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Heat transport through a quantum dot with one-dimensional interacting leads under Coulomb blockade regime

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Abstract. The peculiarities of a low temperature heat transfer through a ballistic quantum dot (a double potential barrier) with interacting leads due to a long-range Coulomb interaction (in the geometrical capacitance approach) are considered. It is found that the thermal conductance K shows periodic peaks as a function of the electrostatic potential of a dot at low temperatures. At the peak maximum it is $K \sim T$ whereas near the minimum it is $K \sim T^3$. Near the peak maximum the dependence $K(T)$ is essentially nonmonotonic at the temperatures correspondent to the level spacing in the quantum dot.

PACS. 72.10.-d Theory of electronic transport; scattering mechanisms – 73.20.Mf Collective excitations (including plasmons and other charge-density excitations) – 73.40.Gk Tunneling

1 Introduction

At low temperatures the charging energy affects considerably the electron transport in the mesoscopic systems connected to an environment by tunnel junctions [1–4]. An electron tunneling through the potential barrier changes the charge of a mesoscopic sample by $1e$ that changes the energy of the system by $E_\mathrm{c} = e^2/(2C)$, where e is an electron charge; C is the geometrical capacitance of a sample. Note that for the mesoscopic system the typical capacitance is $C \leq 10^{-15}$ F, that corresponds to $E_\mathrm{c} \geq 1$ K. At low temperatures

$$T \ll E_\mathrm{c}, \tag{1}$$

in the common case the charge transfer is suppressed (the Coulomb blockade effect) [5–10]. However, at some values of a sample potential V_g the electrostatic energy of the system $E = (q - eN)^2/(2C)$ (where q is the charge of the sample; $N = CV_g/e$) is degenerate in q ($q \leftrightarrow q + e$) that occurs at half-integer values of N [7,11]. In this case the Coulomb blockade is lifted and the conductance of the system increases.

The same effect must be shown by other transport coefficients, in particular, the thermal conductance K which is considered in the present paper. In the case of a one-dimensional thermal transport the main consequence of the Coulomb blockade effect consists in the considerable contribution of (neutral) electron-hole pairs (plasmons) into the heat transfer. This is due to as follows. In reference [12] it was shown for the strong cotunneling regime that tunneling of spinless noninteracting ($g = 1$) electrons through a double barrier (a quantum dot) in the Coulomb blockade regime can be considered as tunneling of interacting electrons described as a Luttinger liquid [13–16] with $g^* = g/2 = 1/2$ (where $g(g^*)$ is Haldane's parameter) through a single barrier [17]. The thermal transport in a Luttinger liquid with a single backscattered impurity was considered in reference [18]. Where it was shown that at low temperatures the heat is carried by electrons as well as by plasmons (electron-hole pairs). The electron contribution (*i.e.*, due to tunneling of electrons) to the thermal conductance is $K_\mathrm{e} \sim T^{\frac{2}{g}-1}$ whereas the plasmon contribution is $K_\mathrm{p} \sim T^3$. At $g = 1/2$ these contributions are of the same order, but at $g < 1/2$ the plasmon contribution dominates at low temperatures. Thus, if the noninteracting electrons tunnel through a double barrier the plasmon and electron contributions to the heat transfer are the same, whereas if the repulsively interacting electrons tunnel through a double barrier (that may be described as tunneling of a Luttinger liquid with $g^* = g/2 < 1/2$ through a single barrier) the plasmon contribution to the thermal conductance will dominate. These circumstances allow us to develop the theory of a thermal transport based on the self-consistent harmonic approximation (SCHA) [19,20] which, in fact, describes the transport of plasmons.

In the present paper we concentrate on the peculiarities of thermal transport in a one-dimensional Luttinger liquid with a double potential barrier (a quantum dot) which are due to the Coulomb blockade effect. Note, that in the realistic case the mesoscopic system (a quantum dot with one-dimensional Luttinger liquid leads) is connected to reservoirs of noninteracting electrons that can significantly modify the transport as electrical [21] as thermal [22–24]. Strictly speaking, it is necessary to take into account such connection. However, to clarify the effect

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of Coulomb blockade on the thermal transport we will follow reference [18] and consider infinite one-dimensional leads (more precisely, we consider an adiabatic (as for electrons as for plasmons) connection between a quantum dot and reservoirs and assume the same electrons in the mesoscopic system and in reservoirs).

We have calculated the thermal conductance K of a quantum dot as a function of both the temperature T and the potential V_g of a dot. The dependence $K(V_g)$ consists of a series of Coulomb blockade peaks with a temperature dependent shape. The dependence $K(T)$ is essentially nonmonotonic when the temperature is of the order of the level spacing in the quantum dot.

We note, that the study of a thermal transport may give evidence for the non-Fermi-liquid behaviour of electron systems [22–24]. On the other hand, the heating effects influence on the behaviour of mesoscopic systems which show the Coulomb blockade effect (see *e.g.*, [25]) that requires of a detailed study of the heat transport in such systems.

2 Model and basic equations

As a model we consider a one-dimensional ballistic channel (lead) containing spinless electrons. Two point barriers with a strength V_1 and V_2 located at $x_1 = -d/2$ and $x_2 = d/2$ model a quantum dot (QD). The potential difference V_g between the QD and the lead may be changed by the gate. The left- ($x < x_1$) and right-hand ($x > x_2$) parts of a lead couple the QD to electron reservoirs with the temperature T_1 and T_2 and with the same electrochemical potentials $\mu_1 = \mu_2 = \mu$. We ignore any inelastic processes in the system (the QD plus one-dimensional leads) and consider the QD as a purely elastic scatterer. This approach is valid at enough low temperatures when the electron phase breaking length $L_\phi(T)$ far exceeds the size of a mesoscopic system. The heat transport (like the charge transport) is considered as the transport between the electron reservoirs (which are far from the QD) where the inelastic processes occur. Such an approach is usual for the mesoscopic physics [26,27]. The thermal conductance we will calculate in the linear regime when the temperature difference is small $\Delta T = |T_1 - T_2| \ll T$.

Such an extreme simple model, nevertheless, allows to take into account the effect of a charging energy as well as the effect of spatial quantization within the quantum dot [28] on the heat transport at low temperatures. Note, that such a model [29] corresponds to the experimental configuration when the QD is connected to electron reservoirs through the adiabatic one-mode quantum point contacts. The transmission coefficients of such contacts can be varied from zero to unit. We assume that at low energies $\Delta\epsilon \sim T \ll \mu$ the potential barrier strengths V_1 and V_2 do not depend on the energy. So, we can put $V_i = \mu\pi^{-1} r_i/t_i$ ($i = 1,2$) [12], where r_i and t_i are the modulus of the reflection amplitude and transmission amplitude, respectively, which describe tunneling of electrons with the Fermi energy through the potential barrier V_i. According to the Landauer-Büttiker approach [30], the electrical conductance of a single barrier for the case of noninteracting ($g = 1$) spinless electrons is $G_i = G_0 t_i^2$, where $G_0 = e^2/(2\pi\hbar)$ is the conductance quantum. Whereas, the interelectron interactions ($g \neq 1$) renormalize the potential strength that makes the conductance temperature dependent [17].

Interesting in the low temperature ($T \ll E_\mathrm{c} \ll \mu$) behaviour of our system we can linearize the electron spectrum near the Fermi energy μ and describe one-dimensional interacting electrons as a Luttinger liquid. The low-energy physics of a Luttinger liquid is well described *via* the method of bozonization [14,15] which allows to account the charging energy (in the geometrical capacitance C approach) exactly. In this method the spinless fermion (electron) field operator is expressed in terms of a bosonic (phase) field $\theta(x,t)$.

Using the standard calculations and integrating out the fluctuations in θ for all x excepting x_1 and x_2 we can obtain an effective action S_eff [28] for our system as follows

$$
\begin{aligned}
S_\mathrm{eff}[\theta,\phi] = \frac{\hbar}{g\beta}\sum_{n=-\infty}^{\infty}\Bigg\{ & \frac{2|\omega_n|}{1+\mathrm{e}^{-|\omega_n|/\Delta\omega}}|\theta_n|^2 \\
& + \frac{|\omega_n|}{2(1-\mathrm{e}^{-|\omega_n|/\Delta\omega})}|\phi_n|^2\Bigg\} \\
& + \int_0^\beta \mathrm{d}\tau\,\Big\{V_1\cos(\pi^{1/2}[2\theta(\tau)-\phi(\tau)]-k_\mathrm{F}d) \\
& + V_2\cos(\pi^{1/2}[2\theta(\tau)+\phi(\tau)]+k_\mathrm{F}d)\Big\} \\
& + \frac{E_\mathrm{c}}{\pi}\int_0^\beta\Big\{\phi(\tau)-\pi^{1/2}N\Big\}^2 . \qquad (2)
\end{aligned}
$$

Here $\tau = \mathrm{i}t$ is an imaginary time; $\beta = 1/T$; $\omega_n = 2\pi n/\beta$ is the Matsubara frequency (n is an integer); $\Delta\omega = v/d$; v and g are Haldane's parameters [16] (for noninteracting electrons $g = 1$; $v = v_\mathrm{F} = \pi\hbar\rho_0/m^*$, where v_F is the Fermi velocity, ρ_0 is the mean density of electrons, m^* is the effective electron mass); $k_\mathrm{F} = \pi\rho_0$ is the Fermi wave number; $\theta(\tau)=(\theta(x_2,\tau)+\theta(x_1,\tau))/2$; $\phi(\tau)=\theta(x_2,\tau)$-$\theta(x_1,\tau)$; θ_n and ϕ_n are the Fourier coefficients: $x(\tau)=\frac{1}{\beta}\sum_n \mathrm{e}^{-\mathrm{i}\omega_n\tau}x_n+\zeta_x$, ($x \equiv \theta,\phi; \zeta_x$ is the zero mode). The last term in equation (2) decries the charging energy of a system. Note that the field $\phi(\tau)$ is related to the excess charge of a quantum dot $\delta q = e\pi^{-1/2}\phi$ [17,28].

Now we obtain the approximate expression for S_eff describing the low-amplitude oscillations of a bosonic field $\theta(\tau)$. To this end we first assume that $V_1, V_2 \to 0$ and average the scattering terms over the fluctuations of a quantum dot charge (over the field ϕ) [12]. At $T \ll E_\mathrm{c}$

equation (1) we obtain

$$S'_{\mathrm{eff}} = \frac{\hbar}{g\beta}\sum_n \left\{ \frac{2|\omega_n|}{1+\mathrm{e}^{-|\omega_n|/\Delta\omega}}|\theta_n|^2 + \left(\frac{|\omega_n|}{2(1-\mathrm{e}^{-|\omega_n|/\Delta\omega})} + \frac{gE_{\mathrm{c}}}{\pi\hbar}\right)|\phi_n|^2 \right\} + V\int_0^\beta \mathrm{d}\tau \cos(2\pi^{1/2}\theta(\tau)), \quad (3)$$

where

$$V = \left(\frac{2g\gamma E_{\mathrm{c}}}{\pi\mu}\right)^{g/2} \left(V_1^2 + V_2^2 + 2V_1V_2\cos(2\pi N + 2k_{\mathrm{F}}d)\right)^{1/2}. \quad (4)$$

Here $\gamma = \mathrm{e}^C$ with $C \approx 0.5772$ being the Euler's constant. Accounting only low-amplitude fluctuations of θ we can write $\cos(2\pi^{1/2}\theta) \approx 1 - 2\pi\theta^2$. In addition we must renormalize the potential $V \to V^*$ [19,20] integrating out the modes θ_n with the energy $\hbar\omega_n$ higher than V^*. This is possible because in the main approximation such fluctuations are not affected by the potential V^* and, therefore, they are fluctuations of a free field. The final expression for the effective action in the self-consistent harmonic approximation is

$$S_{\mathrm{SCHA}} = \frac{\hbar}{g\beta}\sum_n \left\{ \left(\frac{2|\omega_n|}{1+\mathrm{e}^{-|\omega_n|/\Delta\omega}} - \omega_V\right)|\theta_n|^2 + \left(\frac{2|\omega_n|}{1-e^{-|\omega_n|/\Delta\omega}} + \omega_c\right)\frac{|\phi_n|^2}{4} \right\}. \quad (5)$$

Here $\omega_{\mathrm{c}} = 4gE_{\mathrm{c}}/(\pi\hbar)$ and

$$\hbar\omega_V = 2\pi g V (V/\mu)^{g/(2-g)}, \quad (6)$$

where we use the Fermi energy μ as a high-frequency cutoff. Below we apply this action, equation (5) for describing the low-temperature heat transfer through a quantum dot. We assume that the results will be qualitatively correct at arbitrary strength of the potential barriers V_1 and V_2. This approximation describes the dynamics of plasmons whose contribution into the heat transfer dominates in the case of repulsively ($g < 1$) interacting electrons.

In the linear regime the thermal transport is characterized by the thermal conductance $K = -Q/\Delta T$, where Q is the heat current between the reservoirs and $\Delta T = T_2 - T_1$, $|\Delta T| \ll T$ [27]. The thermal conductance may be expressed through the ac electrical conductance $G(\omega)$ [18]

$$K = \frac{\hbar^3}{4ge^2T^2}\int_0^\infty \mathrm{d}\omega \frac{\omega^2 \mathrm{Re}\{G(\omega)\}}{\sinh^2(\hbar\omega/(2T))}, \quad (7)$$

where $\mathrm{Re}\{x\}$ is the real part of x.

To find $G(\omega)$ in the model equation (5) we will use a Kubo formula [31,32]

$$G(\omega) = \frac{-\mathrm{i}}{\hbar\omega}\lim_{\omega_n \to -\mathrm{i}\omega}\int_0^\beta \mathrm{d}\tau \mathrm{e}^{\mathrm{i}\omega_n\tau}\langle I(\tau)I(0)\rangle. \quad (8)$$

In above expression we will use $\langle X \rangle = Z^{-1}\int \mathrm{D}\theta\mathrm{D}\phi X \mathrm{e}^{-S_{\mathrm{SCHA}}/\hbar}$, where Z is the partition function. Expanding the current into a Fourier series and accounting the quadratic nature of action equation (5) we obtain $G(\omega) = -\mathrm{i}/(\hbar\omega\beta)\lim_{\omega_n \to -i\omega}\Phi_n$, where $\Phi_n = \langle I_n I_{-n}\rangle$. Using the current definition (at point $x = x_2$) $I = \mathrm{i}e\pi^{-1/2}\partial\theta(x_2,\tau)/\partial\tau$ [17] (where $\theta(x_2,\tau) = \theta(\tau) + \phi(\tau)/2$), we get $I_n = \frac{-e\omega_n}{\beta\pi^{1/2}}(\theta_n + \phi_n/2)$. Straightforward calculations give

$$\mathrm{Re}\{G(\omega)\} = gG_0 \times \left\{ \frac{\omega^2(1+\cos(\omega/\Delta\omega))}{\omega_V^2(1+\cos(\omega/\Delta\omega)) + 2\omega_V\omega\sin(\omega/\Delta\omega) + 2\omega^2} + \frac{\omega^2(1-\cos(\omega/\Delta\omega))}{\omega_{\mathrm{c}}^2(1-\cos(\omega/\Delta\omega)) + 2\omega_{\mathrm{c}}\omega\sin(\omega/\Delta\omega) + 2\omega^2} \right\}. \quad (9)$$

Substituting equation (9) into equation (7) we obtain the thermal conductance of a quantum dot within the SCHA with respect to the Coulomb blockade effect ($T, \hbar\omega \ll E_{\mathrm{c}}$). The analysis of the dependence $K(T,\omega_V)$ is presented in the following parts.

Notice that the obtained expression $G(\omega)$ describes the plasmon contribution to the electrical conductance of a double barrier (the case of a single barrier was considered in Ref. [19]). Because the plasmons (*i.e.*, the low-amplitude oscillations of a bosonic field) are neutral, this contribution vanishes in the limit $\omega \to 0$. In the used formalism the charge is carried by topological excitations of the bosonic (phase) field [16]. The change of θ by $\pi^{1/2}$ corresponds to the transfer of 1e from the left to right leads [17].

3 Thermal conductance of a symmetrical quantum dot

We assume that $V_1 = V_2$, (*i.e.*, $r_{1(2)} = r; t_{1(2)} = t$). In such a case

$$\omega_V = \omega_{V0}\cos^{\frac{2}{2-g}}(\pi N + k_{\mathrm{F}}d),$$
$$\hbar\omega_{V0} = \mu\left(\frac{\gamma E_{\mathrm{c}}}{\mu}\right)^{\frac{g}{2-g}}\left(\frac{4gr}{\pi t}\right)^{\frac{2}{2-g}}. \quad (10)$$

From above expression we see that at some values of N (V_g) the effective potential vanishes ($\omega_V = 0$), that is due to a degeneration of the electrostatic energy in QD charge ($q \leftrightarrow q+e$) [7,11] In this case the Coulomb blockade is lifted and the heat current increases. So, the thermal conductance shows periodic peaks as a function of N (see Fig. 1). This effect is quite analogous to that in the case of a charge transfer [1–4], excepting the neutral nature of plasmons.

Note that in the case of an asymmetrical quantum dot ($V_1 \neq V_2$) the effective potential barrier V, equation (4) does not vanish at any values of V_g and the Coulomb blockade oscillations in $K(V_g)$ will be slightly pronounced only.

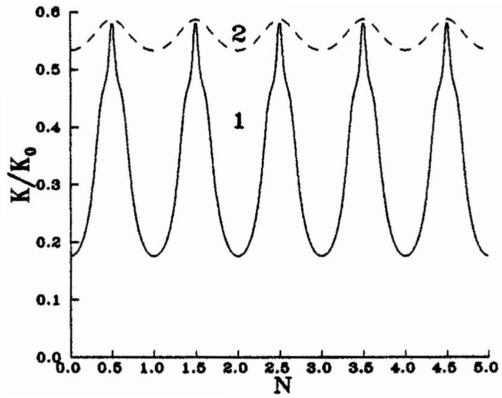

Fig. 1. The dependence of the thermal conductance K in units of $K_0 = \pi T/(6\hbar)$ on the quantum dot potential $N = CV_g/e$ for the symmetrical case ($V_1 = V_2$) at low $\hbar\omega_{V0}/T = 10$ (1) and high $\hbar\omega_{V0}/T = 0.1$ (2) temperatures. The parameters are $g = 1$; $T = \Delta_\mathrm{p}$; $\Delta_\mathrm{p}/E_\mathrm{c} = 0.1$.

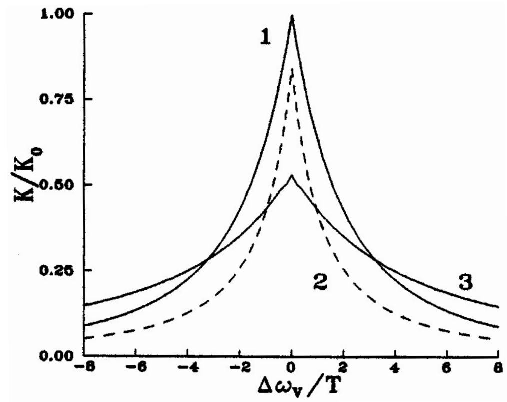

Fig. 2. The dependence of the thermal conductance K on the effective barrier height ω_V. $\Delta\omega_V = \omega_V(N)\ \mathrm{sgn}(N - N_\mathrm{max})$, where N_max corresponds to the peak maximum. The curves correspond to $T \ll T^*$ (1); $T = T^*$ (2); $T = 50T^*$ (3) ($T^* = \Delta_\mathrm{p}/\pi^2$). The parameters are $g = 1$; $\Delta_\mathrm{p}/E_\mathrm{c} = 0.01$.

3.1 Shape of the Coulomb blockade peak

The shape of peaks in $K(V_g)$ essentially depends on the temperature (see Fig. 2). The crossover occurs at $T \sim T^*$, where $T^* = \Delta_\mathrm{p}/\pi^2$; $\Delta_\mathrm{p} = \pi\hbar v/d$ is the level spacing for plasmons in the isolated ($V_{1,2} \to \infty$) quantum dot. For the case of Fermi liquid ($g = 1$) it is $\Delta_\mathrm{p} = \Delta_\mathrm{F}$, where $\Delta_\mathrm{F} = \pi\hbar v_\mathrm{F}/d$ is the level spacing near the Fermi energy μ for electrons in the isolated quantum dot. Note, because the plasmons are neutral the magnitude of Δ_p is not renormalized by the Coulomb energy in contrast with electrons [33]. However Δ_p increases when the inter-electron repulsion strengthens ($\Delta_\mathrm{p} \sim 1/g$; $g < 1$). Near the peak maximum ($\hbar\omega_V \ll T$) the thermal conductance equals

$$K/K_0 = 1 - \frac{3\hbar\omega_V}{2\pi T}, \qquad \hbar\omega_V \ll T \ll T^*, E_\mathrm{c} \quad (11)$$

$$K/K_0 = \frac{1}{2} - \frac{3\hbar\omega_V}{2^{5/2}\pi T}, \qquad T^*, \hbar\omega_V \ll T \ll E_\mathrm{c}. \quad (12)$$

Here $K_0 = \pi T/(6\hbar)$ is the thermal conductance of a one-dimensional ballistic channel [34,35]. Thus, with increasing temperature the height of peak reduces two times. This is due to the charging energy (the capacity C) leading to the dependence of the electrical conductance G on the frequency ω equation (9). Note, that at $E_\mathrm{C} = 0$ and $\hbar\omega_V = 0$ we will obtain $G(\omega) = gG(0)$ that formally follows from equation (9). At low temperatures ($T \ll T^*$) the main contribution to the thermal conductance comes from low-frequency (long wavelength) plasmons which do not "feel" the inner structure of a potential barrier. In this case the heat transfer (in the peak maximum) through the system is defined by the thermal conductance of a single fully transmitting ($t = 1$) barrier. At higher temperatures the thermal conductance reduces two times that is due to a destructive interference of contributions of plasmons with different frequencies. Effectively, it is a result of a noncoherent (at $T \gg T^*$) transmission through a double potential barrier. In this case at peak maximum ($\omega_V = 0$) we have two fully transmitting ($t_1 = t_2 = 1$) barriers connected in series. The thermal conductance of each of two barriers is K_0 so the system conductance is $K = K_0/2$. However, we emphasize that the reduction of a thermal conductance is due to averaging over the temperature in the phase-coherent system without any real inelastic processes (which occur only in the electron reservoirs far from the system). The effect of a destructive interference due to the temperature is enough common for the mesoscopic ballistic systems. The energy scale $T^* \sim \Delta_\mathrm{F}/\pi^2$ is characteristic for the persistent current problem [36]. This scale is also important for describing the transport properties of ballistic mesoscopic samples [37,38].

3.2 Temperature dependence of the thermal conductance

The dependence of the thermal conductance on the temperature at some values of an effective potential is depicted in Figure 3. One can see that for the small barrier

$$\hbar\omega_V \sim T^* \quad (13)$$

the dependence $K(T)$ is essentially nonmonotonic at $T \sim T^*$ that is due to a mutual influence of two effects. On the one hand, the above mentioned destructive interference effect leads to a reduction of K with the temperature. On the other hand, the increase of the number of high-energy (ballistic) plasmons (with $\omega > \omega_V$) causes the increase of K. Note that the condition equation (13) is valid either near the peak maximum or for the barrier with a small reflection amplitude ($r \to 0; t \sim 1$) (the strong tunneling regime). In the last case the dependence $K(V_g)$ slightly oscillates (Fig. 1, the curve 2).

The analytical expressions for $K(T)$ may be obtained either at $\omega_V = 0$ or at $\hbar\omega_V \gg T$. Using equations (9, 7)

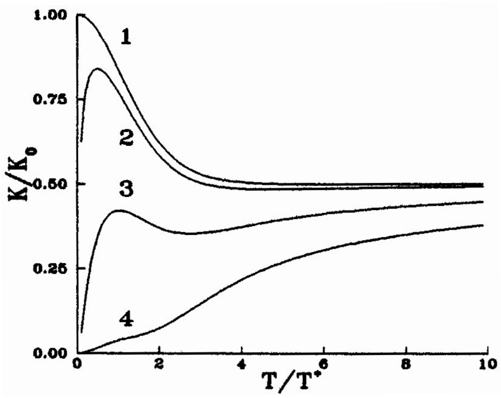

Fig. 3. The temperature dependence of the thermal conductance K at $\omega_V = 0$ (1); $\hbar\omega_V/T^* = 0.1$ (2); 1(3); 10(4). The parameters are $g = 1$; $\Delta_{\rm p}/E_{\rm c} = 0.01$.

we get the Coulomb peak height ($\omega_V = 0$) as follows

$$K/K_0 = \frac{1}{2}\left(1 + 3\frac{T/T^* \tanh(T/T^*) - 1}{\sinh^2(T/T^*)}\right), \qquad (14)$$

where $T^* = \Delta_{\rm p}/\pi^2$. This dependence is depicted in Figure 3 (the curve 1). Note, that the same crossover (at $T \to 0$) for the electrical conductance was obtained in reference [28].

Further we consider the heat transfer far from the Coulomb peak maximum. We assume

$$\hbar\omega_V \gg T, T^*. \qquad (15)$$

In this case the main mechanism of a heat transfer is tunneling of plasmons. At $T \ll T^*$ it follows from equation (9) $\mathrm{Re}\{G(\omega)\} = G_0\{(\omega/\omega_V)^2 + (\omega/\omega_{\rm c})^2\}$. Substituting this expression into equation (7) we obtain

$$K(T) = K_0 \frac{4\pi^2}{5\hbar^2} T^2 \left(\frac{1}{\omega_V^2} + \frac{1}{\omega_{\rm c}^2}\right), \qquad T \ll T^*. \qquad (16)$$

The same temperature dependence was firstly obtained for the case of a single barrier in references [24,18].

At more high temperatures ($T \gg T^*$) the effect of resonant tunneling of plasmons must be accounted. The importance of such an effect was emphasized in references [23,24]. With respect to equation (15) we can expand the dependence $G(\omega)$ equation (9) into the sum of the Breit-Wigner resonances and the quadratic in ω background

$$\mathrm{Re}\{G(\omega)\} = gG_0 \left\{ \sum_{n=0}^{\infty} \frac{\Gamma_{nV}^2}{(\omega - \omega_{nV}^r)^2 + \Gamma_{nV}^2} + \left(\frac{\omega}{\omega_V}\right)^2 + \sum_{n=1}^{\infty} \frac{\Gamma_{n{\rm c}}^2}{(\omega - \omega_{n{\rm c}}^r)^2 + \Gamma_{n{\rm c}}^2} + \left(\frac{\omega}{\omega_{\rm c}}\right)^2 \right\}, \qquad (17)$$

where $\omega_{nV}^{\rm r} = \omega_{nV}(1 + 2\Delta_p/(\pi\hbar\omega_V))$; $\hbar\omega_{nV} = \Delta_{\rm p}(2n+1)$, ($n = 0,1, \ldots$); $\omega_{n{\rm c}}^{\rm r} = \omega_{n{\rm c}}(1 - 2\Delta_{\rm p}/(\pi\hbar\omega_{\rm c}))$; $\hbar\omega_{n{\rm c}} = 2\Delta_{\rm p} n$, ($n = 1,2, \ldots$) and

$$\Gamma_{nV} = \frac{2\Delta_{\rm p}}{\pi}\left(\frac{\omega_{nV}}{\omega_V}\right)^2; \qquad \Gamma_{n{\rm c}} = \frac{2\Delta_{\rm p}}{\pi}\left(\frac{\omega_{n{\rm c}}}{\omega_{\rm c}}\right)^2. \qquad (18)$$

Substituting equation (17) into equation (7) we obtain

$$K = K_0 \sum_{i={\rm c},V} \frac{1}{\hbar^2\omega_i^2} \times \left\{ \frac{4\pi^2}{5} T^2 + \frac{24T\Delta_p}{\pi^2} \sum_{n=0}^{\infty} \frac{(\omega_{ni}/(2T))^4}{\sinh^2(\omega_{ni}/(2T))} \right\}, \qquad (19)$$

(here we neglect an unimportant difference between ω_{ni} and $\omega_{ni}^{\rm r}$). In the limit $T \gg T^*$ one can replace $\sum_n \to \int {\rm d}\omega/(2\Delta_p)$ that gives

$$K(T) = K_0 \frac{8\pi^2}{5\hbar^2} T^2 \left(\frac{1}{\omega_V^2} + \frac{1}{\omega_{\rm c}^2}\right), \qquad T \gg T^*. \qquad (20)$$

From equations (16, 20) it follows that for the tunneling regime it is $K \sim T^3$ at all the temperatures. But with increasing temperature ($T > T^*$) the numerical factor in this relation increases two times that is due to resonant tunneling of plasmons.

4 Discussion and conclusion

The thermal transport through a double potential barrier (a quantum dot) with one-dimensional (one-channel) interacting (Luttinger liquid) leads under Coulomb blockade regime is considered for the case of spinless electrons at low temperatures. The thermal conductance is calculated within the self-consistent harmonic approximation [20,19], which describes the heat transfer by plasmons. It is shown that the dependence of the thermal conductance on the potential of a quantum dot consists of a series of peaks which are due to a lift of the Coulomb blockade.

At the Coulomb blockade peak maximum the thermal conductance is linear in the temperature $K \sim T$ that is due to a ballistic heat transfer through a quantum dot. However, the factor in this relation reduces two times with increasing the temperature $T > T^*$ equation (14) that is a consequence of the transition from the coherent plasmon transfer through a double potential barrier to the incoherent transfer through two barriers connected in series. This is in accordance with the behaviour of an electrical conductance [28,12]. So, because of a ballistic transport at the peak maximum (for the symmetric double barrier) the Lorentz number $L = K/(GT)$ does not depend on the strength of interelectron interactions and the Wiedemann-Franz law holds $L = L_0 = (\pi^2/3)(k_{\rm B}/e)^2$. At the same time, away from the resonance it is $L \neq L_0$ that is due to the fact that the electrical current and the heat current are carried by the different particles (by electrons and plasmons, respectively) [18,22–24].

In the tunneling regime ($\hbar\omega_V \gg T$) the thermal conductance due to plasmons is $K \sim T^3$, equations (16, 20). We compare the plasmon contribution with the electron contribution to the total thermal conductance. Firstly we consider the case of Fermi liquid ($g = 1$). At low temperatures under Coulomb blockade regime the main mechanism of an electron transport through the dot is elastic ($T \ll \Delta_{\rm F}$) or inelastic ($T \gg \Delta_{\rm F}$) cotunneling [8,9]. Inelastic cotunneling in the strong-tunneling regime (when the module of reflection amplitude of a single potential barrier $r \to 0$) [12] as well as in the weak-tunneling regime (when the module of transmission amplitude of a single barrier $t \to 0$) [8,9] leads to a quadratic temperature dependence of the electrical conductance $G \sim T^2$ (at $T \ll \hbar\omega_V$), that corresponds to the cubic temperature dependence of the electron contribution to the total thermal conductance $K_{\rm e} \sim T^3$. The plasmon contribution has the same temperature dependence equation (20). At lower temperatures ($T \ll T^*$) elastic cotunneling leads to the temperature independent electrical conductance $G \sim (\Delta_{\rm F}/E_{\rm c})^2$. In such a case the electron contribution to the thermal conductance exceeds the plasmon one $K_{\rm p} \sim (T/E_{\rm c})^2$ equation (16): $K_{\rm e}/K_{\rm p} \sim (\Delta_{\rm F}/T)^2 \gg 1$. At $T \sim \Delta_{\rm F}$ (more precisely at $T \sim T^*$) the electron and plasmon contributions are of the same order and the nonmonotonic temperature behaviour due to a plasmon transport may be visible. On the other hand, in the case of repulsively interacting electrons ($g < 1$) the electrical conductance of a quantum dot is $G \sim T^{4/g-2}$ at $\Delta_{\rm p} \ll T \ll E_{\rm c}$ and $G \sim T^{2/g-2}$ at $T \ll \Delta_{\rm p}$ [28]. Therefore, at $g < 1/2$ the plasmon contribution ($\sim T^3$) to the thermal conductance of a QD dominates at all the temperatures $0 < T \ll E_{\rm c}$. In the case of moderate repulsive interactions ($1/2 < g < 1$) the plasmon term dominates at $\Delta_{\rm p}(\Delta_{\rm p}/\mu)^{\frac{2-2g}{2g-1}} < T \ll E_{\rm c}$.

In the case of Fermi liquid ($g = 1$) for the strong cotunneling regime ($r \to 0$) the electrical conductance of a quantum dot may be calculated exactly at $\Delta_{\rm F} \ll T \ll E_{\rm c}$ [12]. Using this solution and equation (7) we readily obtain the thermal conductance for the strong cotunneling regime (the subscript "sc") as follows

$$K_{\rm sc} = \frac{\hbar^2}{16\pi T^2}\int_0^\infty {\rm d}\omega \frac{\omega^2}{\sinh^2(\hbar\omega/(2T))} \times \left\{1 + \frac{1}{\hbar\omega}\int_{-\infty}^{\infty} {\rm d}\epsilon \frac{\Gamma_0^2}{\epsilon^2+\Gamma_0^2}\left(f_0(\epsilon+\hbar\omega) - f_0(\epsilon)\right)\right\}, \quad (21)$$

where $f_0(x) = (\exp(\beta x) + 1)^{-1}$ is the Fermi function; $\Gamma_0 = \frac{2\gamma E_{\rm c}}{\pi^2}(r_1^2 + r_2^2 + 2r_1 r_2 \cos(2\pi N + 2k_{\rm F}d))$. Now we compare the thermal conductance obtained in the SCHA equations (7,9) at $g = 1$ with the one defined by equation (21) (note, that $\hbar\omega_V = 2\Gamma_0$). The dependences of both $K_{\rm sc}$ (the curve 2) and the SCHA thermal conductance (the curve 1) on $\hbar\omega_V/T$ are depicted in Figure 4. These dependences are in a good agreement with each other. So, at $\Gamma_0 \ll T$ we have $K_{\rm sc} = \frac{K_0}{2}\left(1 - \frac{3\pi\Gamma_0}{16T}\right)$. Comparing with equation (12) we conclude that in this regime the deviation of a total thermal conductance from the ballistic value is described by SCHA with a relative accuracy of order 15%. At low temperatures $\Delta_{\rm F} \ll T \ll \Gamma_0$ we have $K_{\rm sc} = K_0\frac{3\pi^2}{10}\left(\frac{T}{\Gamma_0}\right)^2$. And as it follows from equation (20) (note that at $r \to 0$ we have $\hbar\omega_V \ll E_c$) the relative accuracy of SCHA is of the order of 30%. We may improve upon the agreement slightly renormalizing the effective potential, equation (6) $\omega_V^* = 1.25\omega_V$. In fact, this means the change of the high-frequency cut-off. The obtained dependence $K(\hbar\omega_V^*/T)$ is depicted in Figure 4 by squares.

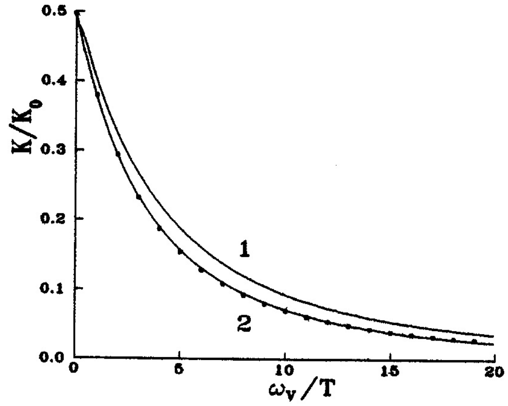

Fig. 4. The dependence of the thermal conductance K on the ratio $\hbar\omega_V/T$ for strong cotunneling ($E_{\rm c} \gg T \gg \Delta_{\rm p}$; $r \to 0$) of noninteracting ($g = 1$) electrons. The curves correspond to the self-consistent harmonic approximation (1); the exact expression for the electrical conductance reference [12] (2); and SCHA with a changed potential height $\omega_V^* = 1.25\omega_V$ (squares).

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