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Persistent current in a mesoscopic ring with resonant tunneling

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Abstract. – In this paper we have studied a persistent current in a ballistic ring containing a potential barrier with a resonant level. It is shown that the current amplitude depends on the change of an electron wave function phase at tunneling.

In ref. [1] the Aharonov-Bohm (AB) oscillations in a mesoscopic ring with a quantum dot (QD) were investigated. In the opinion of the authors, in that system at resonant tunneling the simultaneous measurement of the magnitude and phase of transmission through a potential barrier (quantum dot) is possible. In the above-cited work the oscillations of a ring's conductance in a magnetic field $G(\Phi)$ (the AB oscillations with period $\Phi_0 = h/e$) were measured. When the potential V_P is applied to the QD, the resonant tunneling condition $E_\mathrm{F} = E_n + eV_P$ (E_F is the Fermi energy of an electron in the ring; E_n is the electron level in the QD) is achieved and the ring's conductance G sharply grows (the resonant peaks were observed in the function $G(V_P)$). As was expected, the shift of the phase $\Delta\theta$ of the AB oscillations $G(\Phi)$ should depend on the change of the electron wave function phase at tunneling through the QD. Experimentally a sharp change (on π) of the phase of the AB oscillations within one resonant peak was observed.

The theoretical analysis [2]-[6] has shown that the requirement of symmetry $G(\Phi) = G(-\Phi)$ [7] limits the possible value of the phase shift: $\Delta\theta = 0$ or $\Delta\theta = \pi$. For this reason the conclusion about the impossibility of direct measurement of the phase of the transmission coefficient in a two-terminal interference experiments is made in ref. [3].

It should be noted that the position of the resonant level influences the amplitude of the AB oscillations. The observable change of the oscillations phase on π is due to the vanishing and change of sign of the first-harmonic amplitude [3], [4], [6] at the passing of the quantum dot level $E_n + eV_P$ through the Fermi level. When the first-harmonic amplitude has vanished, the function $G(\Phi)$ has a period $\Phi_0/2$ [3], [6], that is experimentally confirmed in ref. [3].

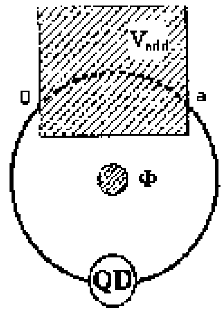

Fig. 1. – One-dimensional ballistic ring threaded by a magnetic flux Φ with a quantum dot (QD) and an additional gate V_{add}.

In the present work we consider a persistent (thermodynamical equilibrium) current in a ballistic ring with a quantum dot and show that the measurement of the phase of the transmission coefficient is possible. However, it is necessary not to use the "magnetic" but the "electrostatic" Aharonov-Bohm effect.

It is known that the amplitude of a persistent current I_0 depends on both the transmission coefficient of a potential barrier in a ring [8], [9] and the change of the phase φ of the wave function of an electron with Fermi energy along the ring [10]. Therefore, on the one hand, at resonant tunneling the amplitude of a current should considerably grow (with increasing transmission coefficient at resonance). On the other hand, the change of phase of an electron wave function at tunneling δ_{F} causes either increase or reduction of the current amplitude I_0 (see eq. (4)) in dependence on the initial (*i.e.* far from resonance) value of the phase φ. However, in a ring, to create a potential step with tunable height V_{add} (for example, by means of an additional gate (see fig. 1)), the amplitude of the current in a ring should oscillate with V_{add}, as is shown in ref. [10]. Furthermore, a significant shift $\Delta\chi$ of the phase of the function $I(V_{\mathrm{add}})$, equal to the change of the electron wave function phase at tunneling, $\Delta\chi = \delta_{\mathrm{F}}$, should be observed near resonance.

Let us consider a one-dimensional ballistic ring of length L ($L \ll L_\varphi$, L_φ is the electron phase coherence length) threaded by a magnetic flux Φ (fig. 1). The ring includes a quantum dot with a potential V_P relatively to its other part. The expression for the persistent current in a ring weakly couples to an electron reservoir with chemical potential E_{F} and temperature T and is $I = -\partial\omega/\partial\Phi$ (ω is the thermodynamic potential of spinless electrons in a ring). It can be written in the form

$$I = -\sum_l f_0\left(\frac{E_l - E_{\mathrm{F}}}{T}\right)\frac{\partial E_l}{\partial \Phi}\,. \tag{1}$$

Here $f_0(E/T)$ is the Fermi distribution function; l is the number of the electron energy level. The energy E_l is calculated by the transfer-matrix method [8] that leads to the following simple equation for the eigenvalues of the electron wave vector k:

$$\mathrm{Re}\left\{t_k^{-1}\exp\left[-i(kL + \delta_k)\right]\right\} = \cos\left(2\pi\frac{\Phi}{\Phi_0}\right), \tag{2}$$

where $\mathrm{Re}(x)$ is the real part of x; t_k and δ_k are the amplitude and phase of the transmission coefficient through a quantum dot: $T_k = t_k \exp[i\delta_k]$. Using the solution of eq. (2) and assuming

$L \gg \lambda_F$ (λ_F is the Fermi wavelength) we obtain the expression for the persistent current:

$$I = \frac{4\pi}{\Phi_0} T \sum_{q=1}^{\infty} t_F \sin\left(2\pi \frac{\Phi}{\Phi_0}\right) \frac{\cos\left(2\pi q \left[\frac{L}{\lambda_F} + \frac{\delta_F}{2\pi}\right]\right) \sin\left(\arccos\left[t_F \cos\left(2\pi \frac{\Phi}{\Phi_0}\right)\right] q\right)}{\mathrm{sh}(qT/T^*)[1 - t_F^2 \cos^2(2\pi\Phi/\Phi_0)]^{1/2}} . \quad (3)$$

Here t_F and δ_F are the amplitude and phase of the transmission coefficient for an electron with Fermi energy $T^* = \Delta_F/(2\pi^2)$ (were Δ_F is the level spacing near the Fermi energy for $\Phi = 0$). Equation (3) is valid when the transmission coefficient T_k changes slowly near the Fermi energy. In our case it means that the width of the resonance δV_P exceeds the level spacing: $e\delta V_P \gg \Delta_F$, *i.e.* sufficiently many levels in a ring are in resonance with the level E_n.

At $t_F = 1$ eq. (3) reproduces the results that were obtained in ref. [8] for a pure ring ($\delta_F = 0$) and in ref. [10] for a ring with a potential step ($\delta_F \neq 0$). We note that in ref. [9] an expression similar to eq. (3) is obtained, however, the dependence on the phase δ_F is obviously not mentioned.

From eq. (3) it follows that the phase δ_F influences the amplitude of the persistent current $I(\Phi)$. We simplify eq. (3). At $T > T^*$ it is enough to take into account only the term with $q = 1$. As a result we have

$$I(\Phi) = I_0 \sin\left(2\pi \frac{\Phi}{\Phi_0}\right) , \qquad I_0 = A(T) t_F \cos(\varphi) , \quad (4)$$

where $A(T) = 8\pi T \exp[-T/T^*]/\Phi_0$; $\varphi = 2\pi L/\lambda_F + \delta_F$ is the change of phase of the electron wave function with Fermi energy along the ring. The amplitude of the current I_0 is proportional to the amplitude of the transmission coefficient t_F and, hence, it considerably grows at resonance. Besides, I_0 depends on the phase of the transmission coefficient δ_F. The dependence on δ_F can be distinguished by means of an additional gate (see fig. 1) which induces a potential step with height $eV_{\mathrm{add}} \ll E_F$ and length a. The existence of a potential step with a small height results in the additional change of the electron wave function phase $\Delta\varphi = -2\pi e V_{\mathrm{add}} a/(L\Delta_F)$ [10]. Thus, for the system shown in fig. 1 we have

$$\varphi = 2\pi \frac{L}{\lambda_F} - 2\pi \frac{V_{\mathrm{add}}}{\Delta V_0} + \delta_F . \quad (5)$$

Here $e\Delta V_0 = \Delta_F L/a$. It is obvious from eqs. (4), (5) that I_0 is periodic in V_{add} with period ΔV_0. Near resonance the phase of the periodic function $I_0(V_{\mathrm{add}})$ will considerably change (in accordance with the change of δ_F).

It is necessary to say that the potential due to the additional gate introduces not only the phase shift of the electron wave $\Delta\varphi$ but also a backscattering, which reduces the magnitude of the persistent current. However in the limit $(eV_{\mathrm{add}}/E_F)^2 \ll 1$ the backscattering does not influence the eigenstates and thus the persistent current [10].

We note that the tunable change of φ by means of a potential step was used in ref. [3] in order that the first harmonic of the function $G(\Phi)$ for a ballistic ring should vanish.

As follows from eqs. (4), (5), the amplitude of the current should also oscillate as a function of E_F with period $\Delta E_F \simeq 2E_F\lambda_F/L$. The phase of the function $I_0(E_F)$ should also change in accordance with the change of δ_F.

We emphasize that in the present paper we consider a ring weakly coupled to an electron reservoir, which defines the chemical potential of a ring. However, the coupling is assumed weak enough and the reservoir does not influence the eigenstates in the ring. This situation corresponds to the coupling parameter $\varepsilon \to 0$ in ref. [11].

In this paper the oscillations of a persistent current in a mesoscopic ring with a quantum dot (the potential barrier with a resonant level) are considered. It is shown that the amplitude of the current I_0 depends on the change of phase of the electron wave function with Fermi energy at tunneling δ_F. The change of phase of the function $I_0(V_{add})$ near resonance is equal to the change of δ_F.

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