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Temperature-induced current in a one-dimensional ballistic ring with contacts

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Abstract. – In this paper we have considered the effect of temperature on the thermoelectric current in a one-dimensional ballistic ring coupled to two electron reservoirs. It is also shown that under some conditions a circulating thermoelectric current highly exceeds the transport current.

The phase-coherent transport is a feature of mesoscopic samples at low temperatures [1]. Hence, it is possible to observe a lot of quantum-mechanical phenomena [2]-[6] including the Aharonov-Bohm (AB) effect [7], [8] in solids.

One of the manifestation of the AB effect is the existence of a thermodynamically in equilibrium (persistent) current in mesoscopic rings threaded by a magnetic flux Φ at low temperatures, which has periodic magnetic flux with period $\Phi_0 = h/e$. For the first time the existence of such a current in real nonsuperconducting metallic one-dimensional rings was predicted theoretically by Büttiker *et al.* [9]. The effect was observed experimentally both at the ensemble of mesoscopic rings [10] and at the single ring [11], [12].

The persistent current exists not only in isolated rings, but also in the so-called "open systems" (*i.e.* in rings coupled to one or several electron reservoirs) [13]-[17]. The persistent current in an open system was observed experimentally in ref. [12]. Besides, the circulating current occurs in a ring coupled to two electron reservoirs with different chemical potentials $\mu_{\rm L} \neq \mu_{\rm R}$ (*i.e.* in the presence of a transport current) even in the absence of a magnetic flux [17]. This nonequilibrium current is due to the injection of carriers [14], [17] and differs from zero only in an asymmetric ring. The existence of this current is a purely quantum-mechanical effect and is due to the interference of an electron wave function in a doubly connected geometry.

The aim of the present paper is to consider the nonequilibrium current in a ballistic one-dimensional "open system" due to the temperature difference of the electron reservoirs. The thermoelectric current differs from zero if the conductance of the system depends on energy [18]. When the transport in a mesoscopic sample is ballistic such a dependence arises as a consequence of the interference. Therefore, the existence in mesoscopic samples of both the

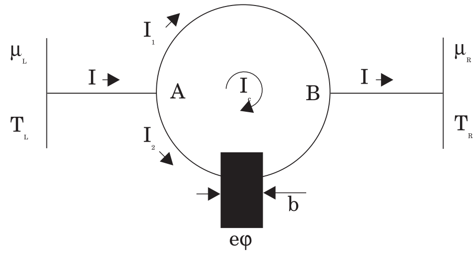

Fig. 1. – The ballistic ring coupled to two electron reservoirs.

circulating current and the transport thermoelectric current is a purely quantum-mechanical effect.

As model of an "open system", we consider a one-dimensional ballistic ring of length L threaded by a magnetic flux Φ and connected via ballistic one-dimensional leads to two electron reservoirs (fig. 1). Let us calculate the current I in such a system. In the ring the current I is split into currents I_1 in the upper arm and I_2 in the lower arm and $I = I_1 + I_2$. The magnitudes of I_1 and I_2 are different owing to the interference of the electron waves, and a circulating current $I_\mathrm{c} = (I_1 - I_2)/2$ arises in the ring [14].

Let us consider the case of a ring with a symmetric arrangement of leads: $L_1 = L_2 = L/2$ (where L_1, L_2 are the lengths of the upper and the lower arms, respectively). Let us place a potential step of height $e\varphi \ll \mu$ and of length b in the lower arm of the ring to take into account the possible asymmetry (see fig. 1). Note, that formally the presence of the potential step is equivalent to the asymmetric arrangement of leads: $L_1 \neq L_2$. Such an asymmetry can be taken into account by the replacements $(kL + \delta_k) \to (kL_1 + kL_2)$ and $\delta_k \to (kL_1 - kL_2)$. However, such replacements are true only at $T = 0$, because the energy dependences of quantities in the left and the right sides of these expressions are different.

Solving the one-dimensional Schroedinger equation for noninteraction electrons in the considered system, we take into account that the scattering of electrons in junctions A and B (see fig. 1) may be described by the scattering matrix $\widehat{S}(\gamma)$ [19], [20], which yields amplitudes of outgoing waves in terms of incoming waves:

$$\widehat{S}(\gamma) = \begin{pmatrix} -(a+b) & \gamma^{1/2} & \gamma^{1/2} \\ \gamma^{1/2} & a & b \\ \gamma^{1/2} & b & a \end{pmatrix} . \tag{1}$$

Here $a = \big((1-2\gamma)^{1/2} - 1\big)/2$; $b = \big((1-2\gamma)^{1/2} + 1\big)/2$; γ is the coupling parameter. For $\gamma = 0$ the lead and the ring are decoupled. $\gamma = 1/2$ is the maximum coupling. Note that $\gamma = 4/9$ corresponds to the approach of the quantum waveguide theory [21], where both the condition of the current conservation and the condition of the electron wave function continuity in the junction are used.

If the coupling parameter in junction A is denoted as γ_A and in junction B as γ_B, expressions for the current I and the circulating current I_c are as follows:

$$I = \frac{2e}{h} \int \mathrm{d}\varepsilon\, g_\varepsilon (f_\mathrm{OL} - f_\mathrm{OR}) , \tag{2}$$

$$I_\mathrm{c} = \frac{2e}{h} \int \mathrm{d}\varepsilon \big(c_\varepsilon (f_\mathrm{OL} - f_\mathrm{OR}) + d_\varepsilon (\gamma_\mathrm{A} b_\mathrm{B}^2 f_\mathrm{OL} + \gamma_\mathrm{B} b_\mathrm{A}^2 f_\mathrm{OR}) \big) , \tag{3}$$

where $f_{\rm OL}$ and $f_{\rm OR}$ are the Fermi functions in the left and the right reservoirs, respectively; $\varepsilon = \hbar^2 k^2/(2m)$; k is the wave number; m is the effective mass. Expressions for functions g_ε, c_ε and d_ε are of the form

$$g_\varepsilon = \frac{4\gamma_{\rm A}\gamma_{\rm B}}{Z_k}\big\{1 - \cos(\delta_k)\cos(\beta_k) + \cos(2\pi\Phi/\Phi_0)\big(\cos(\delta_k) - \cos(\beta_k)\big)\big\}\,, \tag{4}$$

$$c_\varepsilon = -\frac{2\gamma_{\rm A}\gamma_{\rm B}}{Z_k}\sin(\delta_k)\sin(\beta_k)\,, \tag{5}$$

$$d_\varepsilon = -\frac{4}{Z_k}\sin(\beta_k)\sin(2\pi\Phi/\Phi_0)\,. \tag{6}$$

Here $Z_k = (2b_{\rm A}b_{\rm B}\cos(2\pi\Phi/\Phi_0) + 2a_{\rm A}a_{\rm B}\cos(\delta_k) - (K_{\rm AB}+1)\cos(\beta_k))^2 + (K_{\rm AB}-1)^2\sin^2(\beta_k)$; $K_{\rm AB} = ((1-2\gamma_{\rm A})(1-2\gamma_{\rm B}))^{1/2}$; $\beta_k = kL + \delta_k$ is the change of the electron wave function phase along the ring (at $\Phi = 0$); $\delta_k = -2\pi(e\varphi/\Delta_\varepsilon)(b/L)$ is the change of the electron wave function phase due to the potential step [22]; $\Delta_\varepsilon = h(2\varepsilon/m)^{1/2}/L$ is the level spacing in the isolated ring near the energy ε at $\Phi = 0$. Note, that when $(\varepsilon-\mu)/\mu \ll 1$, $\delta_k(\varepsilon)$ is constant and is equal to $\delta_k(\mu)$; in this case it will be denoted δ_μ. According to the purpose of the present paper, such an approach is valid at $T/\mu \ll 1$.

In the equilibrium case, when the chemical potentials and the temperatures of the reservoirs are identical ($\mu_{\rm L} = \mu_{\rm R}$; $T_{\rm L} = T_{\rm R}$), the current through the system equals zero: $I = 0$; however, at $\Phi \neq 0$ there is a circulating current in the ring: $I_{\rm c}(\Phi) \neq 0$ (that is the second term in eq. (3)).

The two terms in eq. (3) correspond to the arising of the circulating current due to the electrostatic ($\varphi \neq 0$) and the magnetic ($\Phi \neq 0$) AB effects. Thus, as far as in the electrostatic AB effect there is no destruction of the electron movement symmetry in the ring in opposite directions, in order to arise the circulating current an external break of such a symmetry is necessary, that takes place in the nonequilibrium case ($\mu_{\rm L} \neq \mu_{\rm R}$ or $T_{\rm L} \neq T_{\rm R}$), *i.e.* when the transport current is present. In the case of the magnetic AB effect the mentioned symmetry is broken down, therefore the circulation current $I_{\rm c}$ can exist even in the absence of the transport current.

In the present paper we consider the nonequilibrium state due to the temperature differences of the reservoirs: $T_{\rm L} \neq T_{\rm R}$. Thus, we shall consider the chemical potentials of the reservoirs to be identical: $\mu_{\rm L} = \mu_{\rm R} = \mu$, and the magnetic flux to be equal to zero: $\Phi = 0$. In this case there is a thermoelectric current $I(\Delta T)$ (where $\Delta T = T_{\rm R} - T_{\rm L}$) between electron reservoirs. This current excites the circulation current $I_{\rm c}(\Delta T)$ in the ring.

As has already been mentioned, the existence of a transport thermoelectric current I is due to the dependence on energy of the conductance of the sample connected to the electron reservoirs. In the considered case the dependence of g_ε and c_ε on the energy, determining thermoelectric transport and circulating currents, is due to the interference of the electron wave functions in the ring and therefore is a purely quantum (mesoscopic) effect, which vanishes in the macroscopic limit (*i.e.* after averaging over the energy in the scale exceeding Δ_μ). Therefore, as will be seen below, the thermoelectric current vanishes at $T_{\rm L}$, $T_{\rm R} \gg \Delta_\mu$. Besides, in the considered system the flow of electrons can move both from a more heated reservoir to a less heated one and vice versa, and this depends on the parameters of the system.

Functions g_ε (4) and c_ε (5) are periodic in k with period $2\pi/L$ and can be expanded in

Fourier series:

$$g_\varepsilon = \bar{g} + \sum_{n=1}^{\infty} A_n \cos(nkL) + B_n \sin(nkL)\,, \tag{7}$$

$$c_\varepsilon = \bar{c} + \sum_{n=1}^{\infty} C_n \cos(nkL) + D_n \sin(nkL)\,. \tag{8}$$

Substituting eqs. (7) and (8) into eqs. (2) and (3), we obtain the expression for the thermoelectric current in the limit of $Lk_\mu \gg 2\pi$ in the following form:

$$I = \frac{I_0}{2\pi} \sum_{n=1}^{\infty} \big(A_n \sin(nLk_\mu) - B_n \cos(nLk_\mu)\big)\big(\Psi_n(T_L/T^*) - \Psi_n(T_\mathrm{R}/T^*)\big)\,, \tag{9}$$

$$I_\mathrm{c} = \frac{I_0}{2\pi} \sum_{n=1}^{\infty} \big(C_n \sin(nLk_\mu) - D_n \cos(nLk_\mu)\big)\big(\Psi_n(T_\mathrm{L}/T^*) - \Psi_n(T_\mathrm{R}/T^*)\big)\,. \tag{10}$$

Here $I_0 = 2ev_\mu/L$ (where v_μ is the Fermi velocity); $T^* = \Delta_\mu/(2\pi^2)$; $k_\mu = (2m\mu)^{1/2}/\hbar$; $\Psi_n(x) = x/\mathrm{sh}(nx)$. It should be noted that coefficients A_n, B_n, C_n and D_n do not depend on the Fermi energy, the length of the ring and the temperature.

The obtained expressions allow us to analyse the temperature dependence of the thermoelectric current in the ballistic ring.

Firstly we consider the regime of the linear response: $\Delta T \ll T = T_\mathrm{L}$. In the low-temperature limit $T \ll T^*$ expressions similar to those obtained in ref. [18] are valid:

$$I(\Delta T) = -\Delta T \frac{\pi^2 T}{3e} \frac{\partial}{\partial \mu} G(\mu)\,, \tag{11}$$

$$I_\mathrm{c}(\Delta T) = -\Delta T \frac{\pi^2 T}{3e} \frac{\partial}{\partial \mu} G_\mathrm{c}(\mu)\,, \tag{12}$$

where $G(\mu) = (2e^2/h) g_\mu$ is the conductance of the ring at $T = 0$; $G_\mathrm{c}(\mu) = (2e^2/h) c_\mu$.

At higher temperatures of $T \gg T^*$ the magnitude of the current decreases and this is due to the averaging of g_ε and c_ε in eqs. (4) and (5). Then in eqs. (9) and (10) one can possibly be limited by the term with $n = 1$, and as a result we obtain

$$I(\Delta T) = -\Delta T \frac{\pi^2 T}{e} \frac{1}{\mathrm{sh}(T/T^*)} \frac{\partial}{\partial \mu} G_1(\mu)\,, \tag{13}$$

$$I_\mathrm{c}(\Delta T) = -\Delta T \frac{\pi^2 T}{e} \frac{1}{\mathrm{sh}(T/T^*)} \frac{\partial}{\partial \mu} G_{\mathrm{c}1}(\mu)\,. \tag{14}$$

Here the index "1" denotes the first harmonic of the quantity that corresponds to the term with $n = 1$ in expansions (7) and (8).

From eqs. (11)-(14) it follows, that in the linear-response regime the thermoelectric current (in absolute magnitude) has its maximum at $T \simeq T^*$. The magnitude of the maximum is $I_{\max}/I_0 \simeq \Delta T/\Delta_\mu$. Note that the sign of the current the low-temperature (11), (12) and the high-temperature (13), (14) limits can be different.

Furthermore, we consider the thermoelectric current at an arbitrary ratio between the temperatures of the reservoirs T_L and T_R. The characteristic feature of this regime is that the averaging over the energy of g_ε and c_ε for electrons moving from the left to the right (with temperature T_{L}) and from the right to the left (with temperature T_{R}) are two independent processes. As a result, reservoir temperatures are well distinguished ($|T_{\mathrm{R}} - T_{\mathrm{L}}| \gg T^*$), the magnitude of the thermoelectric current is determined by the lower of the two temperatures and does not depend on the other temperature. At $T_{\mathrm{R}} > T_{\mathrm{L}} \gg T^*$ the expressions for the currents are

$$I(T_{\mathrm{L}}) = -T^* \frac{\pi^2 T_{\mathrm{L}}}{e} \frac{1}{\mathrm{sh}(T_{\mathrm{L}}/T^*)} \frac{\partial}{\partial \mu} G_1(\mu)\,, \tag{15}$$

$$I_{\mathrm{c}}(T_{\mathrm{L}}) = -T^* \frac{\pi^2 T_{\mathrm{L}}}{e} \frac{1}{\mathrm{sh}(T_{\mathrm{L}}/T^*)} \frac{\partial}{\partial \mu} G_{\mathrm{c}1}(\mu)\,. \tag{16}$$

Let us briefly consider the dependence of the thermoelectric current on the chemical potential. g_ε and c_ε at $Lk_\mu \gg 2\pi$ are periodic in μ with period Δ_μ, therefore the thermoelectric current has the same period. The function $I(\mu)$ (as well as $I_{\mathrm{c}}(\mu)$) contains various harmonic (with period Δ_μ/n, where $n = 1, 2\ldots$). However, at $T > T^*$ only the first harmonic survives (see eqs. (13)-(16)).

By varying the potential step height $e\varphi$, the amplitude of the first harmonic of the function $g(\mu)$ may be vanishing. In this case the function $I(\mu)$ at high temperatures (namely, at $T \gg T^*/2$) has period $\Delta_\mu/2$ (in eq. (8) the term with $n = 1$ is equal to zero, and it is necessary to keep the term with $n = 2$). At the same time the function $I_{\mathrm{c}}(\mu)$ at $T \gg T^*$ has, still, a period Δ_μ. Besides, in this case the magnitude of the transport current decreases faster with the increase of the temperature: $I_{\mathrm{c}}(T)/I(T) \simeq \exp[T/T^*] \gg 1$, at $T \gg T^*$.

In the present paper the thermoelectric current in a ballistic one-dimensional asymmetric ring coupled via one-dimensional ballistic leads to two electron reservoirs, which have identical chemical potentials, is considered. The dependence of the current on the reservoir temperatures is investigated. It is shown, that at a small difference of the reservoir temperatures, $\Delta T \ll T^*$, the dependence of the current on the temperature has its maximum at $T \simeq T^*$. At the same time at $\Delta T \gg T^*$ the magnitude of the current does not depend on the temperature of the more heated reservoir.

It is shown, that under some conditions the magnitude of the circulating current in the ring can greatly exceed the magnitude of the transport current.

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