

Features of level broadening in a ring-stub system

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When a one dimensional (1D) ring-stub system is coupled to an electron reservoir, the states acquire a width (or broadening characterized by poles in the complex energy plane) due to finite life time effects. We show that this broadening is limited by anti-resonances due to the stub. The differences in level broadening in presence and absence of anti-resonance is exemplified by comparison to a 1D ring coupled to an infinite reservoir. We also show that the anti-resonances due to the stub has an anchoring effect on the poles when a magnetic flux through the ring is varied. This will have implication on change in distribution of the poles in disordered multichannel situation as magnetic flux is varied.

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Dephasing in mesoscopic systems is of current research interest and there are various models for dephasing [1] in situations where there is a spatial separation between elastic and inelastic processes. As such it is a difficult problem to model dissipation theoretically because dissipative systems are open systems which are in general more difficult to deal with than closed systems. Real systems are on the other hand most of the time dissipative and it is extremely difficult to obtain a non-dissipative sample in the laboratory. A pioneering idea of Landauer offered a very simple approach to dissipation in electronic systems when he showed that an electron reservoir can introduce some basic features of dissipation like time irreversibility and resistance [2]. This idea evoked a lot of research and resulted in the Landauer-Büttiker conductance formula which is now experimentally realized to be correct [3]. The idea was later extended by Büttiker [4] to introduce level broadening in a ring (or any finite system) penetrated by magnetic flux and there is at least one experiment which verifies the magnitude of persistent current in such a open system [5]. Some more precise testing ground for persistent currents in open systems could be the experimental observation of its directional dependence i.e., if a persistent current loop is coupled to a DC current carrying quantum wire then the magnitude of persistent current will depend on the direction of the DC current [6]. It is to be noted that noise or the DC current itself does not depend on directions. It was also shown that a quantum current magnification effect is possible with a DC bias voltage [7] or with a temperature difference [8] within this theoretical frame work when one can have large circulating currents in the ring in the absence of an Aharonov-Bohm flux.

In the present work, we apply this model for level broadening to study a simple system that consists of a

one dimensional (1D) ring coupled to a 1D side branch (or stub). This elementary system has received some attention in the past where it was shown that one can have persistent currents without parity effect [9], coulomb blockade [10], bistability [11], etc. Parity effect of persistent currents in single channel rings mean consecutive states have opposite slopes [12], whereas in the ring-stub system there are two kinds of eigenstates: parity conserving and parity violating [9]. There are l/u number of consecutive states forming a group with the same slope after which there are l/u consecutive states with slope opposite to the those in the first group [9]. Here l is the length of the stub and u is the length of the ring. Two consecutive states with same slope are called parity violating pair whereas two consecutive states with opposite slope are called parity conserving pair.

When the side branch is adjusted one can have situations when the length of the ring is smaller than the phase coherence length l_ϕ while the length of the side branch is smaller, comparable or greater than l_ϕ . When the side branch is much larger than l_ϕ then the situation is the same as a ring coupled to a reservoir. One can also apply the known mechanism of level broadening of a ring-stub system by coupling to a reservoir which is completely physical as long as the stub length is smaller than l_ϕ but can be much larger than the ring length. We show that in this regime the presence of anti-resonances that always occur in quantum wires [13] has drastic effects on level broadening. Only in the absence of such anti-resonances the persistent current in a ring coupled to a large stub with some level broadening is similar to persistent current in a ring coupled to a reservoir. These anti-resonances will also affect the transition between different universality classes in Random Matrix Theory in disordered multichannel situations.

The ring-stub system coupled to a reservoir is shown in Fig. 1. A flux ϕ penetrates the ring. An electron reservoir can be attached in three ways as shown in (a), (b) and (c). In (b) and (c) there are two junctions in the set up. One is the reservoir-system junction (X) and the other is the ring-stub junction (Y). In (a) the two junctions are merged into one and there is only a single junction (Z). While (a) is a ring-stub system coupled to an electron reservoir, (b) and (c) will have some additional features due to multiple scattering and resonance between points X and Y. However, (b) and (c) will behave similarly to each other which leads us to exclude the situation in (c). In (b) the length of the ring is u , the distance between X and Y is v and the distance between X and the dead end of the stub is w . Hence length of the stub is $l = v + w$. u

is taken to be the unit of length and we will mention the other lengths in numbers without mentioning the units. When $v \rightarrow 0$ the the system in (b) continuously goes over to the system in (a). Free particle quantum mechanical wave function can be written down in different regions of the system in (b) and can be matched at the junctions using the Griffith boundary conditions [14] or the three way splitter S-matrix where ϵ determines the strength of coupling [4]. Analytical expression for the persistent current dI in an wave vector interval dk is given below using the S matrix approach for the system in Fig. 1(b).

$$dI/dk = \frac{e\hbar k}{2\pi m} (-1)^{\epsilon_2 \epsilon_1} \sin(\alpha) \sin(ku) \sin^2(kw)/D$$

$$D = 4a_2^2 \sin^2(kw)A^2 + b_2^2 B^2$$

$$A = b_1 \sin(kv)[\cos(\alpha) - \cos(ku)] + a_1 \sin(ku) \cos(kv)$$

$$B = b_1 \sin(kl)[\cos(\alpha) - \cos(ku)] + a_1 \sin(ku) \cos(kl). \quad (1)$$

Where

$$a_i = \frac{1}{2}(\sqrt{1 - 2\epsilon_i} - 1), \quad b_i = \frac{1}{2}(\sqrt{1 - 2\epsilon_i} + 1) \quad (2)$$

$i = 1$ for the ring-stub junction and 2 for the reservoir-stub junction. Here $\alpha = 2\pi\phi/\phi_0$, ϕ_0 being the flux quantum.

There can be two kinds of processes in the junctions that can lead to reflection of an electron waveguide. First is due to diffraction as the wave front splits up at the junction which disappears for $\epsilon = 0.5$. For smaller values of ϵ this contribution to reflection is always there. $\epsilon = 4/9$ corresponds to Griffith boundary conditions exactly for a free junction. The second is the reflection due to the weak coupling or a potential scatterer at the junction that leads to weak coupling. However, there is a third way of getting a reflection at the junction (X) in (b) and that is due to an interference effect that produces an anti-resonance. Such anti-resonances occur very generally in a quantum wire of finite width due to evanescent modes [13] that can be mapped exactly into the 1D stub model [13]. An electron coming from the reservoir on reaching junction X can go towards the ring or can go towards the dead end of the stub, get reflected back and then go towards the ring. Interference between these two paths can be constructive or destructive depending on the wave vector and can lead to reflection from the junction X. We are here discussing first order reflection from the junction X which determines the strength of coupling between the reservoir and system. Besides this there is always second order reflection (reflection from other junctions in the system partly flow out of the junction X towards the reservoir) that makes the total current in the lead to be always zero.

When the state of the ring-stub system is broadened by the reservoir, each broadened state will have a pole in the reflection amplitude at junction X that behave similarly as the eigen energies as the flux is varied and the persistent current at the broadened areas can be described using the on shell scattering matrix [15]. In Fig. 2 we plot the persistent current in an infinitesimal energy range ($\frac{2\pi\hbar}{e} \frac{dI}{dE}$) versus incident wave vector ku for this system for two different flux values. The solid curve is for $u=1, v=0.01, w=9.99, \alpha = 2\pi\phi/\phi_0=0.1$ and for Griffith boundary conditions at the free junctions. Here ϕ is the flux through the ring and ϕ_0 is the flux quantum. The dotted curve is for $\alpha = 1.5$ with other parameters remaining the same. We have only plotted up to $ku=2\pi$ because at higher energies the curve repeats itself qualitatively. It can be noted that in the solid curve there are 10 diamagnetic peaks (l/u being 10) consecutively, followed by 10 paramagnetic peaks. As the flux is increased we get the dotted curve in which the diamagnetic peaks shift to higher energy and paramagnetic peaks shift to lower energy compared to the solid curve. Diamagnetic states (broadened by the coupling to the reservoir) group together because of a discontinuous phase change as Fermi energy crosses the zero [16] (or anti-resonance) in the persistent current between each broadened peak which arises because of total first order reflection at X due to the interference effect discussed above. Level broadening in this case is limited by the presence of zeroes (or anti-resonances) and the peaks do not overlap with each other. As a result when the stub is made very long, the persistent current in the ring coupled to a stub does not bear any similarity with that of a ring coupled to a reservoir. This is shown in Fig. 3 where dotted curve is for $u=1, v=0.01, w=99.99$ and $\alpha=1.0$ for Griffith boundary conditions. The thick curve is the persistent current in a ring of length u coupled to an infinite reservoir for $\alpha = 1.0$. The two curves have no similarity at all. Also this is a very simple example that show the zeroes anchor the poles and the poles cannot move freely as the magnetic field is varied. The magnetic field has no effect on the zeroes because the zeroes are determined by the localized states of the stub. This anchoring effect of the zeroes on the poles can drastically change the distribution of the poles in disordered systems like that considered in Ref. [17] i.e., a disordered multichannel ring threaded by a magnetic flux. The magnetic flux is known to make the eigen energies rigid which in turn leads to a transition between different universality classes of level statistics in Random Matrix Theory. The anchoring effect of the zeroes, that will be unaffected by the magnetic flux, will substantially add color to this rigidity phenomenon.

The effect of the anti-resonances on transport currents has been studied to some extent [18]. Here we have shown its effects on persistent currents. Transport currents or transmission coefficient being independent of magnetic flux and bounded by unity does not exhibit the drastic effects shown in Figs. 2 and 3. To exemplify this further let us study a situation when there are no anti-resonances.

The features of level broadening and level statistics when a finite size system is coupled to a reservoir is studied in Ref. [19] when there are no anti-resonances. So the features that we will obtain in the following are in accordance with the theory developed in Ref. [19] but completely different from the situation in Figs. 2 and 3. Essentially in the following we will get a situation that can continuously go over to a ring-reservoir system when the stub length becomes large.

The anti-resonances can be removed by a different boundary condition at the dead end of the stub [20] instead of a hard wall boundary condition as used in this work or by a magnetic field in the stub region if the stub is quasi-one dimensional etc.,. In order to show this we use a simple trick to remove the first order total reflection at the junction X. We make a special choice of parameters that is $u=1$, $v=9.99$ and $w=0.01$ for Griffith boundary conditions. In this case the first order total reflection occur at $ku=0, 100\pi, 200\pi$ and so on. At $ku=50\pi, 150\pi, 250\pi$ etc, the first order reflection at X is zero. Hence in an energy regime like $ku=20\pi$ to 22π the first order reflection coefficient at X is approximately 0.6 and almost independent of energy. So in this energy window of 2π the ring-stub system is weakly coupled to a reservoir for such parameters. In Fig. 4 the thin solid curve is the persistent current versus ku for $\alpha=1.5$ and the thick solid curve is that for $\alpha=0.1$. The broadening is already enough to make the resonances overlap with each other. In the absence of the anchoring effect on the resonances, some resonances can shift a lot with the magnetic field as compared to that in Fig. 2. The dashed curve and the dotted curve are the persistent current in a ring coupled to an infinite reservoir at $\alpha=1.5$ and 0.1 , respectively. Keeping all parameters same as in Fig. 4 we plot the same things in Fig. 5 in a different energy window (40π to 42π) where the first order reflection at the junction X is 0.18. Hence this is a situation of strong coupling between the system and the reservoir and the peaks have broadened further compared to Fig. 4. Curve conventions are the same as in Fig. 4. Now keeping the ring length ($u=1$) to be the same we make the stub very long i.e. $v=99.99$ and $w=0.01$ and plot persistent current versus ku in the same energy interval as that in Fig. 5, for a magnetic field that gives $\alpha=1.0$ in Fig. 6. Here we have just interchanged the values of v and w as compared to that in Fig. 3 and we have exactly the same number of poles as that in Fig. 3. Persistent current of a ring of length $u=1$ connected to an infinite reservoir is shown by the solid curve.

From Figs. 4, 5 and 6 it is seen that as the length of the stub is made longer and broadening of the states of the ring-stub system is made larger, the persistent current is rapidly oscillating around a mean value which turns out to be the persistent current in a ring coupled to an infinite reservoir. The rapid oscillations are resonance effects that are again due to the fact that there are no inelastic processes in the stub. A finite number of inelastic processes (which will arise in a situation when the

stub length is comparable to l_ϕ) will smear out these oscillations and then the situation in Fig. 6 will be similar to that of a ring coupled to a reservoir. But the situation in Fig. 3 is different because of the limiting effects of the anti-resonances. Most of the large current carrying peaks occur around a region where the thick solid curve is very small. Inelastic processes are expected to affect the different peaks equally and for a finite number of inelastic processes a difference between the two situations in Fig. 3 and 6 will continue to exist. However, when the stub length becomes much larger than l_ϕ then there will be a saturation of the broadening effect [4] it produces on the resonances. Once this saturation effect sets in, the broadening will affect the larger resonances more than the smaller ones. Such situations cannot be studied in this model. Finally in both cases one will get the situation when the stub becomes a part of the reservoir and the persistent current in the ring will be that of a ring coupled to a reservoir. One can design similar problems with transport current across the stub but as mentioned before, the transmission coefficient being independent of flux and bounded by unity, the features are not so prominent. We hope that further work on this model will help us to understand how a finite number of inelastic process in a spatially separated region compete with elastic processes (like interference and resonance) and affect persistent currents that show similar scaling behavior as transport currents.

We have therefore shown that Fig. 1(b) gives us a situation where we can easily switch off or on the anti-resonances and also strongly or weakly couple a reservoir and thus study the exact effects produced by anti-resonances. The presence or absence of anti-resonances are however an universal feature of finite width quantum wires.

In summary, we have shown that anti-resonances can drastically limit the broadening of eigen energies by an electron reservoir and as a result, when the stub length is made large the system bears no resemblance to a ring-reservoir system. In absence of the anti-resonances, the system can continuously go over to a ring-reservoir system as the stub length is made large. Although this is strictly valid when there is a complete spatial separation of inelastic and elastic processes, the two situation demonstrated in Fig. 3 and 6 are so dramatically different that some of this effect will survive even in presence of a finite number of inelastic processes in the stub. This effect may be relevant in the dephasing effects observed in the experiment of Ref. [21] where the zeroes were also observed. We have also shown that the anti-resonances has an anchoring effect on the poles that can modify the distribution of poles in situations similar to that considered in Ref. [17].

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curves, respectively, which are persistent currents in the same ring coupled to an infinite reservoir.

Fig. 5 Same as in Fig. 4 but the reservoir is coupled strongly.

Fig. 6 Same as in Fig. 5 but stub length is made 100 times the ring length.

Figure captions

Fig. 1 Three ways of attaching a reservoir to a system of a ring coupled to a stub.

Fig. 2 Persistent current versus incident energy at two different fields in a ring coupled to a 10 times longer stub. The reservoir is attached close to the ring-stub junction.

Fig. 3 Persistent current versus incident energy in a ring coupled to a 100 times longer stub (dotted curve). The reservoir is attached close to the ring-stub junction. Thick solid curve gives persistent current in the ring if it were attached to an infinite reservoir.

Fig. 4 Persistent current versus incident energy at two different fields in a ring coupled to a 10 times longer stub (thick and thin solid curves). The reservoir is attached weakly close to the dead end of the stub. Thick and thin solid curves show oscillations about dotted and dashed











