_j^{(l)}|^2, \quad (18)$$

and

$$\xi_n^{(l)} = 1 - \cos(k_n^{(l)}L) \cos\left(2\pi\frac{\Phi}{\Phi_0}\right) + \frac{\sin(k_n^{(l)}L)}{k_n^{(l)}L} \left[\cos\left(2\pi\frac{\Phi}{\Phi_0}\right) - \cos(k_n^{(l)}L) \right]. \quad (19)$$

Next we consider the current carried by the Floquet state $\Psi_{E^{(l)}}$. We will concentrate on the time averaged (dc) current I_{dc} . To this end we integrate the quantum mechanical current

$$I[\Psi] = i \frac{e\hbar}{2m} \left(\Psi \frac{\partial \Psi^*}{\partial x} - \Psi^* \frac{\partial \Psi}{\partial x} \right) - \frac{e^2}{m} A \Psi \Psi^*, \quad (20)$$

(where $A = \Phi/L$ is a vector potential) over the time period $T = 2\pi/\omega$

$$I_{dc}^{(l)} = \frac{1}{T} \int_0^T dt I[\Psi_{E^{(l)}}(x, t)], \quad (21)$$

and obtain

$$I_{dc}^{(l)} = \frac{e\hbar}{m} \sum_{E_n^{(l)} > 0} k_n^{(l)} \left(|a_n^{(l)}|^2 - |b_n^{(l)}|^2 \right). \quad (22)$$

Note that the evanescent modes do not contribute to the current, therefore we sum over the propagating modes only ($E_n^{(l)} > 0$).

Now we can analyze the effect of an oscillating potential on the electron wave function and the corresponding quantum mechanical current. From the discussion given above, it follows that the oscillating potential has in general a weak effect on the wave function which is mainly determined by interference due to the ring geometry. But as we already mentioned this is not the case at resonance when the difference between two Floquet eigenenergies is an integer number of the energy quantum $\hbar\omega$.

The resonant frequencies are determined by the level spacing. In the perfect ring investigated here the spectrum has special features. Especially at zero flux (or multiples of $\Phi_0/2$) we have levels crossing each other. Consequently for small flux there are two energy scales which determine the resonant frequencies. The first scale is the (large) level spacing $\Delta^{(l)} = E^{(l+1)} - E^{(l)}$ that is a common feature of all finite-size systems. For a small enough sample $L \rightarrow 0$ the corresponding resonant frequency is large $\omega_\Delta = \Delta^{(l)}/\hbar \sim L^{-2} \rightarrow \infty$. The second scale is determined by the magnetic flux dependent separation $\delta^{(l)}(\Phi) = E^{(l)}(\Phi) - E^{(-l)}(\Phi)$ between levels which are degenerate (in pairs) at zero flux (or at multiples of $\Phi_0/2$) given by $E^{(l)}(0) = E^{(-l)}(0)$ (see Eq.(16)). This energy scale is a specific feature of ballistic, perfect, ring-like structures. The n^{th} resonance $\delta^{(l)}(\Phi) = n\hbar\omega$ occurs (in the case of a weak potential) at

$$\omega = \frac{2\Delta^{(l)} \Phi}{n\hbar \Phi_0}. \quad (23)$$

Note that at this condition the n^{th} side band $E_n^{(-l)}$ is in resonance with $E^{(l)}$ and vice versa: the $-n^{\text{th}}$ side band $E_{-n}^{(l)}$ is in resonance with $E^{(-l)}$.

From Eq. (23) we can see that at small magnetic flux $\Phi \rightarrow 0$ the resonant frequency is small even for a small ring. Thus we conclude that even a slowly (adiabatically)

$$\omega \ll \Delta/\hbar, \quad (24)$$

oscillating potential can essentially influence electronic properties of a ring at small $\Phi \rightarrow 0$ (or at $\Phi \rightarrow \Phi_0/2$) magnetic fluxes. This regime is of interest in the present

paper. In what follows we describe numerical results concerning the Floquet states in a ring with a small magnetic flux.

Let us consider a ring at zero magnetic flux with a delta-function potential oscillating with a fixed frequency ω . In this case the Floquet eigenenergies $E^{(l)} = (\hbar k_0^{(l)})^2 / (2m)$ (where $k_0^{(l)}$ is a solution of Eq.(15)) are the same as for a pure ring Eq.(16). This is because the time averaged potential is zero.

Further, we choose some pair of degenerate (at $\Phi = 0$) Floquet energies $E^{(l)}$ and $E^{(-l)}$. Because of interference the main component ψ_0 of the Floquet wave function has a considerable amplitude: $b_0^{(l)} \sim 1/L$, $a_0^{(-l)} \sim 1/L$ (all other coefficients are close to zero). Now we increase the magnetic flux which causes the levels $E^{(l)}(\Phi)$ and $E^{(-l)}(\Phi)$ first to follow the energies of a pure ring Eq.(16). However close to the first resonance $\Phi \sim \Phi_0 \hbar \omega / (2\Delta^{(l)})$ they "interact" with the corresponding side bands ($E_{+1}^{(-l)}$ and $E_{-1}^{(l)}$, respectively) and show an avoided crossing behavior.

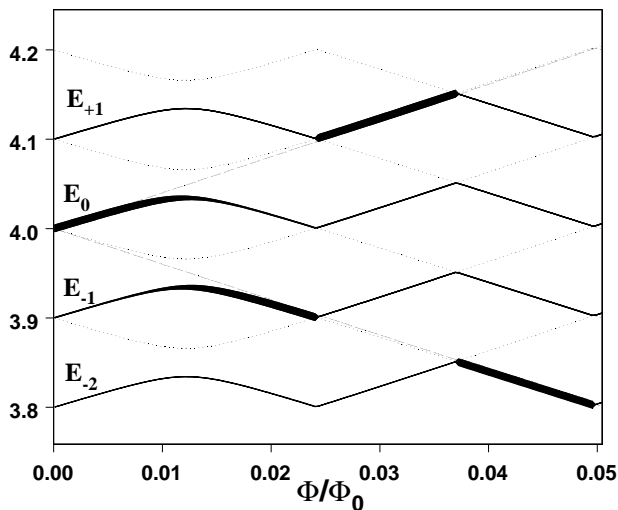


FIG. 1. The dependence of the Floquet eigenenergy $E_0 = E^{(l)}$ and some side bands $E_n = E^{(l)} + n\hbar\omega$ (solid lines; $n = -2, -1, 0, +1$) on the magnetic flux Φ . The width of the solid lines is proportional to the probability of occupation of the corresponding side band. The energy and magnetic flux are given in units of $\epsilon_0 = 4\pi^2\hbar^2/(2mL^2)$ and $\Phi_0 = h/e$, respectively. The parameters are: $l = 2$; $\hbar\omega = 0.1\epsilon_0$; $v = 0.05$. In addition the Floquet eigenenergy $E^{(-l)}$ with side bands ($n = -1; 0; +1; +2$) are also depicted by thin dotted lines. For comparison the eigenenergies $\epsilon_0 (l \pm \Phi/\Phi_0)^2$ of a pure ring are depicted by thin dashed lines.

Close to resonance, because of constructive interference on the ring, the amplitude of the wave function corresponding to the first side band considerably increases²²: $a_{-1}^{(l)} \sim 1/L$ and $b_{+1}^{(-l)} \sim 1/L$. The same occurs at the higher resonances. Thus we can say that at the resonance the Floquet state corresponds to an electron equally dis-

tributed between two states with energies shifted by $n\hbar\omega$ (for the n^{th} resonance). This is evident from Fig.1 where we depict the dependence of the Floquet energy $E^{(l)}$ and some side bands on the magnetic flux. The width of the solid lines is proportional to the square of the absolute value of the amplitude (the probability of occupation) of the side band of the Floquet state.

We see that with increasing magnetic flux the particle belonging to some Floquet eigenstate undergoes a transition between states with an opposite direction of movement. For instance, let us assume that at zero magnetic flux $\Phi = 0$ the particle is in state $E_0^{(l)}$ with $a_0^{(l)} = 0$ and $b_0^{(l)} = 1/L$ and thus it carries a diamagnetic current $I_{dc}^{(l)} < 0$ (see Eq.(22)). Then after the first resonance (see Fig.1) it passes into the state $E_{-1}^{(l)}$ with $a_{-1}^{(l)} = 1/L$ and $b_{-1}^{(l)} = 0$. In this case the particle carries a paramagnetic current $I_{dc}^{(l)} > 0$. Correspondingly, after the second resonance ($\Phi \sim \Phi_0 \hbar \omega / \Delta^{(l)}$) the particle undergoes a transition into the state $E_{+1}^{(l)}$ with $a_{+1}^{(l)} = 0$ and $b_{+1}^{(l)} = 1/L$ and it again carries a diamagnetic current, and so on. Such a behavior has a strong effect on the persistent current carried by the (spinless) electrons on the ring.

Surprisingly, the pair of electrons occupying two Floquet states $E^{(l)}$ and $E^{(-l)}$ carry exactly the same current as in the case of a pure ring:

$$I_{dc}^{(l)} + I_{dc}^{(-l)} = -2I_0 \frac{\Phi}{\Phi_0}, \quad (25)$$

where $I_0 = eh/(mL^2)$. Therefore the oscillating delta-function potential has no effect on the persistent current in a ring with an odd number N_e of (spinless) electrons

$$I_{odd} = -N_e I_0 \frac{\Phi}{\Phi_0}, \quad |\Phi| < \Phi_0/2. \quad (26)$$

However this is not the case for a ring with an even number of electrons. In this case the current I_{even} is mainly determined by the "unpaired" electron in the highest occupied Floquet state. As we discussed above the current carried by this electron oscillates with a large amplitude. In the low frequency limit (see Eq.(24)) of interest here the period $\delta\Phi \sim \Phi_0 \hbar \omega / (2\Delta^{(l)})$ of these oscillations is much smaller than the magnetic flux quantum Φ_0 .

In Fig.2 we depict the dependence of the persistent current on the magnetic flux in a ring with four (I_{even}) and five (I_{odd}) electrons.

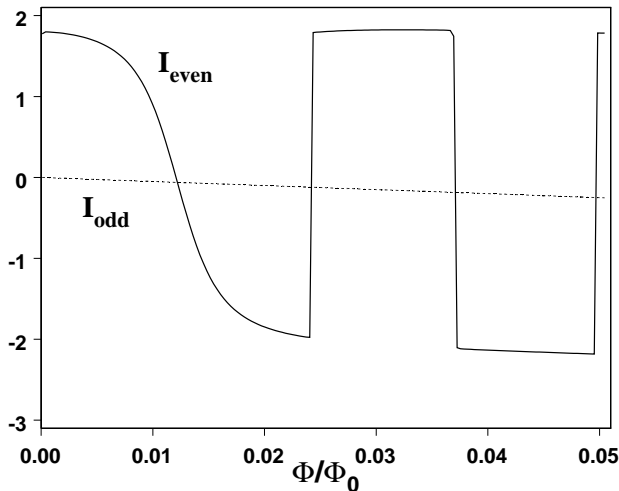


FIG. 2. Persistent current in a ring with an even I_{even} ($N_e = 4$; solid line) and an odd I_{odd} ($N_e = 5$; dashed line) number of electrons. The current is given in units of $I_0 = eh/(mL^2)$. The parameters are the same as in Fig.1.

We would like to emphasize that the behavior of the persistent current in a ring with an even number of electrons is due to an interplay between the interference in a ring and the excitation of side bands by an oscillating potential. The persistent current reflects the behavior of a single Floquet state. The interaction with an oscillating potential can not lead to transitions between the different Floquet states. As a consequence the particle stays in the same Floquet state when the magnetic flux changes. Because of interference the particle losses or gains some energy quanta $\hbar\omega$ which brings it into the appropriate sub state of the Floquet state when the magnetic flux goes through the resonant value.

However the interaction with an environment can lead to transitions between different Floquet states. In this case the particle will relax to the lowest unoccupied Floquet state corresponding to a given value of a magnetic flux and the peculiarities of I_{even} will be diminished. This effect will be considered elsewhere.

We remark on an essential difference between the oscillations of the persistent current investigated here and the oscillations of a persistent current with a period of Φ_0 . In the ballistic case the energy is quadratic in the magnetic flux Eq.(16) and the properties of a ring (in particular, the persistent current) become periodic in Φ (with a period of Φ_0) only because of the relaxation to the state with a minimum energy (for a more detailed discussion see²³). In contrast, the oscillations of interest here (see Fig.2) occur if only the system stays at the same Floquet state (when the magnetic flux changes).

In the present paper we have considered a perfect ring. But the effect under consideration is quite general because it is due to a competition between the quantum mechanical interference and the excitation of side bands by an oscillating scatterer. In particular, in the presence

of disorder there is no level degeneracy¹⁶. However the oscillating scatterer can still generate transitions between the different components of the Floquet state (and thus can affect the persistent current) if only an appropriate resonant condition is fulfilled: $n\hbar\omega = E^{(l)}(\Phi) - E^{(l+1)}(\Phi)$ (here $E^{(l)}(\Phi)$ are energy levels in a ring with disorder). The only difference from the perfect ring case is that (in the low frequency limit) the transitions are due to many photon processes: $n \gg 1$.

In conclusion, within the framework of the Floquet states approach we have considered the effect of an oscillating delta-function potential on the persistent current in a ring of noninteracting spinless electrons threaded by a magnetic flux. We have found an unusual strong parity effect in a weak magnetic flux. The current in a ring with an odd number of electrons is diamagnetic and exactly the same as in a pure ring. In contrast the current in a ring with an even number of electrons oscillates in sign with a large amplitude and with a small (compared with Φ_0) period.

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²² When we consider the energy levels as a function of a magnetic flux it is convenient to use the classification of levels at zero magnetic flux.

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