

Adiabatic quantum pump in the presence of external ac voltages

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We investigate a quantum pump which in addition to its dynamic pump parameters is subject to oscillating external potentials applied to the contacts of the sample. Of interest is the rectification of the ac currents flowing through the mesoscopic scatterer and their interplay with the quantum pump effect. We calculate the adiabatic dc current arising under the simultaneous action of both the quantum pump effect and classical rectification. In addition to two known terms we find a third novel contribution which arises from the interference of the ac currents generated by the external potentials and the ac currents generated by the pump. The interference contribution renormalizes both the quantum pump effect and the ac rectification effect. Analysis of this interference effect requires a calculation of the Floquet scattering matrix beyond the adiabatic approximation based on the frozen scattering matrix alone. The results permit us to find the instantaneous current. In addition to the current generated by the oscillating potentials, and the ac current due to the variation of the charge of the frozen scatterer, there is a third contribution which represents the ac currents generated by an oscillating scatterer. We argue that the resulting pump effect can be viewed as a quantum rectification of the instantaneous ac currents generated by the oscillating scatterer. These instantaneous currents are an intrinsic property of a nonstationary scattering process.

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I. INTRODUCTION

Dynamical transport in mesoscopic structures attracts presently considerable attention^{1–12}. In particular the possibility to vary several parameters at the same frequency but different phases⁷ of a mesoscopic system opens up new prospects for the investigation of quantum transport. Applying two slowly oscillating potentials at frequency ω with fixed phase lag $\Delta\varphi$ to a mesoscopic conductor connected to reservoirs having equal electrochemical potentials one can generate an adiabatic dc current

$$I_{dc} \sim \omega \sin(\Delta\varphi). \quad (1)$$

Such a current was measured experimentally⁷. However the precise origin of the measured current is still unclear. At least two mechanisms considered in the literature can contribute to the experimentally measured current. First, there exists a *quantum pump effect*^{7,13–46} which is due to quantum-mechanical interference and dynamical breaking of time-reversal invariance. Second, there also exists a *rectification of ac currents* by the oscillating scatterer^{12,47,48} if it is part of an external circuit with non-zero impedance. Closely related to this second effect is a pump in the presence of inelastic scattering: in addition to the externally driven pump parameters, inelastic scattering leads to an effective oscillating (electro-)chemical potential of the pump which acts like an additional pump parameter⁴⁹.

The aim of the present paper is to investigate both these mechanisms on the same footing. To this end we consider a phase coherent oscillating scatterer coupled to reservoirs with oscillating potentials (see Fig. 1). We will show that in general the above mentioned mechanisms do not simply add but

interfere between themselves. This leads to a renormalization of both the quantum pump effect as well as the rectification effect in the total dc current.

Theoretically quantum pumps have been investigated mostly under the (implicit) condition that the external circuit exhibits zero-impedance. The work of Brouwer⁴⁷, Polianski

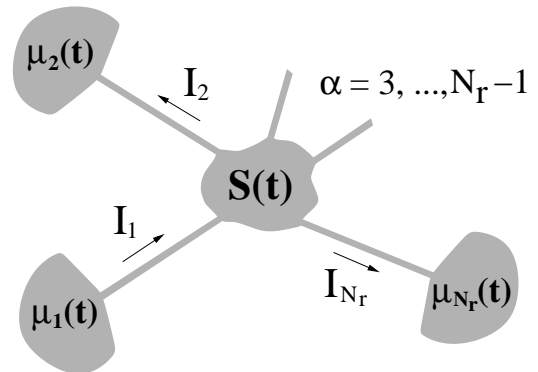


FIG. 1: A mesoscopic pump with scattering matrix $S(t)$ oscillating with frequency ω is coupled to N_r reservoirs with electrochemical potentials $\mu_\alpha(t)$ oscillating with the same frequency ω . A quantum pump effect and a classical rectification effect together result in dc currents I_α flowing through the scatterer. The full current is time-dependent and is needed to characterize pumps in a non-zero impedance external circuit.

and Brouwer⁴⁸, the work on inelastic scattering⁴⁹ mentioned already and the recent work of Martinez-Mares, Lewenkopf and Mucciolo⁵⁰ represent the few exceptions. In reality the zero-impedance condition seems never exactly fulfilled. Coupling an oscillating gate voltage to a scatterer leads, due to the long range nature of the Coulomb interaction, effectively to oscillating voltages at all terminals¹². In addition, in experiments, the pump is investigated with an impedance in series with the oscillating scatterer. Furthermore, a voltage probe, to maintain zero current in the presence of pumping, in effect generates an oscillating potential which acts back on the pump⁴⁹. Therefore, for theory to make contact with experiment, it is necessary to consider the effect of oscillating voltages at the contacts of the conductor.

The paper is organized as follows. In Sec.II we develop the Floquet scattering matrix approach for ac quantum transport through the nonstationary (oscillating) scatterer in the presence of oscillating reservoir potentials. A full theory requires even to first order in frequency an investigation of non-adiabatic corrections to the adiabatic (frozen) scattering matrix. These corrections are discussed in Sec.III. The current to linear order in the reservoir potentials is calculated in Sec.IV. We illustrate the results for a simple one-channel scatterer with two contacts. In Sec.V we present a general expression for the current valid for finite potentials. Sec. VI gives the expression for the instantaneous current.

II. GENERAL APPROACH

For simplicity we consider here the mesoscopic sample, the pump, connected to N_r reservoirs via single channel leads Fig.1. We are interested in the dc and ac currents flowing in the system if this system is subject to a cyclic evolution with period \mathcal{T} . The general situation we want to consider admits the scatterer and the reservoir properties to be oscillating with frequency $\omega = 2\pi/\mathcal{T}$.

We use the scattering matrix approach to ac transport in phase coherent mesoscopic systems¹. According to this approach the currents flowing in the system are determined by the scattering of electrons coming from the reservoirs by the mesoscopic sample^{51,52}. In the present paper we deal with non-interacting electrons. A full theory has eventually to treat the internal potential of the pump in a self-consistent manner.

The scattering properties of a mesoscopic sample oscillating with frequency ω can be described via the Floquet scattering matrix \hat{S}'_F (see, e.g., Ref. 30).

The matrix element $S'_{F,\alpha\beta}(E_n, E)$ is a quantum mechanical amplitude for an electron with energy E entering the scatterer through lead β to leave the scatterer through lead α having energy $E_n = E + n\hbar\omega$. We use Greek letters α, β to number the leads connecting the scatterer to the reservoirs: $\alpha, \beta = 1, \dots, N_r$.

Denoting by \hat{a}' an annihilation operator for incoming particles we can write down the expression for the annihilation operators \hat{b}' for outgoing particles^{26,30,52},

$$\hat{b}'_\alpha(E) = \sum_\beta \sum_{E_n > 0} S'_{F,\alpha\beta}(E, E_n) \hat{a}'_\beta(E_n). \quad (2)$$

By definition the reservoirs are not affected by the coupling to the scatterer and thus they are in an equilibrium

(but not necessary stationary) state. Therefore the properties of incoming particles are independent of the scatterer and are determined by the reservoirs. To be definite we suppose that the cyclic evolution of any reservoir α is due to solely an oscillating electrochemical potential $\mu_\alpha(t)$:

$$\begin{aligned} \mu_\alpha(t) &= \mu_{0,\alpha} + eV_\alpha(t), \\ V_\alpha(t) &= V_\alpha \cos(\omega t + \varphi_\alpha), \\ eV_\alpha &\ll \mu_{0,\alpha}. \end{aligned} \quad (3)$$

We emphasize that we must keep track of the phase shifts φ_α since there are a number of oscillating potentials and we can not eliminate all the phases φ_α simultaneously by merely shifting the time origin.

It is well known (see e.g., Ref. 53) that the wave functions for free electrons in the reservoir (say, α) with an oscillating uniform potential $V_\alpha(t)$ are of the Floquet function type:

$$\psi_\alpha(E, t, \mathbf{r}) = e^{i\mathbf{kr} - iEt/\hbar} \sum_{n=-\infty}^{\infty} J_n\left(\frac{eV_\alpha}{\hbar\omega}\right) e^{-in(\omega t + \varphi_\alpha)}. \quad (4)$$

Here $J_n(x)$ is the Bessel function of the first kind of the n^{th} order; $E = \hbar^2 k^2 / (2m_e)$ (m_e is an electron mass). The corresponding distribution function $f_{0,\alpha} = \langle \hat{a}_\alpha^\dagger(E) \hat{a}_\alpha(E) \rangle$ (here $\langle \dots \rangle$ means quantum-statistical averaging) is independent of the oscillating potential V_α and is the Fermi distribution function

$$f_{0,\alpha}(E) = \frac{1}{1 + \exp\left(\frac{E - \mu_{0,\alpha}}{k_B T_\alpha}\right)}. \quad (5)$$

Here T_α is the temperature of the reservoir α ; k_B is the Boltzmann constant.

In general, to find the Floquet scattering matrix \hat{S}'_F , we have to investigate the transmission and reflection amplitudes of electrons with a wave function $\psi(E, t, \mathbf{r})$ given by Eq.(4) incident on the oscillating scatterer. However if the frequency ω is small compared with the energy of electrons participating in the transport (i.e., with the Fermi energy μ)

$$\hbar\omega \ll \mu, \quad (6)$$

we can reduce the problem to scattering of ordinary plain waves. To this end we use the following trick⁵³. We imagine that in the leads connecting scatterer to the reservoirs the oscillating potentials tend to zero: $V_\alpha = 0$. Then in the leads the electron wave functions are simply plain waves

$$\psi_{0,\alpha}(E, r) = e^{i\mathbf{kr} - iEt/\hbar}. \quad (7)$$

In this region we introduce annihilation operators \hat{a}, \hat{b} for incoming and outgoing particles, respectively. In close analogy with Eq.(2) they are related but through the Floquet scattering matrix $S_{F,\alpha\beta}(E_n, E)$ describing scattering of incident and outgoing plane waves:

$$\hat{b}_\alpha(E) = \sum_\beta \sum_n S_{F,\alpha\beta}(E, E_n) \hat{a}_\beta(E_n). \quad (8)$$

Comparing the wave functions Eq.(4) and Eq.(7) we see that the annihilation operators \hat{a} for particles in the leads

can be expressed in terms of the annihilation operators \hat{a}' for particles in the reservoirs as follows⁵³

$$\hat{a}_\alpha(E) = \sum_{n=-\infty}^{\infty} J_n \left(\frac{eV_\alpha}{\hbar\omega} \right) e^{-in\varphi_\alpha} \hat{a}'_\alpha(E - n\hbar\omega). \quad (9)$$

The above representation is valid for small frequencies Eq.(6). Thus we can put $k(E_n) \approx k(E)$ ignoring the terms of order $\hbar\omega/\mu$ and smaller. In other words we ignore the reflection at the interface between the region with oscillating potential and the region without one.

Using Eqs.(8) and (9) we calculate the distribution functions $f_\alpha^{(out)}(E) = \langle \hat{b}_\alpha^\dagger(E) \hat{b}_\alpha(E) \rangle$ for outgoing and $f_\alpha^{(in)}(E) = \langle \hat{a}'_\alpha^\dagger(E) \hat{a}'_\alpha(E) \rangle$ for incoming electrons in the leads as follows

$$f_\alpha^{(in)}(E) = \sum_{n=-\infty}^{\infty} J_n^2 \left(\frac{eV_\alpha}{\hbar\omega} \right) f_{0,\alpha}(E - n\hbar\omega), \quad (10a)$$

$$f_\alpha^{(out)}(E) = \sum_\beta \sum_{n,m,q=-\infty}^{\infty} S_{F,\alpha\beta}^*(E, E_q) S_{F,\alpha\beta}(E, E_m) \quad (10b)$$

$$\times J_{n+q} \left(\frac{eV_\beta}{\hbar\omega} \right) J_{n+m} \left(\frac{eV_\beta}{\hbar\omega} \right) e^{i(q-m)\varphi_\beta} f_{0,\beta}(E - n\hbar\omega).$$

Now the dc current I_α of spinless electrons, the quantity of interest here, flowing from the scatterer through the lead α can be expressed in terms of these distributions²⁶,

$$I_\alpha = \frac{e}{h} \int_0^\infty dE \left\{ f_\alpha^{(out)}(E) - f_\alpha^{(in)}(E) \right\}. \quad (11)$$

Substituting Eqs.(10) into Eq.(11) we find

$$I_\alpha = \frac{e}{h} \int_0^\infty dE \sum_\beta \sum_{n=-\infty}^{\infty} f_{0,\beta}(E - n\hbar\omega) \left\{ \sum_{m,q=-\infty}^{\infty} S_{F,\alpha\beta}^*(E, E_q) S_{F,\alpha\beta}(E, E_m) J_{n+q} \left(\frac{eV_\beta}{\hbar\omega} \right) J_{n+m} \left(\frac{eV_\beta}{\hbar\omega} \right) e^{i(q-m)\varphi_\beta} - \delta_{\alpha\beta} J_n^2 \left(\frac{eV_\alpha}{\hbar\omega} \right) \right\}. \quad (12)$$

Eq.(12) is the basic result which allows us to analyze the dc currents flowing in the system under consideration. So far we put no restrictions on the reservoirs. Different temperatures of reservoirs as well as different (stationary) electrochemical potentials can by themselves give rise to dc currents. We will not consider the most general situation here. Pumping in the presence of stationary chemical potential differences is investigated by Entin-Wohlman et al.^{25,46}. Here we focus on dynamically oscillating potentials.

In what follows we assume the reservoirs to have equal temperatures and equal dc-components of electrochemical potentials but the oscillating reservoir potentials V_α can be different:

$$T_\alpha = T_0, \quad \mu_{0,\alpha} = \mu_0, \quad \alpha = 1, \dots, N_r. \quad (13)$$

In this case the distribution functions entering Eq.(12) are independent of the lead index: $f_{0,\alpha(\beta)}(X) = f_0(X)$, where f_0 is the Fermi distribution function with temperature T_0 and chemical potential μ_0 .

To calculate the Floquet scattering matrix $\hat{S}_F(E, E_n)$ one needs to solve the time-dependent scattering problem. Generally this can be done only numerically (see e.g., Ref.30).

Here we are interested in the limit of low frequencies. In this limit we can use the adiabatic approximation as a starting point and can express the Floquet scattering matrix in terms of a stationary scattering matrix with time-dependent parameters (the *frozen* scattering matrix): $\hat{S}_0(E, t) \equiv \hat{S}_0(E, \{P(t)\})$. Here $\{P\}$ is a set of parameters $P_i(t) = P_{i,0} + P_{i,1} \cos(\omega t + \phi_i)$, $i = 1, 2, \dots, N_p$ oscillating with frequency ω . The scattering matrix $\hat{S}_0(E, \{P\})$ describes reflection and transmission of particles with energy E at given frozen parameters P_i . This description is valid if the energy scale $\hbar\omega$ dictated by the modulation frequency is small compared with the energy scale δE over which the scattering ma-

trix $\hat{S}(E)$ changes significantly³⁰.

III. ADIABATIC APPROXIMATION

To zero-th order in frequency the elements of the Floquet scattering matrix can be approximated by the Fourier coefficients $\hat{S}_{0,n}$ of the stationary scattering matrix \hat{S}_0 ,

$$\hat{S}_{0,n}(E) = \frac{\omega}{2\pi} \int_0^\tau dt e^{in\omega t} \hat{S}_0(E, t). \quad (14a)$$

$$\hat{S}_0(E, t) = \sum_{n=-\infty}^{\infty} e^{-in\omega t} \hat{S}_{0,n}(E), \quad (14b)$$

as follows³⁰

$$\hat{S}_F(E_n, E) \approx \hat{S}_F(E, E_{-n}) \approx \hat{S}_{0,n}(E). \quad (15)$$

However in general this approximation is not sufficient to calculate the current to order ω . In particular if the oscillating potentials $V_\alpha \neq 0$ are applied to the reservoirs then to calculate the dc current to first order in frequency ω one needs to know the Floquet scattering matrix with the same accuracy.

Note that fortunately in the case of stationary reservoirs ($V_\alpha = 0$) there exists a representation (see Eq.(8) in Ref.30) which allows to calculate the dc current (with accuracy of order ω) using only the zero order approximation Eq.(15). In contrast another representation (see Eq.(9) in Ref.30) for the same dc current requires the knowledge of the Floquet scattering matrix with higher accuracy (i.e., to the first order in frequency).

Note that the nonadiabatic corrections to the scattering states and the corresponding corrections to the pumped cur-

rent were considered in Refs.25,46 in the limit of a small modulating potential. Our approach is valid for an arbitrary oscillating potential since we take into account the effect of all the harmonics of the pump frequency ω .

To calculate the Floquet scattering matrix with an accuracy of order ω we generalize the approach used in Ref.1 and start from the unitarity conditions for the Floquet scattering matrix³⁰

$$\sum_{\alpha} \sum_{n=-\infty}^{\infty} S_{F,\alpha\beta}^*(E_n, E) S_{F,\alpha\gamma}(E_n, E_m) = \delta_{m0} \delta_{\beta\gamma}, \quad (16a)$$

$$\sum_{\beta} \sum_{n=-\infty}^{\infty} S_{F,\alpha\beta}^*(E, E_n) S_{F,\gamma\beta}(E_m, E_n) = \delta_{m0} \delta_{\alpha\gamma}. \quad (16b)$$

Taking into account that Eqs.(15) are a zero-th order approximation we will seek the first order approximation in the following form

$$\hat{S}_F(E_n, E) = \hat{S}_{0,n} \left(\frac{E_n + E}{2} \right) + \hbar\omega \hat{A}_n(E) + O(\omega^2). \quad (17a)$$

$$\hat{S}_F(E, E_{-n}) = \hat{S}_{0,n} \left(\frac{E + E_{-n}}{2} \right) + \hbar\omega \hat{A}_n(E) + O(\omega^2). \quad (17b)$$

Here $\hat{A}_n(E)$ is a matrix of the Fourier coefficients for some matrix $\hat{A}(E, t) \equiv \hat{A}(E, \{P(t)\})$ which is treated as independent of energy on the scale of the order of $n\hbar\omega$; $O(\omega^2)$ denotes the rest which is at least of second order in frequency ω and which we neglect. Note that the first terms in Eqs.(17) should be expanded to the first order in ω

$$\hat{S}_{0,n} \left(\frac{E + E_{\pm n}}{2} \right) \approx \hat{S}_{0,n}(E) \pm \hbar\omega \frac{n}{2} \frac{\partial \hat{S}_{0,n}(E)}{\partial E},$$

and other terms (of higher order in ω) should be ignored.

Based on Eq.(21) we will show that Eq.(17) is, in fact, an expansion in powers of $\hbar\omega/\delta E$. Substituting Eqs.(17) into Eqs.(16) and keeping the terms of order ω^0 and ω^1 we get the required relations which can be used to calculate the current Eq.(12).

In particular the diagonal part ($m = 0, \beta = \gamma$) of Eqs.(16) gives

$$\begin{aligned} \sum_{\alpha(\beta)} \sum_{n=-\infty}^{\infty} S_{0,\alpha\beta,n}^*(E) A_{\alpha\beta,n}(E) + c.c. = \\ \mp \frac{1}{2} \frac{\partial}{\partial E} \sum_{\alpha(\beta)} \sum_{n=-\infty}^{\infty} n |S_{0,\alpha\beta,n}(E)|^2. \end{aligned} \quad (18)$$

Here *c.c.* denotes complex conjugate terms. The sign $-(+)$ corresponds to the summation over $\alpha(\beta)$.

In what follows we concentrate on the case without magnetic fields and suppose that the stationary scattering matrix is symmetric in lead indexes:

$$S_{0,\alpha\beta} = S_{0,\beta\alpha}. \quad (19)$$

It follows from Eq.(18) that in this case the matrix \hat{A} is anti-symmetric:

$$A_{\alpha\beta} = -A_{\beta\alpha}. \quad (20)$$

Since $A_{\alpha\alpha} = 0$, we can immediately conclude that the reflection ($\alpha = \beta$) coefficients are with accuracy of order ω defined by the first terms on the RHS of Eqs.(17). This fact justifies our representation for the elements of the Floquet scattering matrix in Eqs.(17).

We next need to determine the off-diagonal elements of \hat{A} . The detailed calculation is given in Appendix 1. The central result is the relation (valid to first order in ω),

$$\hbar\omega \left(\hat{S}_0^\dagger(E, t) \hat{A}(E, t) + \hat{A}^\dagger(E, t) \hat{S}_0(E, t) \right) = \frac{1}{2} \mathcal{P} \{ \hat{S}_0^\dagger; \hat{S}_0 \}, \quad (21a)$$

$$\mathcal{P} \{ \hat{S}_0^\dagger; \hat{S}_0 \} = i\hbar \left(\frac{\partial \hat{S}_0^\dagger}{\partial t} \frac{\partial \hat{S}_0}{\partial E} - \frac{\partial \hat{S}_0^\dagger}{\partial E} \frac{\partial \hat{S}_0}{\partial t} \right). \quad (21b)$$

Here i is the imaginary unit. Since the scattering matrix is unitary $\hat{S}_0^\dagger \hat{S}_0 = \hat{I}$ (where \hat{I} is a unit matrix) the matrix $\mathcal{P} \{ \hat{S}_0^\dagger; \hat{S}_0 \}$ is traceless: $\sum_{\alpha=1}^{N_r} \mathcal{P} \{ \hat{S}_0^\dagger; \hat{S}_0 \}_{\alpha\alpha} = 0$. (see Appendix 2 for the detailed proof).

Note Avron et al.⁴⁰ consider a closely related matrix $\hat{\Omega} = \mathcal{P} \{ \hat{S}_0; \hat{S}_0^\dagger \}$. This matrix is the commutator $\hat{\Omega} = \frac{i}{\hbar} [\hat{\mathcal{T}}, \hat{\mathcal{E}}]$ of the Wigner time-delay matrix^{54,55}: $\hat{\mathcal{T}} = -i\hbar \frac{\partial \hat{S}_0}{\partial E} \hat{S}_0^\dagger$, and the matrix of the energy shift^{22,40}: $\hat{\mathcal{E}} = i\hbar \frac{\partial \hat{S}_0}{\partial t} \hat{S}_0^\dagger$. Note however, that on the RHS of Eq. (21a) the commutator appears with a different sequence of S^\dagger and S as compared to $\hat{\Omega}$. For this reason (and other reasons to be become clear later on, we have introduced a separate notation, the Poisson bracket \mathcal{P} . As we will show (see Eq.(34)) the diagonal elements $(e/h)\mathcal{P}_{\alpha\alpha}$ of this commutator are just spectral current densities (current per energy).

From Eq.(21a) we find an expression for the product of the frozen scattering matrix with elements of \hat{A} ,

$$4\hbar\omega \text{Re} [S_{0,\alpha\beta}^* A_{\alpha\beta}] = \frac{1}{N_r} \left(\mathcal{P} \{ \hat{S}_0^\dagger; \hat{S}_0 \}_{\beta\beta} - \mathcal{P} \{ \hat{S}_0^\dagger; \hat{S}_0 \}_{\alpha\alpha} \right). \quad (22)$$

Note that the scattering matrix \hat{S}_0 is a unitary matrix of dimension N_r . Evidently Eq.(22) is consistent with Eq.(20).

Below we use Eqs.(17) and (22) to evaluate the current Eq.(12) with an accuracy of order ω .

IV. LINEAR RESPONSE ADIABATIC CURRENT

Now we use the adiabatic approximation introduced in the previous section and calculate the zero-frequency, dc-current Eq.(12) to linear order in the oscillating potentials $V_\alpha \rightarrow 0$ of the reservoirs at finite temperature T_0 . We assume that the following conditions hold:

$$\hbar\omega \ll k_B T_0, \quad (23a)$$

$$eV_\alpha \ll k_B T_0. \quad (23b)$$

The first inequality ($\hbar\omega \ll k_B T_0$) is relevant for experiments on adiabatic ($\omega \rightarrow 0$) quantum transport. The second inequality defines nothing but the linear response regime.

In Eq.(12) the sum over n contains approximately $n_{max} \sim \frac{eV}{\hbar\omega}$ terms. Therefore, $\hbar\omega n \leq eV$ and because of Eq.(23b)

we have $\hbar\omega n \ll k_B T$. Hence we can expand the Fermi function entering Eq.(12). Taking into account Eq.(13) this expansion (up to second order in ω) is: $f_{0,\beta}(E - n\hbar\omega) \approx f_0(E) - n\hbar\omega \frac{\partial f_0(E)}{\partial E} + \frac{1}{2}n^2 \hbar^2 \omega^2 \frac{\partial^2 f_0(E)}{\partial E^2}$. Substituting this distribution into Eq.(12) and take the sum over n . We use the summation formulae for the Bessel functions⁵⁶

$$\begin{aligned} \sum_{n=-\infty}^{\infty} J_{n+m}(X)J_{n+q}(X) &= \delta_{mq}, \\ \sum_{n=-\infty}^{\infty} nJ_{n+m}(X)J_{n+q}(X) &= -m\delta_{mq} + \frac{X}{2}(\delta_{m(q+1)} + \delta_{m(q-1)}), \\ \sum_{n=-\infty}^{\infty} n^2 J_{n+m}(X)J_{n+q}(X) &= \left(m^2 + \frac{X^2}{2}\right) \delta_{mq} \\ &- X ([m - 0.5]\delta_{m(q+1)} + [m + 0.5]\delta_{m(q-1)}) \\ &+ \frac{X^2}{4}(\delta_{m(q+2)} + \delta_{m(q-2)}). \end{aligned} \quad (24)$$

$$I_\alpha = \int_0^\infty dE \left(-\frac{\partial f_0(E)}{\partial E} \right) \left\{ I_\alpha^{(pump)} + I_\alpha^{(rect)} + I_\alpha^{(int)} \right\}, \quad (25a)$$

$$I_\alpha^{(pump)}(E) = i \frac{e}{2\pi} \overline{\left(\frac{\partial \hat{S}_0(E, t)}{\partial t} \hat{S}_0^\dagger(E, t) \right)_{\alpha\alpha}}, \quad (25b)$$

$$I_\alpha^{(rect)}(E) = G_0 \sum_\beta \overline{(V_\beta(t) - V_\alpha(t)) |S_{0,\alpha\beta}(E, t)|^2}, \quad (25c)$$

$$I_\alpha^{(int)}(E) = \frac{G_0}{2N_r} \sum_\beta \overline{V_\beta(t) \left(\mathcal{P}\{\hat{S}_0^\dagger; \hat{S}_0\}_{\beta\beta} - \mathcal{P}\{\hat{S}_0^\dagger; \hat{S}_0\}_{\alpha\alpha} - N_r \mathcal{P}\{S_{0,\alpha\beta}^*; S_{0,\alpha\beta}\} \right)}. \quad (25d)$$

Here the bar denotes the time average $\overline{X} = \frac{1}{T} \int_0^T dt X(t)$ over a time period $T = \frac{2\pi}{\omega}$; $G_0 = e^2/h$ is the spinless conductance quantum; the function $\mathcal{P}\{X; Y\}$ is defined in Eq.(21b). To arrive at Eq.(25c) we used the unitarity condition $\sum_\beta |S_{0,\alpha\beta}|^2 = 1$ and the fact that the average potential is zero: $\overline{V_\alpha(t)} = 0$.

We emphasize that in the above expressions we omitted all the terms which are of the second (and higher) order in frequency ω and/or in potentials V_α . Next we characterize briefly the three contributions to the current I_α .

The current $I_\alpha^{(pump)}$ is due to solely the oscillating scatterer. It determines the quantum pump effect when the reservoirs are stationary. It is the formula obtained by Brouwer¹³.

The current $I_\alpha^{(rect)}$ is a consequence of the rectification of ac currents flowing in the system under the influence of ac potentials V_α applied to the reservoirs. In context of pumping this effect was considered by Brouwer in Ref. 47. Note that this rectified current depends on the conductances $G_{\alpha\beta} = -G_0 |S_{\alpha\beta}|^2$ and the corresponding potential differences

After that, substituting Eqs.(17), (22) and applying the inverse Fourier transformation Eq.(14b), we sum over q and m . Finally we represent the dc current I_α flowing in lead α under the action of an oscillating scatterer and oscillating reservoir potentials in the following way:

$\Delta V_{\alpha\beta}(t) = V_\beta(t) - V_\alpha(t)$ in close analogy with the dc current flowing in response to a dc voltage. We stress that here $\Delta V_{\alpha\beta}(t)$ depends not only on the amplitudes of the corresponding potentials but also on the phase lag $\Delta\varphi_{\alpha\beta} = \varphi_\beta - \varphi_\alpha$ as well. In particular if the amplitudes of two oscillating potentials are equal $V_\alpha = V_\beta = V_0$ then the potential difference reads

$$\Delta V_{\alpha\beta}(t) = -2V_0 \sin(\Delta\varphi_{\alpha\beta}) \sin\left(\omega t + \frac{\varphi_\alpha + \varphi_\beta}{2}\right). \quad (26)$$

This equation [together with Eq.(25c)] shows clearly that the rectification of ac currents can depend on the phase lag between the applied ac potentials and, hence, it can mimic a quantum pump effect⁴⁷.

The third term $I_\alpha^{(int)}$ is novel. Interestingly, as we will see, this current renormalizes both $I_\alpha^{(pump)}$ and $I_\alpha^{(rect)}$. The current $I_\alpha^{(int)}$ is a consequence of the *interference* between the ac currents produced by the external voltages and the ac currents produced by the nonstationary scatterer. Remarkably, it is essentially determined by commutator expressions. An

"oscillating" scatterer is much richer in physics than expressed by Eq. (25c). The expression for $I^{(rect)}$ is widely used but this is only a part of a correct answer. The part ($I^{(rect)}$) is due to a rectification of external currents caused by the time-dependence of the conductances. The oscillating scatterer is much richer: It generates its own ac currents which can interfere with the external ac currents. This interference effect leads to $I^{(int)}$.

Before proceeding we check the current conservation. To this end we sum I_α over the lead index α . Note that each of the currents $I_\alpha^{(pump)}$, $I_\alpha^{(rect)}$, and $I_\alpha^{(int)}$ is separately conserved. This fact supports the current decomposition introduced above.

For the pump currents $I_\alpha^{(pump)}$, using the Birman-Krein formula⁵⁷ we find

$$\sum_\alpha I_\alpha^{(pump)} \sim Tr \left(\frac{\partial \hat{S}_0}{\partial t} \hat{S}_0^\dagger \right) = \frac{\partial}{\partial t} \overline{\ln(\det \hat{S}_0)} = 0.$$

$$\sum_{\alpha=1}^{N_r} I_\alpha^{(int)} = \frac{G_0}{2N_r} \sum_\beta V_\beta(t) \sum_{\alpha=1}^{N_r} \left(\mathcal{P}\{\hat{S}_0^\dagger; \hat{S}_0\}_{\beta\beta} - \mathcal{P}\{\hat{S}_0^\dagger; \hat{S}_0\}_{\alpha\alpha} - N_r \mathcal{P}\{S_{0,\alpha\beta}^*; S_{0,\alpha\beta}\} \right) \sim N_r \mathcal{P}\{\hat{S}_0^\dagger; \hat{S}_0\}_{\beta\beta} - N_r \mathcal{P}\{\hat{S}_0^\dagger; \hat{S}_0\}_{\beta\beta} = 0.$$

To shed more insight onto the nature of the new contribution $I^{(int)}$ we consider a simple but a quite generic example.

A. Two terminal single channel scatterer

Consider a nonstationary scatterer connected to only two reservoirs $\alpha = 1, 2$ via single channel leads. For such a scatterer, assuming there are no magnetic fields, the stationary scattering matrix \hat{S}_0 is a symmetric 2×2 unitary matrix.

$$\hat{S}_0 = e^{i\gamma} \begin{pmatrix} \sqrt{R}e^{-i\theta} & i\sqrt{T} \\ i\sqrt{T} & \sqrt{R}e^{i\theta} \end{pmatrix}. \quad (27)$$

Here R and T are the reflection and the transmission probability, respectively ($R + T = 1$). The phase θ characterizes the asymmetry between the reflection to the left and to the right. The phase γ relates to the change of the overall charge δQ on the scatterer (for instance a dot) via the Friedel sum rule⁵⁸: $\delta\gamma = \pi\delta Q/e$ (where e is the electron charge), or in different notation $\delta Q = e/(2\pi i)\delta[\ln \det \hat{S}]$. We assume that $R, T = 1 - R, \theta, \gamma$ are functions of the electron energy E and the external parameters $P_i(t)$ varying with frequency ω . Before proceeding we remark that for the case $N_r = 2$ the current $I_\alpha^{(int)}$ Eq. (25d) can be simplified

$$I_\alpha^{(int)} = \frac{G_0}{2} \left(\overline{V_\beta(t) \mathcal{P}\{S_{0,\beta\beta}^*; S_{0,\beta\beta}\}} - \overline{V_\alpha(t) \mathcal{P}\{S_{0,\alpha\alpha}^*; S_{0,\alpha\alpha}\}} \right), \quad \alpha \neq \beta. \quad (28)$$

Substituting the scattering matrix Eq.(27) into Eqs.(25) and (28) we find the currents $I_1 = -I_2$ flowing between the scatterer and the reservoirs:

$$I_1^{(pump)}(E) = \frac{e}{2\pi} R(E, t) \frac{\partial \theta(E, t)}{\partial t}, \quad (29a)$$

Here we take into account that the average of a time derivative is identically zero: $\frac{\partial \overline{X(t)}}{\partial t} \equiv 0$; Tr denotes the trace of a matrix: $Tr \hat{S} = \sum_\alpha S_{\alpha\alpha}$.

The conservation of the rectification currents $\sum_\alpha I_\alpha^{(rect)} = G_0 \sum_{\alpha,\beta} \overline{(V_\beta(t) - V_\alpha(t)) |S_{0,\alpha\beta}|^2} = 0$ follows from the unitarity condition $\sum_\alpha |S_{0,\alpha\beta}|^2 = \sum_\beta |S_{0,\alpha\beta}|^2 = 1$.

The current $I_\alpha^{(int)}$ is conserved as well. Since the matrix $\mathcal{P}\{\hat{S}_0^\dagger; \hat{S}_0\}$ Eq.(21b) is traceless we get from Eq.(25d)

$$I_1^{(rect)}(E) = G_0 \overline{T(E, t) [V_2(t) - V_1(t)]}, \quad (29b)$$

$$I_1^{(int)}(E) = \frac{e^2}{4\pi} \overline{\left(\frac{\partial \theta}{\partial t} \frac{\partial R}{\partial E} - \frac{\partial \theta}{\partial E} \frac{\partial R}{\partial t} \right) [V_2(t) + V_1(t)]} + \frac{e^2}{4\pi} \overline{\left(\frac{\partial \gamma}{\partial t} \frac{\partial R}{\partial E} - \frac{\partial \gamma}{\partial E} \frac{\partial R}{\partial t} \right) [V_2(t) - V_1(t)]}. \quad (29c)$$

These expressions demonstrate that the current $I^{(int)}$ has common features with both the rectification current $I^{(rect)}$ and the pumped current $I^{(pump)}$. Like the former, the current $I^{(int)}$ depends on the potential difference ΔV_{12} . Like the latter, the current $I^{(int)}$ can exist even at equal reservoir potentials $V_1(t) = V_2(t)$. In this case, the conditions necessary for the existence of $I^{(int)}$ and $I^{(pump)}$ are the same³⁰: First, the scatterer has to be asymmetric, i.e., $\theta \neq 0$, and, second, the time reversal symmetry (TRS) has to be broken. We note that the current $I^{(int)}$ depends on both the oscillating reservoir potentials $V_\alpha(t) = V_\alpha \cos(\omega t + \varphi_\alpha)$ and the oscillating scattering parameters $P_i(t) = P_{i,0} + P_{i,1} \cos(\omega t + \phi_i)$. Therefore analyzing the presence/absence of the TRS we have to consider all the phases, namely φ_α as well as ϕ_i .

We have here treated only non-interacting electrons. As a consequence sums of potentials appear in Eq.(29c). This is in contrast with an electrically self-consistent theory which permits only the appearance of voltage differences. If interactions are switched on¹ then the (self-consistent) potential $U \neq 0$ inside the scatterer becomes dependent on external potentials V_α and the differences $V_\alpha - U$ should appear instead of V_α . U is in general a function of all the oscillating parameters $P_i(t)$, all the external potentials V_α and also of the potentials at the gates which influence the electrostatic potential inside the scatterer. Our expressions do, however, conserve current.

We see that the first term on the RHS of Eq.(29c) renormalizes the pumped current $I^{(pump)}$ and the second one renor-

malizes the rectification current $I^{(rect)}$. The latter is due to nonadiabatic (first order in ω) corrections to the conductances arising from the corresponding corrections Eq.(17) to the scattering matrix. Note that the analogous corrections are discussed in Refs.25,46 in context of pumping in the presence of a dc bias.

Since the pump effect is the main topic of this work we consider now the case with $V_1(t) = V_2(t)$ in more detail. This case corresponds to an experimental setup in which the scatterer and a large portion of the reservoirs to which it is connected are subject to long wavelength radiation. The effect of such radiation can be modeled via an oscillating uniform potential $V(t)$ which is the same at different reservoirs: $V_1(t) = V_2(t) \equiv V(t)$. In this case the rectification current Eq.(29b) is absent $I^{(rect)} = 0$, and the whole dc current I_α can be reduced to the simple form

$$I_\alpha = \frac{e}{2\pi} \int_0^\infty dE \left(-\frac{\partial f_0(E)}{\partial E} \right) \overline{R(\mathcal{E}, t) \frac{d\theta(\mathcal{E}, t)}{dt}}, \quad (30)$$

$$\mathcal{E} = E + eV(t).$$

To obtain this result we have used the following identity: $-\overline{A \frac{\partial R}{\partial t}} = \overline{R \frac{\partial A}{\partial t}}$ with $A = eV \frac{\partial \theta}{\partial E}$. We have also introduced the full time derivative: $\frac{d}{dt} = \frac{\partial}{\partial t} + e \frac{dV}{dt} \frac{\partial}{\partial \mathcal{E}}$.

This result can be understood in the following way: For stationary reservoirs ($V(t) = 0$) the pumped current is described by equations (25a) and (29a) with the quantities R and θ taken at the energy E of incident electrons. However, if the chemical potential $\mu(t) = \mu_0 + eV(t)$, $V(t) \neq 0$ oscillates slowly ($\omega \rightarrow 0$) then we can consider incident electrons having energy $\mathcal{E} = E + eV(t)$ following adiabatically the reservoir's potential $V(t)$. Substituting in Eq.(29a) \mathcal{E} instead of E and replacing a partial time derivative by a full time derivative we get Eq.(30).

It should be noted that the above substitution $\mathcal{E} = E + eV(t)$ implies that the potential inside the scatterer ($U = 0$) is independent of the external potentials V_α . This is correct for noninteracting electrons but it should be modified if the interactions are present¹.

From equation (30) we can conclude that the effect of an oscillating external potential $V(t)$ is like the effect generated by an oscillating parameter of the scatterer (i.e., an oscillating internal potential). Therefore to analyze the ability of an open system (the scatterer plus reservoirs) to generate adiabatic dc currents we have to consider the full set of oscillating parameters $\{V_\alpha(t), P_i(t)\}$, ($\alpha = 1, 2, \dots, N_r$; $i = 1, 2, \dots, N_p$).

V. DC CURRENT AT FINITE AC VOLTAGES

Now we go beyond linear response theory. We suppose that the potentials V_α can be large compared to the temperature. Thus we calculate the current Eq.(12) with accuracy up to the first order in ω and with an arbitrary ratio of the potentials V_α to the temperature:

$$\hbar\omega \ll k_B T_0, \quad (31a)$$

$$eV_\alpha \ll \mu_{0,\alpha}. \quad (31b)$$

Since the potentials V_α are not necessarily small compared with the temperature T we can not expand the Fermi function $f_{0,\beta}(E - n\hbar\omega)$ entering Eq.(12). Nevertheless Eq.(31a) allows us to sum over n and to simplify Eq.(12).

To this end we go from the energy representation over to the time representation. We express the Fermi function $f_{0,\beta}(E)$ Eq.(5) and the Bessel functions $J_n(x)$ as follows:

$$f_{0,\beta}(E - n\hbar\omega) = \int_{-\infty}^{\infty} d\tau f_{0,\beta}(\tau) e^{i(E - n\hbar\omega)\frac{\tau}{\hbar}},$$

$$J_{n+q} \left(\frac{eV_\beta}{\hbar\omega} \right) e^{i\varphi_\beta(n+q)} = \frac{1}{T} \int_0^T dt W_\beta^*(t) e^{-i(n+q)\omega t},$$

$$J_{n+m} \left(\frac{eV_\beta}{\hbar\omega} \right) e^{-i\varphi_\beta(n+m)} = \frac{1}{T} \int_0^T dt_1 W_\beta(t_1) e^{i(n+m)\omega t_1},$$

$$f_{0,\beta}(\tau) = \frac{ik_B T_0}{2\hbar \sinh(\pi k_B T_0 \frac{\tau}{\hbar})} e^{-i\mu_{0,\beta} \frac{\tau}{\hbar}},$$

$$W_\beta(t) = e^{-i \frac{eV_\beta}{\hbar\omega} \sin(\omega t + \varphi_\beta)}.$$

Substituting these equations into Eq.(12) and summing over n we obtain a delta-function $\delta(t_1 - t - \tau)$ which allows us to perform one additional integration. At $\tau > 0$ ($\tau < 0$) we integrate over t_1 (t). This leads to the substitution $t_1 = t + \tau$ ($t = t_1 + |\tau|$). Further we expand $\sin(\omega\tau + \omega t + \varphi_\beta)$ to first order in $\omega\tau$. We can do this because for the relevant $\tau \leq \frac{\hbar}{k_B T_0}$ Eq. (31a) gives $\omega\tau \ll 1$. Next we integrate over τ and finally get the dc current as follows:

$$I_\alpha = \frac{e}{h} \int_0^\infty dE \frac{1}{T} \int_0^T dt \left\{ \sum_\beta \sum_{m,q=-\infty}^{\infty} f_0 \left(E + (q+m) \frac{\hbar\omega}{2}; \mu_\beta(t) \right) S_{F,\alpha\beta}^*(E, E_q) S_{F,\alpha\beta}(E, E_m) e^{i(m-q)\omega t} - f_0(E; \mu_\alpha(t)) \right\}. \quad (32)$$

Here we have introduced the Fermi function with time-dependent chemical potential $\mu_\alpha(t) = \mu_{0,\alpha} + eV_\alpha(t)$ Eq.(3): $f_0(E; \mu_\alpha(t)) = \left[1 + \exp \left(\frac{E - \mu_\alpha(t)}{k_B T_0} \right) \right]^{-1}$.

Note that Eq.(32) is valid both for the adiabatic as well as for the nonadiabatic case. The only restriction is that the frequency has to be small compared with the temperature

Eq.(31a).

Next we use the adiabatic approximation of Sec.III and calculate the current I_α to first order in frequency ω under the conditions of Eq.(13). To this end we substitute Eqs.(17) and (22) into Eq.(32) and expand the Fermi function in powers of ω . Next we use the inverse Fourier transformation Eq.(14b) and after a little manipulation (we integrate by parts on en-

ergy and dropped the contribution arising from $E = 0$; in addition we exploit the unitarity of the frozen scattering ma-

trix $\sum_{\alpha} |S_{0,\alpha\beta}(E, t)|^2 = 1$) and find the current:

$$I_{\alpha} = \frac{e}{h} \int_0^{\infty} dE \frac{1}{T} \int_0^T dt \left\{ \sum_{\beta} f_0(E; \mu_{\beta}(t)) \left[|S_{0,\alpha\beta}(E, t)|^2 + \frac{\mathcal{P}\{\hat{S}_0^{\dagger}; \hat{S}_0\}_{\beta\beta} - \mathcal{P}\{\hat{S}_0^{\dagger}; \hat{S}_0\}_{\alpha\alpha} - N_r \mathcal{P}\{S_{0,\alpha\beta}^*; S_{0,\alpha\beta}\}}{2N_r} \right] - f_0(E; \mu_{\alpha}(t)) \right\}. \quad (33)$$

The above equation generalizes Eqs.(25) to the case of finite voltages. Current conservation $\sum_{\alpha} I_{\alpha} = 0$ can easily be proven

in analogy with Eqs.(25).

Next we concentrate on the pump effect and consider the case with reservoirs having equal oscillating potentials: $\mu_{\alpha}(t) \equiv \mu(t) = \mu_0 + eV \cos(\omega t + \varphi)$, $\alpha = 1, \dots, N_r$. Since the Fermi functions entering Eq.(33) become independent of the lead index we can sum up over β and obtain

$$I_{\alpha} = \int_0^{\infty} dE \frac{1}{T} \int_0^T dt f_0(E; \mu(t)) \frac{dI_{\alpha}(E, t)}{dE}, \quad (34)$$

$$\frac{dI_{\alpha}}{dE} = \frac{e}{h} \mathcal{P}\{\hat{S}_0; \hat{S}_0^{\dagger}\}_{\alpha\alpha} \equiv i \frac{e}{2\pi} \left(\frac{\partial \hat{S}_0}{\partial t} \frac{\partial \hat{S}_0^{\dagger}}{\partial E} - \frac{\partial \hat{S}_0}{\partial E} \frac{\partial \hat{S}_0^{\dagger}}{\partial t} \right)_{\alpha\alpha},$$

Here we used an obvious equality: $\mathcal{P}\{\hat{S}_0^{\dagger}; \hat{S}_0\}_{\alpha\alpha} = -\mathcal{P}\{\hat{S}_0; \hat{S}_0^{\dagger}\}_{\alpha\alpha}$.

The quantity $dI_{\alpha}(E, t)/dE$ is the spectral current density at energy E and time t , (i.e., the current within the energy interval dE) produced by the adiabatically evolving scatterer towards the reservoir α . This definition seems reasonable because of a conservation law $\sum_{\alpha} dI_{\alpha}/dE(E, t) = 0$ which is valid at any energy E and at any time moment t . Note that in the case of stationary reservoirs the same interpretation was given in Ref. 40.

These currents (or more precisely, the ability to produce them) are an intrinsic property of a time-dependent scatterer. This property differentiates between a nonstationary scatterer and a "frozen" one. Note that the Fermi distribution function in Eq.(34) describe the filling of (potentially) existing "current" states of a nonstationary scatterer.

At $V = 0$ the Eq.(34) reproduces Brouwer's result Eq.(25b) and agrees with that obtained in Ref.40. At small voltages $V \rightarrow 0$ for the scattering matrix Eq.(27) we get Eq.(30).

Equation (34) determines the dc-current to the first order in ω pumped by the slowly oscillating scatterer between the reservoirs having equal (possibly zero) oscillating potentials $V_{\alpha}(t) = V(t)$. Formally in the adiabatic case under consideration the effect of oscillating chemical potentials is only the change of an energy of electrons falling upon the scatterer. However, in fact, the phase φ of an oscillating potential $V(t) = V \cos(\omega t + \varphi)$ is of a great importance because of the following. An adiabatically pumped current $I_{\alpha} \neq 0$ is generated already if the time reversal symmetry is broken in the *whole system* including the scatterer and the reservoirs. At $V \neq 0$ analyzing this question we have to take into account a possible phase shift between the potentials of reservoirs and the oscillating parameters $P_i(t) = P_{i,0} + P_{i,1} \cos(\omega t + \phi_i)$ of a scatterer. In particular even a scatterer with a *single* oscillating parameter can produce an adiabatic dc current if only

$\phi_1 \neq \varphi$.

VI. INSTANTANEOUS CURRENT

In this Section we derive an expression for the instantaneous current of an adiabatic quantum pump simultaneously subject to oscillating external potentials. We first clarify the physical meaning of the (diagonal elements of the) quantity $\mathcal{P}\{\hat{S}_0^{\dagger}; \hat{S}_0\}$ defining (antisymmetric in lead indices) nonadiabatic corrections Eqs.(21) to the scattering matrix. From the geometrical point of view⁴⁰ this quantity is a curvature in the time-energy plane. The physical interpretation is based on Eq.(34). We can consider the quantity $dI_{\alpha}/dE(E, t) = \frac{e}{h} \mathcal{P}\{\hat{S}_0; \hat{S}_0^{\dagger}\}_{\alpha\alpha}$ as an instantaneous *spectral current* which is pushed by the oscillating scatterer into the lead α . A more detailed partitioning of the current follows from Eq.(33): We can say that the scatterer drives the following spectral currents from the lead β into the lead α :

$$\frac{dI_{\alpha\beta}}{dE} = \frac{e}{h} \frac{\mathcal{P}\{\hat{S}_0; \hat{S}_0^{\dagger}\}_{\alpha\alpha} - \mathcal{P}\{\hat{S}_0; \hat{S}_0^{\dagger}\}_{\beta\beta} + N_r \mathcal{P}\{S_{0,\alpha\beta}; S_{0,\alpha\beta}^*\}}{2N_r}. \quad (35)$$

The above spectral currents are subject to the following conservation law: $\sum_{\alpha=1}^{N_r} dI(E, t)_{\alpha\beta}/dE = 0$. This property supports the point of view that these currents arise "inside" the scatterer (they are generated by the nonstationary scatterer) without any external current source. Thus we can consider the pump as a source of currents rather than a source of voltages⁴⁴.

For a scatterer with scattering matrix Eq.(27), we obtain the spectral currents

$$\frac{dI_{11}(E, t)}{dE} = -\frac{e}{4\pi} \left(\frac{\partial(\gamma - \theta)}{\partial t} \frac{\partial R}{\partial E} - \frac{\partial(\gamma - \theta)}{\partial E} \frac{\partial R}{\partial t} \right), \quad (36a)$$

$$\frac{dI_{22}(E, t)}{dE} = -\frac{e}{4\pi} \left(\frac{\partial(\gamma + \theta)}{\partial t} \frac{\partial R}{\partial E} - \frac{\partial(\gamma + \theta)}{\partial E} \frac{\partial R}{\partial t} \right). \quad (36b)$$

Note that above currents depend on the phase γ related to the charge of a scatterer.

Strictly speaking if we are dealing with time dependent currents (instead of only the time averaged currents) then we need to show that these currents satisfy the continuity equation for the charge currents:

$$\sum_{\alpha} I_{\alpha}(t) + \frac{\partial Q(t)}{\partial t} = 0. \quad (37)$$

Here $I_{\alpha}(t)$ is the full time-dependent current flowing through the scatterer to the lead α ; $Q(t)$ is a charge of a scatterer.

To calculate $I_\alpha(t)$ we first calculate the Fourier transformed current $I_{\alpha,l} = \frac{\omega}{2\pi} \int_0^\tau dt e^{i\omega t} I_\alpha(t)$ which reads⁵²:

$$I_{\alpha,l} = \frac{e}{h} \int_0^\infty dE \left\{ \langle \hat{b}_\alpha^\dagger(E) \hat{b}_\alpha(E + l\hbar\omega) \rangle - \langle \hat{a}_\alpha^\dagger(E) \hat{a}_\alpha(E + l\hbar\omega) \rangle \right\}.$$

$$I_\alpha(t) = \int_0^\infty dE \sum_\beta \left\{ \frac{e}{h} [f_0(E; \mu_\beta(t)) - f_0(E; \mu_\alpha(t))] |S_{0,\alpha\beta}(E, t)|^2 - e \frac{\partial}{\partial t} \left[f_0(E; \mu_\beta(t)) \frac{dN_{\alpha\beta}(E, t)}{dE} \right] + f_0(E; \mu_\beta(t)) \frac{dI_{\alpha\beta}(E, t)}{dE} \right\}. \quad (38)$$

Here we have introduced the partial density of states¹ for a "frozen" scatterer,

$$\frac{dN_{\alpha\beta}}{dE} = \frac{i}{4\pi} \left(\frac{\partial S_{0,\alpha\beta}^*}{\partial E} S_{0,\alpha\beta} - S_{0,\alpha\beta}^* \frac{\partial S_{0,\alpha\beta}}{\partial E} \right).$$

These density of states define the charge $Q(t)$ of a "frozen" scatterer as follows:

$$Q(t) = e \sum_\alpha \sum_\beta \int_0^\infty dE f_0(E; \mu_\beta(t)) \frac{dN_{\alpha\beta}(E, t)}{dE} \quad (39)$$

The quantity $I_\alpha(t)$ Eq.(38) and $Q(t)$ Eq.(39) do satisfy the continuity equation (37).

The three terms in the curly brackets on the RHS of Eq.(38) can be interpreted as follows. The first term defines the currents flowing under the action of external voltages through a "frozen" scatterer. The second one defines currents attributed to an oscillating charge of a "frozen" scatterer. The third term can not be entirely viewed just as a nonadiabatic correction of either the "frozen" conductance nor of the "frozen" density of states. It is more naturally to consider it as the ac currents generated by the oscillating scatterer. The ability to generate these ac currents differentiates a nonstationary dynamical scatterer from a merely "frozen" scatterer.

VII. DISCUSSION

We have investigated the nonstationary adiabatic charge transport through a time-dependent mesoscopic scatterer coupled to reservoirs subject to oscillating voltages. The external voltages applied to the reservoirs induce ac currents flowing through the scatterer. In addition the oscillating scatterer itself is a source of ac currents flowing between the reservoirs. In general these two types of currents interfere with themselves. This gives rise to renormalization of the rectification (i.e., proportional to the potential difference) contribution to the dc current and gives rise to a renormalization of the quantum pump current.

To analyze this interference effect we calculated the Floquet scattering matrix beyond the adiabatic approximation. We investigated the first order in ω corrections to the (adiabatic) scattering matrix and found that the dc currents of both the zeroth and the first order in ω can be expressed in terms of a stationary scattering matrix with time-dependent parameters. Within this approximation, within a non-interacting theory,

The operators \hat{b}_α and \hat{a}_α are defined in Eqs.(8) and (9), respectively. The calculations analogous to those leading to Eq.(33) give us $I_{\alpha,l}$. Performing the inverse Fourier transformation Eq.(14b) we finally get the time-dependent current $I_\alpha(t)$ flowing in the system as follows:

the oscillating potentials $V_\alpha(t)$ of reservoirs can be accounted for by allowing the energy E of incident particles to follow adiabatically the reservoir potential: $E \rightarrow \mathcal{E} = E + eV_\alpha(t)$.

We emphasize the importance of the phases of all the cyclically evolving quantities (the potentials of reservoirs and the parameters of a scatterer) for generating a dc current. In particular even when all the reservoirs have the same oscillating potential $V_\alpha(t) = V(t)$ and the rectification effect is ineffective the dc currents at $V = 0$ and at $V \neq 0$ can nevertheless differ significantly.

The analysis allows us to perform a current partition that clarifies the physical meaning of the (diagonal elements of the) quantity $\mathcal{P}\{\hat{S}_0^\dagger; \hat{S}_0\}$ (the Floquet corrections to the "frozen scattering" matrix) and show that they correspond to spectral current densities generated by a dynamic scatterer. The instantaneous current contains a contribution from such self-generated ac currents in addition to the currents from the "frozen" charge and the ac currents generated by the external potentials.

We emphasize that the results presented in this work, the effect of external ac-potentials on a quantum pump, are of importance whenever the pump is not part of an ideal zero-impedance external circuit. In particular, if the pump is in series with a resistance used to measure the voltage generated by the pump, or if the circuit is a multiterminal circuit with probes used to measure voltages, the results presented here will be needed.

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APPENDIX

1. The matrix \hat{A}

The matrix \hat{A} defines the first order in frequency corrections to the adiabatic Floquet scattering matrix Eqs.(17), (20). It is a matrix antisymmetric in lead indexes.

To obtain Eq.(21) we substitute the adiabatic expansion Eq.(17a) in the current conservation condition Eq.(16a).

Keeping terms of order ω^0 and ω^1 we get the following:

$$\begin{aligned}
& \sum_{\alpha} \sum_{n=-\infty}^{\infty} S_{F,\alpha\beta}^*(E_n, E) S_{F,\alpha\gamma}(E_n, E_m) \\
& \approx \sum_{\alpha} \sum_n \left(S_{0,\alpha\beta,n}^*(E) + \hbar\omega \frac{n}{2} \frac{\partial S_{0,\alpha\beta,n}^*(E)}{\partial E} + \hbar\omega A_{\alpha\beta,n}^*(E) \right) \\
& \times \left(S_{0,\alpha\gamma,n-m}(E) + \hbar\omega \frac{n+m}{2} \frac{\partial S_{0,\alpha\gamma,n-m}(E)}{\partial E} + \hbar\omega A_{\alpha\gamma,n-m}(E) \right) \\
& \approx \sum_{\alpha} \sum_n S_{0,\alpha\beta,n}^*(E) S_{0,\alpha\gamma,n-m}(E) \\
& + \sum_{\alpha} \sum_n S_{0,\alpha\beta,n}^*(E) \left(\hbar\omega \frac{n+m}{2} \frac{\partial S_{0,\alpha\gamma,n-m}(E)}{\partial E} + \hbar\omega A_{\alpha\gamma,n-m}(E) \right) \\
& + \sum_{\alpha} \sum_n \left(\hbar\omega \frac{n}{2} \frac{\partial S_{0,\alpha\beta,n}^*(E)}{\partial E} + \hbar\omega A_{\alpha\beta,n}^*(E) \right) S_{0,\alpha\gamma,n-m}(E) \\
& = \delta_{m0} \delta_{\beta\gamma}.
\end{aligned}$$

Applying the inverse Fourier transformation Eq.(14b) and introducing corresponding matrixes we rewrite above equation as follows:

$$\begin{aligned}
& \left(|\hat{S}_0(E, t)|^2 + \hbar\omega \hat{S}_0^\dagger(E, t) \hat{A}(E, t) + \hbar\omega \hat{A}^\dagger(E, t) \hat{S}_0(E, t) \right. \\
& \left. + \frac{i\hbar}{2} \hat{S}_0^\dagger \frac{\partial^2 \hat{S}_0}{\partial E \partial t} - i\hbar \frac{\partial}{\partial t} \left[\hat{S}_0^\dagger \frac{\partial \hat{S}_0}{\partial E} \right] - \frac{i\hbar}{2} \frac{\partial^2 \hat{S}_0^\dagger}{\partial E \partial t} \hat{S}_0 \right)_{\beta\gamma, -m} = \delta_{m0} \delta_{\beta\gamma}.
\end{aligned} \tag{A.1}$$

To simplify further this equation we use the unitarity condition for the frozen scattering matrix: $\hat{S}_0(E, t) \hat{S}_0^\dagger(E, t) = \hat{I}$. First, from this condition it follows that

$$\left(|\hat{S}_0(E, t)|^2 \right)_{\beta\gamma, -m} = \delta_{m0} \delta_{\beta\gamma}. \tag{A.2}$$

And second, we can write $\frac{\partial^2}{\partial E \partial t} \left[\hat{S}_0^\dagger \hat{S}_0 \right] = 0$ and, correspondingly,

$$-\frac{\partial^2 \hat{S}_0^\dagger}{\partial E \partial t} \hat{S}_0 - \hat{S}_0^\dagger \frac{\partial^2 \hat{S}_0}{\partial E \partial t} = \frac{\partial \hat{S}_0^\dagger}{\partial E} \frac{\partial \hat{S}_0}{\partial t} + \frac{\partial \hat{S}_0^\dagger}{\partial t} \frac{\partial \hat{S}_0}{\partial E}. \tag{A.3}$$

Substituting Eqs.(A.2) and (A.3) in Eq.(A.1) we arrive at the Eq.(21).

2. The commutator matrix \mathcal{P}

The matrix $\mathcal{P}\{\hat{S}_0^\dagger; \hat{S}_0\}$ defined in Eq.(21b) is self adjoint

$$\mathcal{P}\{\hat{S}_0^\dagger; \hat{S}_0\} = \mathcal{P}^\dagger\{\hat{S}_0^\dagger; \hat{S}_0\}, \tag{A.4}$$

and traceless

$$Tr[\mathcal{P}\{\hat{S}_0^\dagger; \hat{S}_0\}] = 0. \tag{A.5}$$

To demonstrate the latter property we use the equality $d[\hat{S}] \hat{S}^\dagger = -\hat{S} d[\hat{S}^\dagger]$ following from the unitarity of the scattering matrix $\hat{S} \hat{S}^\dagger = \hat{I}$ and the invariance of trace to the cyclic rearrangements of the matrices. As a result from Eq.(21b) we get

$$\begin{aligned}
Tr[\mathcal{P}] &= i\hbar Tr \left[\frac{\partial \hat{S}_0^\dagger}{\partial t} \frac{\partial \hat{S}_0}{\partial E} - \frac{\partial \hat{S}_0^\dagger}{\partial E} \hat{S} \hat{S}^\dagger \frac{\partial \hat{S}_0}{\partial t} \right] \\
&= i\hbar Tr \left[\frac{\partial \hat{S}_0^\dagger}{\partial t} \frac{\partial \hat{S}_0}{\partial E} - \hat{S}^\dagger \frac{\partial \hat{S}_0}{\partial E} \frac{\partial \hat{S}_0^\dagger}{\partial t} \hat{S} \right] \\
&= i\hbar Tr \left[\frac{\partial \hat{S}_0^\dagger}{\partial t} \frac{\partial \hat{S}_0}{\partial E} - \frac{\partial \hat{S}_0^\dagger}{\partial t} \frac{\partial \hat{S}_0}{\partial E} \right] = 0.
\end{aligned}$$

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