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Coulomb blockade of the persistent current in a one-dimensional ballistic Luttinger liquid ring

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Abstract. The effect of an electrostatic energy (in the geometrical capacitance approach) on a persistent current is considered. It is shown that at high temperatures the current amplitude shows periodic dips as a function of the potential difference between a ring and a reservoir. These dips correspond to a lift of the Coulomb blockade. In a minimum of a dips a current is periodic in a magnetic flux with a period $\Phi_0/2$ at any temperatures.

PACS. 72.10.-d Theory of electronic transport; scattering mechanisms – 73.20.Dx Electron states in low-dimensional structures (superlattices, quantum well structures and multilayers)

1 Introduction

Mesoscopic physics [1] studies the properties of systems which manifest quantum effects beyond the atomic realm. At low temperatures the inelastic mean free path (or phase breaking length) L_φ exceeds the size of a sample L, therefore thermodynamic [2] as well as kinetic [3] properties of such systems are sensitive to the change of an electron wave function phase. This allows, in particular, the study of the Aharonov-Bohm (AB) effect [4] in solids.

As was shown in references [5,6], the free energy F of doubly connected systems containing an AB flux Φ is periodic in Φ with a period of $\Phi_0 = h/e$. The derivative of the free energy over the magnetic flux determines an equilibrium persistent current $I = -\partial F/\partial \Phi$ which exists in normal (nonsuperconducting) samples at low temperatures. The existence of such a current (or a magnetic moment) was predicted in reference [7] for a thin-walled ballistic cylinder and in reference [8] for a one-dimensional metallic ring with disorder. The persistent current was observed experimentally in the ensemble of many mesoscopic rings [9] as well as in single rings in diffusive [10] and ballistic [11]regimes. It should be noted that the current amplitude, measured in a ballistic low-channel ring which was formed in the $Al_xGa_{1-x}As/GaAs$ semiconductor heterostructure [11] is in good agreement with the prediction of a theory based on a model of noninteracting spinless particles [12].

The problem of persistent currents has facilitated the study of some fundamental problems of statistical physics (in particular of the role of statistical averaging [13,14]). If the ring is connected to an electron reservoir, which fixes the chemical potential of electrons in a ring (μ = const.); then averaging within the framework of the grand canonical distribution is suitable. At the same time for an isolated ring with the fixed number of particles (N_e = const.) the canonical distribution should be applied. The various nature of averaging affects the current in a single ring as well as the current averaged over an ensemble of macroscopically identical rings. For an isolated ring [14,15] the crossover temperature is two times higher than the one for a ring coupled to an electron reservoir [12,16]. The ensemble averaged current is exponentially small in the case of the grand canonical ensemble [12,17–19] and has a finite amplitude for the canonical ensemble [12,20–27].

It should be noted that the temperature affects upon current in a twofold way [12]. Firstly, at $T > 0$ thermal excitations, such as phonons, are present and an interaction with them destroys a current coherent state. Secondly, with increasing temperature, the occupation of various quantum levels is changed, which does not destroy a coherent state, but does result in a reduction of current magnitude [7,12,16]. Below we are limited to the temperature range where we can neglect inelastic processes: $L_\varphi(T) \gg L$.

In a open system (a ring is connected to a reservoir) the charge transfer from one region into another region is important. For mesoscopic systems the typical capacitance C of the system can be very small; therefore the Coulomb interaction between the electrons affects considerably the charge transfer. At low temperatures the charging energy $E_C = e^2/(2C)$ associated with the transfer of the elementary charge e can significantly exceed the temperature of the system T and therefore strongly suppress the charge transfer (the Coulomb blockade) [28–30]. As a result the mesoscopic system, though coupled to a reservoir, should be considered as isolated, *i.e.* within the framework of the canonical distribution. At the same time, at some values of the potential V_g of a mesoscopic sample the charging energy is degenerate and the Coulomb blockade is lifted

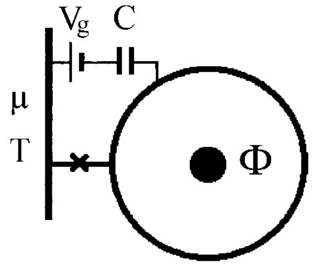

Fig. 1. One-dimensional ring threaded by a magnetic flux Φ and weakly connected to an electron reservoir with the chemical potential μ and the temperature T. V_g and C are the potential difference and the geometrical capacitance between a ring and a reservoir, respectively.

[31]. This results, for instance, in series of sharp peaks in the conductance of a system as a function of V_g.

In view of the persistent current problem the effect of Coulomb blockade on the coherent charge transfer at $T = 0$ was considered in references [32–35].

The purpose of the present paper is to consider the persistent current in a one-dimensional ballistic ring near points of a degeneracy of the charging energy. In such a case the persistent current is periodic in Φ with a period $\Phi_0/2$. Besides, at high temperatures the current amplitude is exponentially small compared to the one in the Coulomb blockade regime. Thus, at high temperatures the dependence of a current amplitude on V_g consists of a series of Coulomb dips, which correspond to a degeneracy of a charging energy.

2 Formulation of the problem and basic equations

We consider a one-dimensional ballistic ring coupled to an electron reservoir by a tunnel junction (Fig. 1). We assume that the transparency of a tunnel barrier (denoted by a cross in Fig. 1) is small and the reservoir does not influence the electron spectrum in a ring but both the energy exchange and the exchange of particles with the reservoir are allowed. However, at low temperatures in the general case the particles exchange is suppressed by the Coulomb blockade.

In the case of a small capacitance C (see Fig. 1) it is necessary to take into account the charging energy

$$\Delta E = E_C(N_e - N)^2, \tag{1}$$

where $E_C = e^2/(2C)$; N_e is the number of electrons in a ring; $N = CV_g/e$ is a parameter proportional to the potential difference between the ring and the reservoir V_g and characterizing the effective charge of a positive background in a ring.

The change of the number of electrons in a ring $\Delta N_e = \pm 1$ increases the energy of the system by E_C. Therefore at $E_C \gg T$ the number of electrons in a ring is fixed: $N_e = \text{const}$. At the same time at half-integer values of N the charging energy (1) is degenerate in N_e [31]. In this case the exchange of particles between the ring and the reservoir is allowed.

It should be noted, when the charging energy is degenerate an effect similar to the Kondo-effect [36] leads to a significant increasing of the barrier transparency [37]. This leads, generally speaking, to the broadening of energy levels in a ring and, as a consequence, to the decreasing of the persistent current amplitude [16]. This effect requires additional consideration. However, in the present paper we neglect this effect; since, for the small enough mesoscopic system; the level spacing exceeds the level broadening Γ and the last can not change qualitatively the results.

We consider spinless electrons in a one-dimensional ballistic ring within the framework of a Luttinger liquid model [38]. The choice of such a model is due to two reasons. Firstly, this model includes non-perturbatively electron-electron interaction, which affects considerably the persistent current at $T > 0$ [39]. Secondly, the charging energy (1) is similar to part of the Hamiltonian of the Luttinger liquid [38] and also can be included non-perturbatively.

In this model, the low-energy excitations around the Fermi surface of the spinless interacting electron system are expressed in terms of a scalar bosonic field $\varphi(x,t)$ [38]. The Lagrangian of a Luttinger liquid L_{LL} in a bosonic form is [15,39]

$$L_{LL}(x,t) = \hbar K \left\{ \frac{1}{v}\left(\frac{\partial\varphi}{\partial t}\right)^2 - v\left(\frac{\partial\varphi}{\partial x}\right)^2 \right\}. \tag{2}$$

Here K and v are Haldane's parameters [38] which depend on the interelectron interaction in a ring. Within the free-electron gas approach they are $K = 0.5$ and $v = v_F$, where $v_F = \hbar k_F/m^*$ is the Fermi velocity (k_F is the Fermi wave number; m^* is the electron effective mass). The spatial derivative of the field φ determines the deviation of the particle density $\rho(x,t)$ from the mean density in the ground state $\rho_F = k_F/\pi$

$$\rho(x,t) = \rho_F + \pi^{-1/2}\partial\varphi/\partial x. \tag{3}$$

The Lagrangian corresponding to a charging energy (1) is

$$L_C(t) = -\frac{E_C}{L}\left(\int_0^L dx\rho(x,t) - N(V_g)\right)^2. \tag{4}$$

This Lagrangian is quadratic in φ and therefore the charging energy can be taken into account exactly.

At last, the Aharonov-Bohm interaction of electrons in a ring with the magnetic flux Φ is described by the Lagrangian L_{AB} [15]

$$L_{AB}(x,t) = \frac{\hbar}{L}\pi^{1/2}\frac{\partial\varphi}{\partial t}\left(k_j + \frac{2\Phi}{\Phi_0}\right), \tag{5}$$

Where the topological number k_j depends on the parity of a number of particles in a ring N_e.

The partition function Z determines the free energy $F = -T\text{Ln}(Z)$ and may be presented in the form of a path integral over the field $\varphi(x,t)$ [15]

$$Z = \int D\varphi \, \exp\left(-S_E/\hbar\right). \tag{6}$$

Here the Euclidean action is

$$S_E = -\int_0^L dx \int_0^\beta d\tau \left(L_{LL}(x,\tau) + L_C(\tau) + L_{AB}(x,\tau)\right), \tag{7}$$

where $\beta = \hbar/T$; τ is the imaginary time.

The field $\varphi(x,\tau)$ obeys twisted boundary conditions on a torus [15]

$$\varphi(x + k_1 L, \tau + k_2\beta) = \varphi(x,\tau) + k_1\pi^{1/2}(2m + k_M) + k_2\pi^{1/2}n. \tag{8}$$

Here k_1, k_2, n, m are integers; k_M is the topological number, describing the parity of the additional number (over the number in the ground state) of particles in a ring. For an isolated ring there is $m = 0$ and $k_M = 0$. Otherwise topological numbers k_j and k_M depend on the parity of the number of particles in the ground state $N_0 = [\rho_F L]$ (where $[x]$ is the integer part of x) [15]

$$\begin{aligned} &k_j = k_M, \text{ if } N_0 \text{ is odd} \\ &k_j = 1;\ k_M = 0 \text{ and } k_j = 0;\ k_M = 1, \text{ if } N_0 \text{ is even.} \end{aligned} \tag{9}$$

Considering Lagrangian $L = L_{LL} + L_C + L_{AB}$ is quadratic in φ; therefore the extremal trajectories obeying the boundary conditions (8) and determining the flux-dependent part of the free energy $\Delta F(\Phi)$ are linear functions of both x and τ

$$\varphi_{mn}(x,\tau) = \pi^{1/2}\left((2m + k_M)\frac{x}{L} + n\frac{\tau}{\beta}\right). \tag{10}$$

3 Calculation of both the free energy and the persistent current

By substituting (2–5, 10) into (6, 7) and performing the summation over n and m (like the one in [15]) we obtain the following expression for the free energy

$$\Delta F(\Phi) = -T\mathrm{Ln}\left(\sum_{k_j,k_m}\Theta_3\left(\frac{k_j}{2} + \frac{\Phi}{\Phi_0}, \exp\left(-\frac{T}{T^*}\right)\right) \times\Theta_3\left(\frac{k_M}{2} + \delta_C, \exp\left(-\frac{\pi^2 T}{4T_C}\right)\right)\right). \tag{11}$$

Here the summation over k_j and k_M is performed with respect to the topological constraint (9); $T^* = \hbar v/(\pi K L)$ is the crossover temperature for an isolated ring (N_e= const.); $\Theta_3(v,q)$ is the Jacobi Θ function [40]. The parameters T_C and δ_C depend on the charging energy E_C and are

$$T_C = T_{C0} + E_C, \tag{12}$$

where $T_{C0} = \pi\hbar K v/L$;

$$\delta_C = E_C\, N(V_g)/(2T_C). \tag{13}$$

Thus, the charging energy (1) results in the following. Firstly, it renormalizes the characteristic temperature T_{C0} for arising of topological excitations in the spatial sector [15], *i.e.* excitations which lead to a change of the number of electrons in a ring. At $T \ll T_C$ such excitations do not arise, therefore the number of electrons in a ring is conserved: N_e = const. In such a case a ring is effectively isolated from a reservoir. In the limit of a large charging energy $E_C \gg T^*$ (the Coulomb blockade regime) the condition N_e = const. is true throughout the temperature range $T \leq T^*$ where the persistent current exists. Therefore, in this case the dependence of a current on the temperature is like the one for an isolated ring [15].

We note, that for an appropriate experimental situation, namely, for the ballistic ring in the $Al_xGa_{1-x}As/GaAs$ semiconductor heterostructure [11], $T_{C0} = \Delta_F/4 \leq 0.2$ K (within the free electron gas model). Thus, if the capacitance $C < 10^{-15}$ F then the charging energy $E_C \geq 1$ K, and the last significantly renormalizes the characteristic temperature T_{C0}.

Secondly, the degeneracy of a charging energy (1) in N_e: $\Delta E(N_e) = \Delta E(N_{e+1})$ affects considerably the free energy (11) (and the persistent current). This degeneracy condition can be satisfied by varying V_g (see Fig. 1). The parameter δ_C (13) characterizes the deviation of the state of a system from such a degeneracy state which occurs at $\delta_{C0} = 0.25$. At $\{\delta_C\} = \pm\delta_{C0}$ (where $\{x\}$ is the fractional part of x) the Coulomb blockade is lifted and a ring couples to a reservoir.

3.1 The persistent current in points of a degeneracy of the charging energy

As it follows from equation (11) at $\{\delta_C\}= \pm\delta_{C0}$ the current in a ring $I = -\partial F/\partial\Phi$ is

$$I(\Phi) = \frac{4\pi}{\Phi_0}T\sum_{q=1}^{\infty}(-1)^q\frac{\sin\left(4\pi q\Phi/\Phi_0\right)}{\sinh\left(4qT/T^*\right)}. \tag{14}$$

The persistent current (14) is periodic in Φ with a period $\Phi_0/2$. This result is in agreement with reference [33] where it was shown that the odd harmonics of a persistent current vanish at V_g corresponding to the charge transfer resonance.

3.2 Dependence of a persistent current on V_g

From both equation (11) and periodicity of the Θ_3-function it follows that the free energy (and the persistent current) is periodic in V_g with a period of

$$\Delta(eV_g) = 4T_C. \tag{15}$$

This period depends on the charging energy (see Eq. (12)) and corresponds to a change of a parameter δ_C by 1.

At the same time both the parameter N and the number of electrons in the ground state of a ring N_e are changed by two: $\Delta N = \Delta N_e = 2$. We note, that the change of N_e by 1 leads to an alteration of sign of a current (for the spinless particles) pursuant to the parity effect [5,15,41].

It should be noted that the relation between ΔN and $\Delta(V_g)$ is determined by an effective capacitance C^* which includes the part depending on the density of states in a ring [42,43]: $1/C^* = 1/C + 1/C_0$, where $C_0 = e^2/(2T_{C0})$.

Within the free electron gas model ($T_{C0} = \Delta_F/4$ without the charging energy $C \to \infty$ ($E_C = 0$) the dependence on V_g with the period (15) corresponds to a well-known dependence of the persistent current on the chemical potential μ of a reservoir with a period of $\Delta(eV_g) = \Delta(\mu) = \Delta_F$ [7,12]. At the same time, at $C \to 0(E_C \to \infty)$ the charging energy fixes the number of electrons in a ring and cancels out such a dependence (at $T \ll E_C$): $\Delta(eV_g) \to \infty$. A similar result was obtained previously in references [32, 43,44] at $T = 0$.

3.3 The persistent current away from the degeneracy points

Further we consider the properties of a persistent current in a Coulomb blockade regime. By definition we assume that N_e is odd and $-\Phi_0/2 < \Phi < \Phi_0/2$; $-0.5 < \delta_C < 0.5$. If N_e is even it is necessary to make the replacement $\Phi \to \Phi + \Phi_0/2$ in the equations obtained below.

3.3.1 Low temperatures: $T \ll T^*, T_C$

By making use of the asymptotic expression for the Θ_3-function we obtain the following expression for the persistent current

$$I = -\frac{\pi^2}{\Phi_0} T^* \left\{ 2\frac{\Phi}{\Phi_0} - \text{sgn}(\Phi) \frac{B(\pi^2 T^*, \Phi/\Phi_0) B(4T_C, \delta_C)}{1 + B(\pi^2 T^*, \Phi/\Phi_0) B(4T_C, \delta_C)} \right\}, \quad (16)$$

where $B(x,y) = \exp\left(-\frac{x}{4T}(1-4|y|)\right)$. At the points $\delta_C = \pm 1/4$ the odd harmonics of a current vanish (see Eq. (14)). We designate $I_{odd} = |I(\Phi = \Phi_0/4)|$ and obtain from equation (16)

$$I_{odd} = \frac{\pi^2}{2\Phi_0} T^* A_0(\delta_C),$$
$$A_0(\delta_C) = \left| \frac{1 - \exp(-(1-4|\delta_C|)T_C/T)}{1 + \exp(-(1-4|\delta_C|)T_C/T)} \right| \cdot \quad (17)$$

The dependence $I_{odd}(\delta_C)$ is depicted in Figure 2. This dependence consists of periodic sharp dips located at $\delta_C = q/2 + 1/4$ (where q is an integer). The width of a dip $\Delta_{1/2}$ is

$$\Delta_{1/2} = 0.55 \frac{T}{T_C} \cdot \quad (18)$$

Thus, at low temperatures the lift of the Coulomb blockade leads to the halving of a period of a dependence $I(\Phi)$ only.

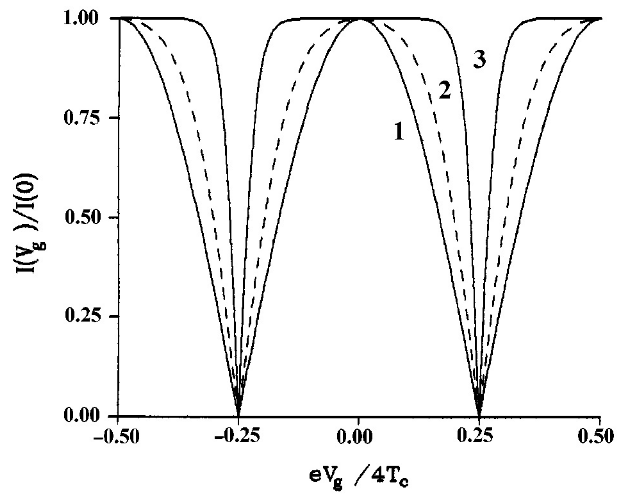

Fig. 2. Persistent current I as a function of the dimensionless potential difference $\delta_C = eV_g/(4T_C)$ for several values of the Coulomb energy $4T_C/(\pi^2 T^*) = 1$ (1); 10 (2); 30 (3) at both the temperature $T = \pi^2 T^*/2$ and the magnetic flux $\Phi = \Phi_0/4$.

3.3.2 High temperatures: $T \gg T^*$

In this case the asymptotic expression for the current is

$$I = -\frac{4\pi}{\Phi_0} T \exp\left(-\frac{T}{T^*}\right) A(\delta_C) \sin\left(2\pi \frac{\Phi}{\Phi_0}\right),$$
$$A(\delta_C) = \frac{\Theta_3(\delta_C, q_C) - \Theta_3(\delta_C - 1/2, q_C)}{\Theta_3(\delta_C, q_C) + \Theta_3(\delta_C - 1/2, q_C)}, \quad (19)$$

where $q_C = \exp\left(-\pi^2 T/(4T_C)\right)$. The function $A(\delta_C)$ describes the form of a Coulomb dips and is

$$A(\delta_C) = \begin{cases} A_0(\delta_C), T \ll T_C \\ 2 \exp\left(-\frac{\pi^2 T}{4T_C}\right) \cos(2\pi\delta_C), T \gg T_C. \end{cases} \quad (20)$$

We note, that at high enough temperatures ($T \gg T_C$) the width of a dip does not depend on the temperature.

At high temperatures the amplitude of a current at $\delta_C = \pm 1/4$ (the Coulomb blockade is lifted) is exponentially small compared to the one in the Coulomb blockade regime. Since, in this case the current is

$$I(\pm\delta_{C0}) = -\frac{8\pi}{\Phi_0} T \exp\left(-\frac{4T}{T^*}\right) \sin\left(4\pi \frac{\Phi}{\Phi_0}\right). \quad (21)$$

and $I(\delta_{C0})/I(\delta_C) \simeq \exp(-3T/T^*)$. While at low temperatures such a relation is about unity.

4 Conclusion

In the present paper, the influence of the Coulomb blockade on the persistent current in a one-dimensional ballistic ring weakly coupled to an electron reservoir is considered. Spinless electrons in a ring are described within the framework of the Luttinger liquid model with respect to the electrostatic (charging) energy of a system in a geometrical capacitance approach.

It is shown that the Coulomb energy $E_C = e^2/(2C)$ renormalizes the characteristic temperature T_{C0} for topological excitations arising in a spatial sector (*i.e.* excitations which change the number of electrons in a ring): $T_C = T_{C0}+E_C$. In a limit $E_C \gg T$ such excitations do not arise and a ring is effectively isolated from the reservoir ($N_e = \text{const.}$)

At the certain values of V_g (see Fig. 1) the charging energy is degenerate in N_e therefore the Coulomb blockade is lifted and a ring couples to a reservoir. At low temperatures ($T \ll T^*$) this leads to a halving of a period of a dependence $I(\Phi)$. At the same time at high temperatures such a change of a period of oscillations $I(\Phi)$ is accompanied by an exponential reduction of the current amplitude.

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