Hidden quantum pump effects in quantum coherent rings

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Time periodic perturbations of an electron system on a ring are examined. For small frequencies periodic small amplitude perturbations give rise to side band currents which in leading order are inversely proportional to the frequency. These side band currents compensate the current of the central band such that to leading order no net pumped current is generated. In the non-adiabatic limit, larger pump frequencies can lead to resonant excitations: as a consequence a net pumped current arises. We illustrate our results for a one channel ring with a quantum dot whose barriers are modulated parametrically.

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I. INTRODUCTION

Recently Switkes et al.[1] demonstrated that a phase-coherent mesoscopic sample subjected to a cyclic two parameter perturbation can produce a directed current. Of interest is a quantum pump effect which arises solely due to quantum-mechanical interference and dynamical breaking of time-reversal invariance. This pump effect can be elegantly expressed with the help of scattering matrices[2,3] in close analogy to ac-transport in mesoscopic structures[4,5]. Research in this field is currently very active. We refer the reader only to a few recent related works, on charge quantization[6], the role of dephasing[7,8], heat generation by pumps[9,10,11] and noise[9,10,12] and the transition from adiabatic to non-adiabatic transport[13,14]. Additional related problems addressed are for example, adiabatic pumping in hybrid super-conducting normal structures[15], Cooper pair pumps[16], spin-pumping[17], the magnetic field symmetry[18]. For an extensive list of references to earlier work we refer the reader to Refs. 5 and 10. Since a pump current results even in the limit of a slow variation of the pump parameters and in the limit of small amplitude variation of these parameters, the system under consideration is still close to its equilibrium state. Parametric pumping provides therefore an approach to examine near-equilibrium properties of the system which can not be obtained by conductance measurements.

Of most interest have been open systems, like the one investigated in Ref.1, for which the particle spectrum is continuous. As a consequence even at small frequencies $\omega \to 0$ pumped electron current is in a strictly quantum-mechanical sense *non-adiabatic*. This is because an oscillating scatterer induces transitions between electron states separated by one or several modulation quanta $\hbar\omega$ and in a system with continuos spectrum this implies that the system is driven out of equilibrium at arbitrarily small frequencies.

In this work we consider an electron system on a ring. At sufficiently low temperatures, when the phase coherence length is much larger than the diameter of the ring, such a system exhibits a spectrum which is essentially discrete. It is well known that in such a system purely static breaking of the left-right (L-R) symmetry by a magnetic field (by an Aharonov-Bohm flux) can generate at *equilibrium* directed currents. These are the well known "persistent currents"[19,20] which were observed experimentally[21,22,23]. Here we are interested in the generation of directed currents in such a ring due to pumping in the absence of a symmetry breaking magnetic field. Instead of the magnetic field the left-right symmetry

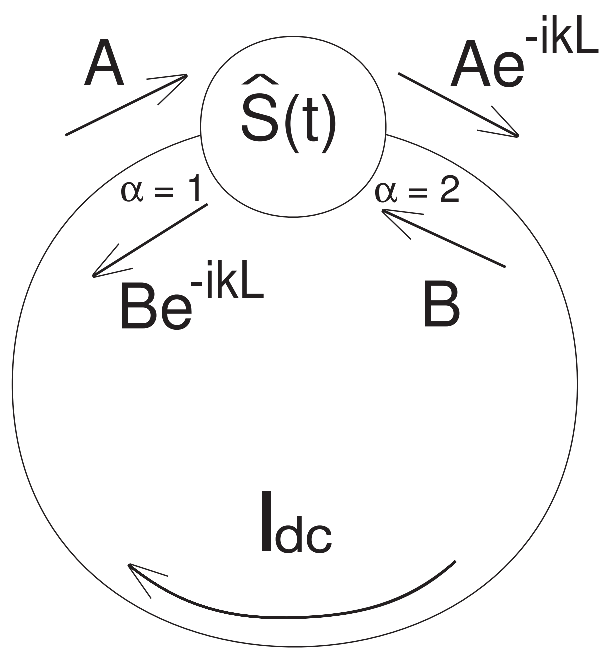

FIG. 1: A quantum dot with scattering matrix $\hat{S}$ is embedded in a one-dimensional ring. The Greek letter α numbers the scattering channels. $\Psi(x) = Ae^{ik(x-L)} + Be^{-ikx}$ is an electron wave function. $x = 0$ and $x = L$ correspond to the right and left sides of a dot, respectively. If the scattering matrix depends on time $\hat{S} = \hat{S}(t)$ then a directed circulating current I_{dc} can arise.

is dynamically broken by the oscillating scatterer. It is the purpose of this work to investigate in more detail the physical processes underlying quantum pumping of electrons along a ring. We use the Floquet function representation for an electron wave function in the periodically driven systems. We show that if the potentials oscillate out of phase (the time reversal symmetry is *dynamically* broken) then each component of the Floquet state carries a current. This is the reason why a net circulating current (a pumped current) arises even if the potentials oscillate with a small amplitude. In the present paper we concentrate on this small amplitude limit.

Note that employing the Floquet approach allows us to consider the pump effect in closed systems and in open systems[13] on the same footing. The scattering approach has been useful in the discussion of parametric pumping in open systems and we demonstrate here its applicability to pumping in closed systems. Other approaches, more closely related to a linear response approach are also possible[24,25]. The Floquet approach used here, allows us to show that a pumped current exists even in the absence of near degeneracies, at pumping amplitudes which are so small that they do not lead to level crossings.

For small frequencies and small amplitudes, we find that a parametric oscillation of the scatterer generates a Floquet state with a current both in the main branch of the wave function as well as in its side bands. Interestingly, these currents are in leading order *inversely* proportional to frequency. Moreover, to leading order they compensate one another. Thus at small frequencies and small amplitudes pumping generates currents at different energies within a Floquet state without (to leading order) generating a net total pumping current. We term this phenomena a "hidden pump effect". We present an analytical (exact) discussion of this effect. We also support the analytical discussion with a numerical calculation of a specific model and present results for the nonadiabatic case.

II. A GENERAL FLOQUET SCATTERING APPROACH

We consider a one-channel ring of length L with embedded scatterer (a quantum dot) of a small size $w \ll L$ as shown in Fig.1. We suppose that the scattering matrix of the quantum dot oscillates in time with frequency ω. Then according to the Floquet theorem we can write the single-electron wave functions $\Psi(x,t)$ as follows

$$\Psi_E(x,t) = e^{-iEt/\hbar} \sum_{n=-\infty}^{\infty} \psi_n(x) e^{-in\omega t}. \qquad (1)$$

Here E is the Floquet energy. Each Floquet state can be occupied by only one electron (because of the Pauli principle) and thus the wave function Ψ_E must be normalized

$$\frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} dt \int_0^{L_r} dx |\Psi_E|^2 \equiv \sum_n \int_0^{L_r} dx |\psi_n|^2 = 1. \qquad (2)$$

Here $\mathcal{T} = 2\pi/\omega$. For the ring problem under consideration we choose functions $\psi_n(x)$ in the following form

$$\psi_n(x) = A_n e^{ik_n(x-L)} + B_n e^{-ik_n x}. \qquad (3)$$

Here $k_n = \sqrt{2m_e E_n/\hbar^2}$ with $Re[k_n] \geq 0$ and $Im[k_n] \geq 0$. Furthermore, $E_n = E + n\hbar\omega$ is the side band energy.

The coefficients A_n and B_n with different index n are coupled through the cyclic boundary conditions at the oscillating scatterer. We express them in terms of the Floquet scattering matrix $\hat{S}_F$ relating the incoming waves A_m, B_m to outgoing ones $A_n e^{-ik_n L}, B_n e^{-ik_n L}$ (see Fig.1). The matrix element $S_{F,\alpha\beta}(E_n, E)$ defines the quantum mechanical amplitude $\mathcal{A}_{\alpha\beta}(E_n, E)$ for the particle coming from the channel β with energy E to be scattered into the channel α after the emission ($n < 0$) or the absorption ($n > 0$) of n energy quanta $\hbar\omega$:

$$\mathcal{A}_{\alpha\beta}(E_n, E) = \sqrt{\frac{k}{k_n}} S_{F,\alpha\beta}(E_n, E). \qquad (4)$$

Numbering the scattering channels as it is shown in Fig.1 we find that the boundary conditions imply:

$$\begin{aligned} A_n e^{-ik_n L} &= \sum_{m=-\infty}^{\infty} \sqrt{\tfrac{k_m}{k_n}} \\ &\times \left[A_m S_{F,21}(E_n, E_m) + B_m S_{F,22}(E_n, E_m)\right], \\ B_n e^{-ik_n L} &= \sum_{m=-\infty}^{\infty} \sqrt{\tfrac{k_m}{k_n}} \\ &\times \left[A_m S_{F,11}(E_n, E_m) + B_m S_{F,12}(E_n, E_m)\right]. \end{aligned} \qquad (5)$$

Thus we obtain an infinite system of uniform linear equation for the coefficients A_n and B_n. To have a nontrivial solution the corresponding (infinite range) determinant must be equal to zero. This (dispersion) equation determines the allowed values of the Floquet energy $E^{(l)}$ (where l is an integer) and the corresponding set of coefficients $A_n^{(l)}$ and $B_n^{(l)}$ of the Floquet wave function Eqs.(1) and (3).

In practice only a finite number of sidebands have to be taken into account. For instance, in the case of weak pumping (when the corresponding potentials oscillate with small amplitudes) only the first side bands are essential. In this case we can put $n = 0, \pm 1$ in Eq.(1) and Eq.(5) reduces to a system of only six equations.

Next we consider the current carried by the particular Floquet state $\Psi_{E^{(l)}}$. We will concentrate on the time averaged (dc) current I_{dc}. To this end we integrate the quantum mechanical current (in what follows the star denotes complex conjugation)

$$I[\Psi] = -\frac{e\hbar}{m_e} \mathrm{Im}\left[\Psi \frac{\partial \Psi^*}{\partial x}\right], \qquad (6)$$

over the time period $\mathcal{T} = 2\pi/\omega$

$$I_{dc}^{(l)} = \frac{1}{\mathcal{T}} \int_0^{\mathcal{T}} dt I[\Psi_{E^{(l)}}], \qquad (7)$$

and obtain

$$\begin{aligned} I_{dc}^{(l)} &= \sum_{E_n^{(l)} > 0} I_n^{(l)}, \\ I_n^{(l)} &= \tfrac{e\hbar}{m_e} k_n^{(l)} \left(|A_n^{(l)}|^2 - |B_n^{(l)}|^2\right). \end{aligned} \qquad (8)$$

Here we have restricted the summation over the propagating modes ($E_n^{(l)} > 0$) only, since the bounded states ($E_n^{(l)} < 0$) do not contribute to the current.

To solve Eq.(5) and calculate the current Eq.(8) one needs to know the Floquet scattering matrix $\hat{S}_F$. Therefore in what follows we consider some particular cases. Furthermore, we concentrate on the current carried by a single electron state. To find the full circulating current we have to sum Eq.(8) over all the occupied levels in the ring.

III. CURRENTS GENERATED BY AN OSCILLATING ENERGY INDEPENDENT SCATTERER

The Floquet scattering matrix depends in general on both the energy of the incident carriers and the energy of the exiting carriers [see Eq. (4)]. However, for frequencies which are small compared to the typical energy over which the scattering matrix varies significantly, the Floquet scattering matrix can be expressed in terms of one energy argument only, i.e. in terms of the on-shell scattering matrix of the stationary problem[13]. To be definite we assume that the stationary scattering matrix depends on two parameters $\hat{S} = \hat{S}(p_1, p_2)$ which oscillate with frequency ω and with phase lag φ:

$$\begin{aligned} p_1 &= p_{01} + 2p_{11}\cos(\omega t + \varphi/2); \\ p_2 &= p_{02} + 2p_{12}\cos(\omega t - \varphi/2). \end{aligned} \tag{9}$$

Then the corresponding Floquet scattering matrix $\hat{S}_F$ can be expressed in terms of the Fourier coefficients of $\hat{S}$ as follows[13]

$$\begin{aligned} \hat{S}_F(E_n, E) &= \hat{S}_{n\omega}, \\ \hat{S}_{n\omega} &= \tfrac{\omega}{2\pi}\int\limits_0^{2\pi/\omega} dt e^{in\omega t}\hat{S}(p_1(t), p_2(t)). \end{aligned} \tag{10}$$

Note that we restrict our consideration to scatterers which, in the absence of the time-dependent perturbations, are time reversal invariant: $S_{12} = S_{21}$. In particular, this implies that there is no stationary circulating current.

In what follows we concentrate on the small amplitude

$$p_{1i} \ll p_{0i}, \quad i = 1, 2. \tag{11}$$

and the small frequency case:

$$\hbar\omega \ll \Delta, \tag{12}$$

where Δ is the level spacing of the ring (near the level $E^{(l)}$ under consideration). In the small amplitude limit Eq.(11) only the side bands with $n = \pm 1$ are excited. Their intensities are proportional to the following Fourier coefficients

$$\hat{S}_{\pm\omega} = p_{11}e^{\mp i\frac{\varphi}{2}}\frac{\partial \hat{S}}{\partial p_{01}} + p_{12}e^{\pm i\frac{\varphi}{2}}\frac{\partial \hat{S}}{\partial p_{02}}, \tag{13}$$

and, hence, they are small. Therefore to find the Floquet eigen energies $E^{(l)}$ in the lowest order in the oscillating amplitudes p_{1i} it is sufficient to consider only the two equations of Eq.(5) which correspond to the coefficients A_0 and B_0. The other amplitudes can be taken to be zero. The resulting dispersion equation coincides with that of the time-independent problem, i.e., the dispersion equation for a ring with a static quantum dot having the scattering matrix $\hat{S} = \hat{S}(p_{01}, p_{02})$. In terms of the components of the scattering matrix the dispersion equation reads

$$D[k] \equiv \left(e^{-ikL} - S_{12}\right)^2 - S_{11}S_{22} = 0. \tag{14}$$

This equation defines the allowed set of eigenvectors $k^{(l)}$ and the corresponding Floquet eigen energies $E^{(l)} = (\hbar k^{(l)})^2/(2m_e)$.

Apparently only the main component ($n = 0$) of the Floquet state Ψ_E Eq.(1) is subject to constructive interference on the ring. In contrast, the side bands with energies $E^{(l)}_{\pm 1} = E^{(l)} \pm \hbar\omega$ are subject to destructive interference. That is another reason why their intensities are small. Note that this is correct if the frequency ω is small enough Eq.(12) and the side bands are not close to other eigen energies of the ring.

We choose some particular energy level $E^{(l)}$ and consider the currents $I^{(l)}_n$ carried by the different components ($n = -1, 0, +1$) of the Floquet state. In what follows we drop the upper index $^{(l)}$. To calculate these currents, induced by the oscillating scatterer, we have to take into account the processes of absorption and emission of energy quanta $\hbar\omega$. In the weak amplitude limit these processes being unimportant for the eigen energy problem nevertheless lead to a noticeable change in the wave function: the resulting different components of the Floquet state give rise to circulating currents.

First we calculate the current I_{+1} [see Eq.(8)] associated with the upper side band of the Floquet state. To find the current in the lowest order in oscillating amplitudes p_{1i} we use Eq.(5) and express the coefficients A_{+1} and B_{+1} in terms of A_0 and B_0:

$$\begin{aligned} A_{+1}e^{ik_{+1}L}\left(e^{-ik_{+1}L} - S_{12}\right) - B_{+1}S_{22} \\ = A_0e^{ikL}S_{12,+\omega} + B_0S_{22,+\omega}; \\ -A_{+1}e^{ik_{+1}L}S_{11} + B_{+1}\left(e^{-ik_{+1}L} - S_{12}\right) \\ = A_0e^{ikL}S_{11,+\omega} + B_0S_{12,+\omega}. \end{aligned} \tag{15}$$

We neglect other side bands and use the following equation (which holds in the zero order in p_{1i})

$$\begin{aligned} B_0 &\approx A_0e^{ikL}\tfrac{e^{-ikL} - S_{12}}{S_{22}} \\ &= A_0e^{ikL}\tfrac{S_{11}}{e^{-ikL} - S_{12}}. \end{aligned} \tag{16}$$

In addition to determine the coefficient A_0 we use the normalization condition

$$|A_0|^2 + |B_0|^2 \approx \frac{1}{L}. \tag{17}$$

Note that for any solution of Eq.(14) ($k = k^{(l)}$) the equation (16) gives $|A_0|^2 = |B_0|^2$.

Substituting Eq.(16) into Eq.(15) we get

$$\begin{aligned} A_{+1}e^{ik_{+1}L} &= A_0e^{ikL}\tfrac{e^{-ikL} - S_{12}}{D[k_{+1}]} \\ &\times \left\{\Pi[S_{22}]\left(e^{-ik_{+1}L} - S_{12}\right)\right. \\ &\left. + \Pi[S_{11}]\left(e^{-ikL} - S_{12}\right)\right\}, \\ B_{+1} &= B_0\tfrac{e^{-ikL} - S_{12}}{D[k_{+1}]} \\ &\times \left\{\Pi[S_{11}]\left(e^{-ik_{+1}L} - S_{12}\right)\right. \\ &\left. + \Pi[S_{22}]\left(e^{-ikL} - S_{12}\right)\right\}, \end{aligned} \tag{18}$$

where

$$\Pi(S_{ii}) = \frac{S_{ii,+\omega}}{S_{ii}} + \frac{S_{12,+\omega}}{e^{-ikL} - S_{12}}. \tag{19}$$

The function $D[k_{+1}]$ entering the denominators in Eq.(18) is defined in Eq.(14). At small frequency ω [see Eq.(12)] we can expand $k_{+1} \approx k + \omega/v$ (here $v = \hbar k/m_e$ is an electron velocity) and obtain (since $D[k] = 0$)

$$D[k_{+1}] \approx -2i\frac{\omega}{\omega_0}e^{-ikL}\left(e^{-ikL} - S_{12}\right). \tag{20}$$

Here $\omega_0 = v/L$. This equation is of the lowest order in the ratio ω/ω_0.

The function $D[k_{+1}]$ describes the effect of a destructive interference on the side band ($n = +1$). On the other hand the main component ($n = 0$) is subject to constructive interference. Hence the smaller the frequency ω (and, thus, the closer the energy $E_{+1} = E + \hbar\omega$ of the side band to the eigen energy E) the weaker is the destructive interference. As a result the amplitude of a wave function $\psi_{+1} \sim \frac{1}{D[k_{+1}]}$ and, correspondingly, the current I_{+1} *increases* with decreasing ω.

Such a dependence $A_{+1}, B_{+1}, I_{+1} \sim \omega^{-1}$ holds while the wave function ψ_{+1} is still small. At extremely small ω the effect of the oscillating parameters has to be taken into account exactly (not perturbatively). Numerical calculations (see[13]) show that in this case higher side bands are excited. The limiting case of such small frequencies will be considered elsewhere.

Using Eq.(18) we calculate the current $I_{+1} \sim Re[(A_{+1} + B_{+1})(A_{+1} - B_{+1})^*]$. After simple manipulations we can express the current in terms of quantities $\Pi_{(\pm)} = \Pi[S_{22}] \pm \Pi[S_{11}]$. To find $\Pi_{(+)}$ we use the dispersion equation (14) and take into account that the eigenvector k depends on the parameters: $k = k(p_{01}, p_{02})$. Differentiating Eq.(14) with respect to either p_{01} or p_{02} we obtain

$$\Pi_{(+)} = \frac{-2iLk_{+\omega}e^{-ikL}}{e^{-ikL} - S_{12}}. \tag{21}$$

Here $k_{+\omega}$ is defined in the same fashion as $\hat{S}_{+\omega}$ Eq.(13). To find $\Pi_{(-)}$ we use the identity $|S_{11}|^2 = |S_{22}|^2$ and get

$$\Pi_{(-)} = 2i\theta_{+\omega} \tag{22}$$

where $\theta_{+\omega}$ is a Fourier coefficient of the phase

$$\theta = \frac{i}{2}ln\left(\frac{S_{11}}{S_{22}}\right). \tag{23}$$

Note that the (real) phase θ characterizes the asymmetry in the reflection of particles incident from the left (S_{11}) and from the right (S_{22}). This asymmetry is due to the spatial asymmetry of a quantum dot. Finally the current I_{+1} reads as follows

$$\begin{aligned} I_{+1} &= I_\omega \sin\varphi, \\ I_\omega &= \frac{ev}{L}\frac{p_{11}p_{12}}{\hbar\omega}\left(\frac{\partial\theta}{\partial p_{01}}\frac{\partial E}{\partial p_{02}} - \frac{\partial\theta}{\partial p_{02}}\frac{\partial E}{\partial p_{01}}\right). \end{aligned} \tag{24}$$

Here $E = E(p_{01}, p_{02})$ is an eigen energy for the static problem.

From Eq.(24) it follows that if two parameters oscillate out of phase $\varphi \neq 0, \pi$ then a dc current I_{+1} arises. This is similar to the case of an open quantum cavity where an oscillating scatterer pumps a dc current between external reservoirs[2]. However the dependence on the frequency ω is strikingly different. This indicates that this is a new phenomena which is specific for closed systems.

Like the pumped current in open systems[13] the current I_{+1} under considerations is due to dynamical breaking of the time reversal symmetry ($\varphi \neq 0, \pi$) by the oscillating scatterer. Note that there is another necessary condition for the existence of dynamically generated dc currents which is general for open and closed systems: The varying parameters must affect the spatial asymmetry of the scatterer, i.e., $\partial\theta/\partial p_i \neq 0$.

In addition the general conditions mentioned above, there exists a particular condition which is specific for the ring problem under consideration. The current I_ω depends on the sensitivity of eigen energies to the varied parameters. If we have (accidentally) $\partial E/\partial p_i = 0$, then the current is zero. In particular, in the limit of an extremely small scatterer ($w \to 0$) one can classify the eigenstates $\psi_{E^{(l)}}(x)$ in a ring according to their spatial symmetry. If the scatterer is at $x = 0$ then for the (anti-) symmetric states we have $\psi(x) = (-)\psi(-x)$. The antisymmetric states are insensitive to the presence of a small quantum scatterer (because $\psi_{(anti)}(x = 0) \approx 0$) and their energies do not depend on p_i. Thus in such a case the antisymmetric states do not exhibit the pump effect discussed here.

Now we calculate the currents carried by the other components of the Floquet state. To obtain the current $I_{-\omega}$ we need to replace $\omega \to -\omega$ and $\varphi \to -\varphi$. From Eq.(24) it follows that $I_{-\omega} = I_{+\omega}$. A similar calculation shows that $I_0 = -(I_{+\omega} + I_{-\omega})$. Thus within the approximation used the full circulating current

$$I_{dc} = I_0 + I_{-1} + I_{+1} \tag{25}$$

is zero. Because it is impossible to measure the current carried by only one side band (only a full current is a measurable quantity) we call the effect under consideration a *"hidden pump effect"*.

The disappearance of the full circulating current is a consequence of the symmetry between the side bands corresponding to absorption ψ_{+1} and to emission ψ_{-1} of modulation quanta $\hbar\omega$. This symmetry can be broken if the energy of one of the side bands (either E_{+1} or E_{-1}) lies close to another eigen energy in the ring. In this case a net circulating current can arise: $I_{dc} \neq 0$. Thus if the adiabaticity condition Eq.(12) is violated then the effect under consideration can be measured. In the next section we present the results of a numerical calculation confirming such a conclusion.

It is important to note that the pumped currents discussed here are inversely proportional to the frequency, $I \sim \omega^{-1}$. Therefore even in the non-adiabatic regime the pumping effect discussed here differs strongly from the pump effect in open systems where $I \sim \omega^{1,2}$. Nevertheless in addition to the *"hidden pump effect"* there exists a true pump effect[26] (for which the circulating current is proportional to ω) and which is fully analogous to the pump effect in open systems. To obtain this current one needs to carry out the expansions in powers of the frequency to a higher order then is done here. We will present the results of such a higher order expansion in a separate work[26].

IV. CURRENTS GENERATED BY AN OSCILLATING DOUBLE BARRIER

In this section we use a simple model for the quantum dot to calculate numerically the currents generated in the ring. We

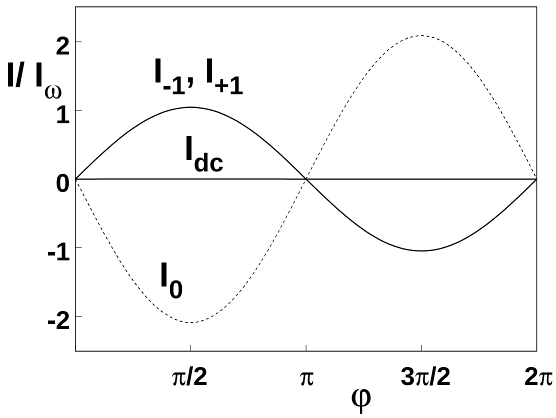

FIG. 2: Adiabatic case. The off resonance currents carried by the main component I_0 and by the first two side bands $I_{+1} = I_{-1}$ of the Floquet state are given as a function of the phase lag $\varphi = \varphi_1 - \varphi_2$. The full current $I_{dc} = \sum_n I_n$ and currents carried by the higher side bands are vanishingly small. The currents are given in units of I_ω Eq.(27). The parameters of the oscillating double barrier are: $V_{01} = V_{02} = 400$; $V_{11} = V_{12} = 0.04$. $k = 9.546$; $\omega = 0.1$; $L_r = 10\pi$; $w = \pi/20$. We use the units: $2m_e = \hbar = e = 1$.

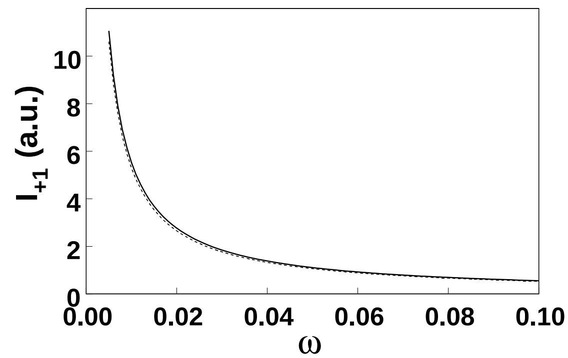

FIG. 3: Adiabatic case. The dependence of the current I_{+1} at $\varphi = \pi/2$ on the frequency ω: (i) numerical calculations (solid line); (ii) estimation according to Eq.(27) (dashed line). The parameters are the same as in Fig.2.

consider both the adiabatic case Eq.(12) and the nonadiabatic case to confirm the conclusions of the previous section.

We model a quantum dot by two delta-function potentials separated by the distance w and choose the strength of these potentials V_1 and V_2 as varying parameters.

Appendix A gives an exact solution of the model under consideration. In the numerical calculations we use the units $2m_e = \hbar = e = 1$ and put $\varphi_1 = -\varphi_2 = \varphi/2$, where φ is the phase lag. Note that at $\varphi = 0$ two potentials oscillate in phase. We concentrate on the case of opaque ($V_{1(2)} \gg k\hbar^2/m_e$) closely placed ($w \ll L_r$) barriers which correspond to a quantum dot only weakly coupled to a ring.

A. Adiabatic case

We consider small frequencies Eq.(12) and calculate the current carried by some energy level E in the ring which is far from any energy level in the dot (off resonance case). In this case the oscillating potentials have only a weak effect on the wave function amplitudes which are mainly determined by the interference due to the ring geometry[27].

In Fig.2 we depict the dependence of the currents carried by the main component I_0 and by the first two side bands I_{+1} and I_{-1} of the Floquet state on the phase lag φ between the oscillating potentials V_{11} and V_{12} (see Appendix A). The full current I_{dc} (which is vanishingly small) is shown as well.

At small amplitudes of the oscillating potentials only the lowest side bands ($n = \pm 1$) are excited. In the case under consideration the symmetry between the absorption and the emission is preserved. As a result both side bands E_{+1} and E_{-1} have the same amplitudes and carry the same currents:$I_{+1} = I_{-1}$. The current I_0 is twice as large as $I_{\pm 1}$ and opposite to $I_{\pm 1}$. This is in agreement with the previous section.

To estimate the magnitude I_ω of a current we note that far from the resonance and at small frequencies the double barrier scattering matrix can be considered as energy independent[13]. Thus we can use Eq.(24) with the scattering matrix of a double barrier potential:

$$\hat{S}(p_{01}, p_{02}) = \frac{e^{ikw}}{\Delta} \begin{pmatrix} \xi + 2\frac{p_{02}}{k}\sin(kw) & 1 \\ 1 & \xi + 2\frac{p_{01}}{k}\sin(kw) \end{pmatrix}. \tag{26}$$

Here $p_{0i} = V_{0i} m_e/\hbar^2$ (i = 1,2); $\xi = (1 - \Delta)e^{-ikw}$; $\Delta = 1 + \frac{p_{01}p_{02}}{k^2}(e^{2ikw} - 1) + i\frac{p_{01}+p_{02}}{k}$.

Far from the resonance (and for $k \ll p_0$) we get: $\partial k/\partial p_i \sim k/(2p_0^2 L)$; $\partial\theta/\partial p_{01} = -\partial\theta/\partial p_{02} \sim k/(2p_0^2)$. Substituting these estimates into Eq.(24) we obtain

$$I_\omega \approx 2eT\frac{\omega_0^2}{\omega}\left(\frac{p_1}{k}\right)^2. \tag{27}$$

Here $T = k^4/(4p_0^4) \ll 1$ is an off resonance probability for tunneling through the double barrier potential. We put $p_{01} = p_{02} = p_0$ and $p_{11} = p_{12} = p_1$.

The above equation [see also Eq.(24)] predicts that the amplitude of a current carried by the individual side band scales as ω^{-1}. This is illustrated in Fig.3. We can see that the analytical results Eq.(24) and Eq.(27) are in a good agreement with the results of numerical calculations.

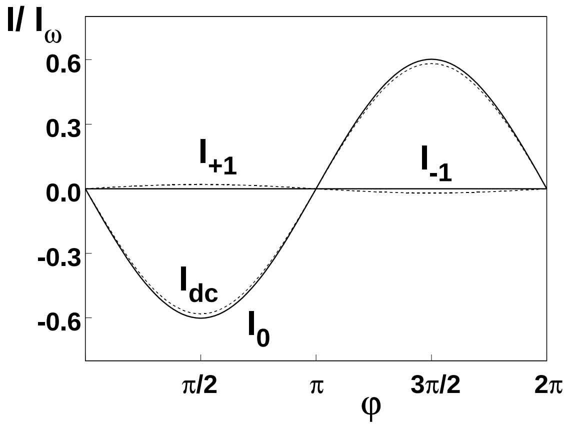

FIG. 4: Nonadiabatic case. The currents carried by the individual side bands I_{-1}, I_0, I_{+1} and the full circulating current $I_{dc} = \sum_n I_n$ are given as a function of the phase lag $\varphi = \varphi_1 - \varphi_2$ close to the resonance. The currents are given in units of I_ω Eq.(26) with T being an actual transmission probability at energy $E = 89.84$ and ω replaced by $\omega_r - \omega$, where $\omega_r = (E_r - E)/\hbar = 2.34$ is a resonance frequency. The transmission resonance through the quantum dot occurs at $E_r = 92.18$. The width of the resonance is $\Gamma = 0.13 \ll \omega$. The side band $E_{+1} \equiv E + \hbar\omega = 92.15$ is close to the transmission resonance whereas E and E_{-1} are far from the resonance. The parameters of the oscillating double barrier are: $V_{01} = V_{02} = 400$; $V_{11} = V_{12} = 0.04$. $k = 9.4785$; $\omega = 2.31$; $L_r = 2\pi$; $w = \pi/9.75$. We use the units: $2m_e = \hbar = e = 1$.

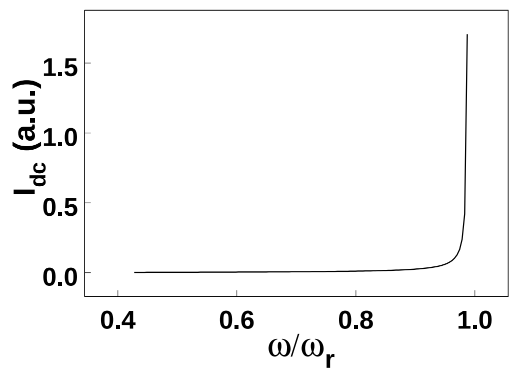

FIG. 5: Nonadiabatic case. The dependence of the circulating current I_{dc} on the frequency ω at $\varphi = -\pi/2$. The parameters are the same as in Fig.4.

B. Nonadiabatic case

In this subsection we consider the conditions under which a net circulating current arises in a ring with oscillating double barrier potential: $I_{dc} \neq 0$. As we pointed out already if the symmetry between the emission and the absorption holds the full current carried by the Floquet state is zero. However if this symmetry is destroyed then a net circulating current can arise. In particular the symmetry between the absorption and the emission is destroyed if one of the side bands (say E_{+1}) is close to the transmission resonance through the quantum dot. In this case tunneling after absorbing of a modulation quantum $\hbar\omega$ dominates over tunneling after emitting $\hbar\omega$.

In Fig.4 we depict the dependence of the net pumped current I_{dc} and currents carried by the individual side bands I_0, I_{+1}, and I_{-1} on the phase difference $\varphi = \varphi_1 - \varphi_2$. At given parameters the side band $E_{+1} = E + \hbar\omega$ is close to the transmission resonance through the quantum dot whereas the basic energy level E and the side band $E_{-1} = E - \hbar\omega$ are out of resonance: $E - E_r \gg \Gamma$, where E_r is the resonance energy and $\Gamma \approx \frac{\hbar^2}{2m_e}\frac{2k^3}{wp_0^2}$ is the width of the transmission resonance. When ω approaches the resonance frequency $\hbar\omega_r = E_r - E$ then the circulating current increases. This is illustrated in Fig.5.

From Fig.4 we see that in the nonadiabatic case under consideration the circulating current I_{dc} is carried, in fact, by the main component ψ_0 of the Floquet state. The mechanism which generates this current is as follows. An electron (mainly staying at the level with energy E) can absorb an energy $\hbar\omega$ and tunnel resonantly ($E_{+1} \approx E_r$) through the quantum dot. After tunneling an electron emits an energy $\hbar\omega$ and returns to its original energy level. Because of the phase lag φ between the oscillating potentials $V_1(t)$ and $V_2(t)$ the amplitudes for tunneling to the left and to the right are different. Thus an electron tunnels more frequently to one side and a net circulating current arises. Note that both the emission and the absorption affect the current. However if $E < E_r$ then only the processes where the emission follows the absorption (or vice versa if $E > E_r$) contribute to the current I_{dc}.

V. CONCLUSION

We have considered a quantum pump in a mesoscopic ring with embedded quantum dot. If two (or more) parameters affecting the scattering properties of a quantum dot change periodically but out of phase then a circulating dc current can be generated. We have emphasized the small frequency and small amplitude case when the oscillations do not affect the spectrum (the positions of energy levels are unchanged) but rather break dynamically the time reversal symmetry of the system. We have examined the features of parametric pumping which are specific for closed doubly connected systems. The resulting pumped current is due to a competition between exciting an electron system by an oscillating scatterer and interference due to the ring geometry.

The effect of an oscillating scatterer on an electron wave function is twofold. On the one hand, because of the oscillations the system is nonstationary and the electron is in a Floquet state. This state is characterized by the set of substates (side bands) with energies $E_n = E + n\hbar\omega$, $n = 0, \pm 1, \pm 2, \ldots$. On the other hand, if the time reversal symmetry is broken then each substate carries a current. Interference results in an unusual ω^{-1} dependence of the current amplitude carried by the individual side band.

A main feature of the pump effect under consideration is that in the adiabatic case the currents carried by the different sub states of a given Floquet state compensate each other. Therefore we term this effect "hidden". Nevertheless in the nonadiabatic case the current carried by the main component of the Floquet state dominates and a net circulating current I_{dc} appears.

We again emphasize that, in addition to the effect considered here, there exists a usual adiabatic pump effect with a circulating current proportional to ω. This effect, analogous to the pump effect in open systems, will be considered elsewhere[26].

Acknowledgments

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APPENDIX A: A RING WITH TWO OSCILLATING BARRIERS.

In this Appendix we determine the single-particle eigen energies and the coefficients of the corresponding Floquet wave function Eq.(1) for a ring of length L_r with two oscillating delta function potentials separated by a distance w.

The electron wave functions $\Psi(x,t)$ is a solution of the time-dependent Schrödinger equation

$$\begin{aligned} & i\hbar \frac{\partial \Psi(x,t)}{\partial t} = \hat{H}(x,t)\Psi(x,t), \\ & \hat{H}(x,t) = -\frac{\hbar^2}{2m_e}\frac{\partial^2}{\partial x^2} + V(x,t), \\ & V(x,t) = V_1(t)\delta(x) + V_2(t)\delta(x-w), \\ & V_i(t) = V_{0i} + 2V_{1i}\cos(\omega t + \varphi_i), i = 1,2. \end{aligned} \tag{A1}$$

According to the Floquet theorem the wave function is given by Eq.(1). In addition to Eq.(3) we define the functions $\psi_n(x)$ inside the dot as well

$$\psi_n(x) = \begin{cases} a_n e^{ik_n x} + b_n e^{-ik_n x}, 0 \le x \le w, \\ A_n e^{ik_n x} + B_n e^{-ik_n x}, w \le x \le L_r. \end{cases} \tag{A2}$$

The function $\Psi_E(x,t)$ is periodic in x with the period of L_r. Hence the boundary conditions at $x = 0$ and $x = w$ read as follows

$$\begin{aligned} & \Psi_E(L_r,t) = \Psi_E(0,t), \\ & \Psi_E(w-0,t) = \Psi_E(w+0,t), \\ & \frac{\partial \Psi_E(x,t)}{\partial x}\Big|_{x=0} - \frac{\partial \Psi_E(x,t)}{\partial x}\Big|_{x=L_r} \\ & = \frac{2m}{\hbar^2} V_1(t)\Psi_E(L_r,t), \\ & \frac{\partial \Psi_E(x,t)}{\partial x}\Big|_{x=w+0} - \frac{\partial \Psi_E(x,t)}{\partial x}\Big|_{x=w-0} \\ & = \frac{2m}{\hbar^2} V_2(t)\Psi_E(w,t). \end{aligned} \tag{A3}$$

These boundary conditions determine the discrete set of the Floquet eigen energies $E^{(l)}$ (where l is an integer) and corresponding Floquet eigenfunctions $\Psi_{E^{(l)}}$.

Note that because of the Floquet theorem the time-dependent problem with oscillating potential Eq.(A1) is reduced to a time-independent one. The cost we need to pay is a splitting of each energy level $E^{(l)}$ into a ladder $E^{(l)} + n\hbar\omega$ ($n = 0, \pm 1, \pm 2, \ldots$)[27].

To find the Floquet eigen energies $E^{(l)}$ we apply the method described in[13,27]. We substitute Eq.(1) and Eq.(A1) into the Eq.(A2) and write the result in a matrix form

$$\hat{C}_n(L_r,w)\begin{pmatrix} A_n \\ B_n \end{pmatrix} - \hat{C}_n(0,w)\begin{pmatrix} a_n \\ b_n \end{pmatrix} = 0, \tag{A4}$$

$$\begin{aligned} & \hat{U}_{0n}(L_r,w)\begin{pmatrix} A_n \\ B_n \end{pmatrix} + \hat{U}^*_{0n}(0,-w)\begin{pmatrix} a_n \\ b_n \end{pmatrix} \\ & = -2\hat{F}^{(+)}\hat{C}_{n+1}(L_r,w)\begin{pmatrix} A_{n+1} \\ B_{n+1} \end{pmatrix} \\ & - 2\hat{F}^{(-)}\hat{C}_{n-1}(L_r,w)\begin{pmatrix} A_{n-1} \\ B_{n-1} \end{pmatrix}. \end{aligned} \tag{A5}$$

Here we have introduced the matrices

$$\hat{C}_n(L_r,w) = \begin{pmatrix} e^{ik_n L_r} & e^{-ik_n L_r} \\ e^{ik_n w} & e^{-ik_n w} \end{pmatrix}, \tag{A6}$$

$$\hat{U}_{0n}(L_r,w) = \begin{pmatrix} e^{ik_n L_r}(p_{01} + ik_n) & e^{-ik_n L_r}(p_{01} - ik_n) \\ e^{ik_n w}(p_{02} - ik_n) & e^{-ik_n w}(p_{02} + ik_n) \end{pmatrix}, \tag{A7}$$

$$\hat{F}^{(\pm)} = \begin{pmatrix} p_{11}e^{\pm i\varphi_1} & 0 \\ 0 & p_{12}e^{\pm i\varphi_2} \end{pmatrix}, \tag{A8}$$

where the parameters are: $p_{ji} = V_{ji}m_e/\hbar^2$, $j = 0, 1$, $i = 1, 2$. Note that the system of equations (A3) and (A4) represents an infinite number of linear equations. To simplify it we introduce new matrixes $\hat{X}_n$ as follows

$$-2\hat{F}^{(\pm)}\hat{C}_{n\pm1}(L_r.w)\begin{pmatrix} A_{n\pm1} \\ B_{n\pm1} \end{pmatrix} = \hat{X}_{n\pm1}\begin{pmatrix} A_n \\ B_n \end{pmatrix}. \tag{A9}$$

Here and hereafter the upper (lower) sign is for $n > 0$ ($n < 0$). Substituting Eq.(A4) and Eq.(A9) into Eq.(A5) we get the following recursive equation for $\hat{X}_n$

$$\hat{X}_n = 4\hat{F}^{(\pm)}\hat{C}_n(L_r, w)\left(\hat{U}_n - \hat{X}_{n\pm1}\right)^{-1}\hat{F}^{(\mp)}\hat{C}_{n\mp1}(L_r, w), \tag{A10}$$

where

$$\hat{U}_n = \hat{U}_{0n}(L_r, w) + \hat{U}_{0n}(0, w)\hat{C}_n^{-1}(0, w)\hat{C}_n(L_r, w).$$

The advantage of Eq.(A10) is in the following. In each particular case we need to take into account only the limited number $|n| < n_{max}$ of side bands and thus we can put $\hat{X}_{\pm(n_{max}+1)} \approx 0$. After that we can easily calculate $\hat{X}_n$ at $|n| \le n_{max}$ and can express all the coefficients A_n, B_n in terms of A_0 and B_0 only. Thus we have used all the equations of the system Eqs.(A4) and (A5) except those for $n = 0$.

The remaining part is a system of uniform equations for the coefficients a_0, b_0, A_0, and B_0. To have a nontrivial solution the corresponding determinant (4×4) must be equal to zero. This condition determines the set of the allowed Floquet eigen energies $E^{(l)} = (\hbar k^{(l)})^2/(2m_e)$ and corresponding side bands $E_n^{(l)} = E^{(l)} + n\hbar\omega$. To calculate the coefficients entering the corresponding Floquet wave function $\Psi_{E^{(l)}}$ Eqs.(1),(A2) we use the normalization condition Eq.(2).

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