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SOLIDS
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Influence of the Coulomb Blockade Effect on Heat Transfer in a One-Dimensional System of Spin-Zero Electrons

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Abstract—An analysis is made of some characteristics of the low-temperature thermal conductivity of a ballistic quantum dot, attributed to the influence of long-range Coulomb interaction in the geometric capacitance approximation. It is shown that at fairly low temperatures the thermal conductivity K exhibits Coulomb oscillations as a function of the electrostatic potential of the quantum dot. At the maximum of the Coulomb peak we find $K \propto T$ whereas at the minimum $K \propto T^3$. The dependence $K(T)$ is essentially nonmonotonic at temperatures corresponding to the characteristic spacing between the size-quantization levels in the quantum dot. © *2000 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

Electrostatic energy strongly influences charge transport in mesoscopic systems connected by tunneling junctions with the surroundings (supply conductors) at low temperatures [1–4]. Electron tunneling through a potential barrier is accompanied by a change in the charge of the mesoscopic sample by $1e$ and a change in the system energy by $E_c = e^2/(2C)$, where e is the electron charge and C is the electrostatic capacitance of the sample. At temperatures $T \ll E_c$ charge transport is generally strongly suppressed (Coulomb blockade effect) [5–10].[1] However, for certain values of the potential difference V_g between the sample and the surroundings the electrostatic energy of the system $E = (q - eN)^2/(2C)$ (where q is the charge of the sample; $N = CV_g/e$) is degenerate with respect to change in the charge by $1e$: $q \longleftrightarrow q + e$ (this occurs for half-integer values of N) [7, 11]. In this case, the Coulomb blockade is broken and this is manifest as an appreciable increase in the conductance of the system.

A similar effect should be observed for other kinetic coefficients, in particular for the thermal conductivity K which is studied in the present paper. In a one-dimensional system the nontrivial appearance of the Coulomb blockade effect in the heat transport case consists in (neutral) electron–hole pairs making a considerable contribution to the heat transfer because of the following circumstance. It was shown in [12] that for a quantum dot (i.e., a phase-coherent mesoscopic sample connected by two quantum point contacts to supply conductors) in the strong tunneling regime, integration of the charge fluctuations of the quantum dot at low temperatures $T \ll E_c$ can reduce the problem of tunneling of spin-zero Fermi electrons through a double barrier to the tunneling of a Luttinger liquid [13–16] with $g = 1/2$ (where g is the Haldane parameter) through an isolated impurity [17]. Heat transfer in a Luttinger liquid with an isolated impurity was considered in [18]. It was shown that at low temperatures in addition to the electron (caused by electron tunneling) contribution to the thermal conductivity $K_e \propto T^{2/g-1}$, electron–hole pairs [plasmons, i.e., small-amplitude fluctuations of the boson (phase) field describing a Luttinger liquid] also make a significant contribution $K_p \propto T^3$. In the case $g = 1/2$ these contributions are of the same order of magnitude. It should be noted that the important role of plasmons in heat transfer in a Luttinger liquid was noted in [19–21].

Thus, heat transfer across a double potential barrier (quantum dot) under conditions when the electrostatic energy is substantial (i.e., when $T \ll E_c$) is accomplished by electrons and by neutral particles (plasmons).[2] At low temperatures both contributions are of the same order of magnitude. This factor can be used to develop a theory of heat transport based on a self-consistent harmonic approximation [22, 23]. This approximation in fact describes plasmon propagation. However, electron tunneling processes are also partly taken into account by renormalizing the potential barrier height.

It should be noted that studies of heat transport are important first from the point of view of observing the non-Fermi liquid behavior of an electron system [19–21]. Second, heating effects also influence the proper-

[1] For mesoscopic samples the capacitance may reach $C \leq 10^{-15}$ F which corresponds to $E_c \geq 1$ K.

[2] The present study only takes into account the long-range Coulomb interaction of electrons in the quantum dot described in the approximation of the geometric capacitance C. Allowance for short-range interelectron interaction ($g \neq 1$) like the spin will modify the dependence $K(T, V_g)$ and is not considered here. Note that the spin-zero electron model can be used in the presence of a strong magnetic field which polarizes the electron gas near the quantum dot.

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ties of mesoscopic systems which exhibit a Coulomb blockade effect (single-electron transistors and so on) [24], which requires a study of heat transfer processes in these systems.

In the present study we calculate the thermal conductivity K of a quantum dot as a function of temperature T and potential V_g. The dependence $K(V_g)$ contains peaks corresponding to destruction of the Coulomb blockade. The form of the Coulomb peak depends on temperature. The dependence $K(T)$ is essentially nonmonotonic at temperatures corresponding to the spacing between the size-quantization levels in a quantum dot.

2. FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

We shall consider a one-dimensional ballistic channel containing spin-zero noninteracting electrons as our model. Two point potential barriers of height V_1 and V_2 positioned at points $x_1 = -d/2$ and $x_2 = d/2$ simulate the quantum dot. With respect to the rest of the channel, the quantum dot has the potential V_g which can be varied by using an additional metal electrode (gate). One-dimensional conductors corresponding to $x < x_1$ and $x > x_2$ connect the quantum dot to remote reservoirs having the temperature T and chemical potential μ. We shall neglect any inelastic processes in the system (quantum dot plus supply conductors) and we shall consider the quantum dot as a purely elastic scatterer. This holds at fairly low temperatures when the phase coherence length of the electrons $L_\phi(T)$ is greater than the distance between the electron reservoirs. Heat transport (like charge transport) is understood in the usual meaning for mesoscopic physics [25, 26] as transport between (remote) reservoirs where electron energy relaxation takes place. In calculations of the thermal conductivity in the linear approximation we assume that the electron reservoirs have the same chemical potentials $\mu_1 = \mu_2 = \mu$ and their temperatures differ by the small amount $\Delta T = |T_1 - T_2| \ll T$.

This highly simplified model can nevertheless allow for the influence of the electrostatic energy and the spatial quantization in the quantum dot [27] on the electron transport at low temperatures. Note that this model [28] corresponds to the experimental situation where a ballistic quantum dot is connected to supply conductors using single-mode quantum point contacts whose transmission coefficient may vary between zero and one.

As we know, in the one-dimensional case the electrostatic energy (in the geometric capacitance approximation) can be taken into account exactly (beyond the limits of perturbation theory) using the bosonization method [14, 15]. In order to describe the low-energy properties of the system ($\Delta\epsilon \ll \mu$) we can linearize the electron spectrum near the Fermi energy μ. In this case, the Lagrangian density of the spin-zero electrons has the form [17]

$$L_0 = \frac{\hbar v}{2g}\left\{\frac{1}{v^2}\left(\frac{\partial\theta(x,t)}{\partial t}\right)^2 - \left(\frac{\partial\theta(x,t)}{\partial x}\right)^2\right\}, \quad (1)$$

where v, g are the Haldane parameters [16]. In the present case we confine our analysis to noninteracting electrons: $g = 1$; $v = v_F = \pi\hbar\rho_0/m^*$, where v_F is the Fermi velocity, ρ_0 is the average electron density, and m^* is the effective electron mass. The boson (phase) field $\theta(x, t)$ determines the deviation $\delta\rho$ of the electron density from the average density and the electron current j:

$$\delta\rho = \frac{1}{\pi^{1/2}}\frac{\partial\theta(x,t)}{\partial x}, \quad j = \frac{e}{\pi^{1/2}}\frac{\partial\theta(x,t)}{\partial t}.$$

The presence of potential barriers is taken into account by the following Lagrangian [17]:

$$\begin{aligned} L_V = & -V_1\cos(2\pi^{1/2}\theta(x,t) - k_F d)\delta(x + d/2) \\ & - V_2\cos(2\pi^{1/2}\theta(x,t) + k_F d)\delta(x - d/2), \end{aligned} \quad (2)$$

where $k_F = \pi\rho_0$ is the Fermi wave number. We shall assume that in the energy range of interest to us $\Delta\epsilon \sim T \ll \mu$, the values of V_1 and V_2 do not depend on the electron energy. Following [12], we can assume $V_i = \mu\pi^{-1}r_i/t_i$ ($i = 1, 2$), where r_i and t_i are the moduli of the reflection coefficient and the transmission coefficient for electrons having the Fermi energy, which characterize a point potential barrier at the point x_i. Note that according to the Landauer–Büttiker approach [29], the conductance of an isolated barrier for the case of one-dimensional spin-zero noninteracting electrons is $G_i = G_0 t_i^2$, where $G_0 = e^2/(2\pi\hbar)$ is the conductance quantum. For interacting electrons ($g \neq 1$) this is not the case which leads to a temperature dependence of the conductance at low temperatures [17].

The electrostatic energy associated with the capacitance C of the quantum dot is described by

$$\int_{-\infty}^{\infty} L_C(x,t)dx = -E_c\left\{\int_{x_1}^{x_2}\delta\rho(x,t)dx - N\right\}^2. \quad (3)$$

The partition function Z required to describe the properties of the system may be expressed as a functional integral:

$$Z = \int\exp\left(-\frac{S_E}{\hbar}\right)D\theta. \quad (4)$$

The Euclidean (calculated at imaginary time $\tau = it$) action is

$$S_E = -\int_{-\infty}^{\infty} dx \int_0^{\beta} d\tau (L_0 + L_V + L_C),$$

where $\beta = \hbar/T$.

Since the nonquadratic part of the Lagrangian (L_V) with respect to θ only depends on the fields at two fixed points, we can integrate over the fluctuations of the field θ at all points apart from $x = x_1$ and $x = x_2$. As a result, we obtain the effective Euclidean action [27]:

$$\begin{aligned} S_{eff}[\theta, \phi] = {}& \frac{\hbar}{\beta}\sum_{n=-\infty}^{\infty}\left\{\frac{2|\omega_n||\theta_n|^2}{1+\exp(-|\omega_n|/\Delta\omega)} + \frac{|\omega_n||\phi_n|^2}{2[1-\exp(-|\omega_n|/\Delta\omega)]}\right\} \\ & + \int_0^{\beta} d\tau \{ V_1 \cos(\pi^{1/2}[2\theta(\tau) - \phi(\tau)] - k_F d) \\ & + V_2 \cos(\pi^{1/2}[2\theta(\tau) + \phi(\tau)] + k_F d) \} \\ & + \frac{E_c}{\pi}\int_0^{\beta}\{\phi(\tau) - \pi^{1/2}N\}^2. \end{aligned} \tag{5}$$

Here we introduce the following notation: $\omega_n = 2\pi n/\beta$ is the Matsubara frequency (n is an integer); $\Delta\omega = v_F/d$; $\theta(\tau) = [\theta(x_2, \tau) + \theta(x_1, \tau)]/2$; $\phi(\tau) = \theta(x_2, \tau) - \theta(x_1, \tau)$; θ_n and ϕ_n are the coefficients of the Fourier series expansion:

$$x(\tau) = \frac{1}{\beta}\sum_n \exp(-i\omega_n\tau)x_n + \zeta_x$$

($x \equiv \theta, \phi$; ζ_x is the zeroth-order mode). Note that the field $\phi(\tau)$ determines the excess charge of the quantum dot $\delta q = e\pi^{-1/2}\phi$ and the field $\theta(\tau)$ determines the transmitted current:

$$I = \frac{ie}{\pi^{1/2}}\frac{\partial\theta(\tau)}{\partial\tau}$$

[17, 27].

We then obtain an approximate expression for $S_{\rm eff}$ which corresponds to small-amplitude oscillations of the boson field $\theta(\tau)$. For this we first assume that V_1 and V_2 are small (i.e., V_1, $V_2 \ll \mu$) and we integrate over charge fluctuations of the quantum dot (over fluctuations of the field ϕ).

If the condition

$$T \ll E_c \tag{6}$$

is satisfied, we obtain the effective action in the following form:

$$\begin{aligned} S'_{eff}[\theta] = {}& \frac{\hbar}{\beta}\sum_n \frac{2|\omega_n|}{1+\exp(-|\omega_n|/\Delta\omega)}|\theta_n|^2 \\ & + V\int_0^{\beta} d\tau \cos(2\pi^{1/2}\theta(\tau)). \end{aligned} \tag{7}$$

The value of V is defined as

$$\begin{aligned} V = {}& \left(\frac{2\gamma E_c}{\pi\mu}\right)^{1/2} \\ & \times [V_1^2 + V_2^2 + 2V_1V_2\cos(2\pi N + 2k_F d)]^{1/2}, \end{aligned} \tag{8}$$

where $\gamma = e^C$, $C \approx 0.5772$ is the Euler constant. We shall subsequently only allow for small fluctuations of the field θ and set $\cos(2\pi^{1/2}\theta) \approx 1 - 2\pi\theta^2$. We then need to renormalize the potential $V \longrightarrow V^*$ [22, 23] after integrating over high-frequency fluctuations whose energy $\hbar\omega_N$ exceeds the renormalized potential V^*. This is possible since these fluctuations are not sensitive to the potential V^* and may be considered as free-field fluctuations. The final expression for the effective action in the self-consistent harmonic approximation has the following form:

$$S_{SCHA}[\theta] = \frac{\hbar}{\beta}\sum_n\left\{\frac{2|\omega_n|}{1+\exp(-|\omega_n|/\Delta\omega)} - \omega_V\right\}|\theta_n|^2. \tag{9}$$

The value of ω_V is obtained from

$$\hbar\omega_V = 2\pi V^2/\mu, \tag{10}$$

where the Fermi energy μ plays an rf cutoff role. We shall use this action (9) to describe heat transport across a quantum dot assuming that the results will be valid for any (and not only small) value of the scattering potentials V_1 and V_2. As was noted in the Introduction, this approximation describes plasmon transport which in this particular case yields the same temperature dependence of the thermal conductivity as for electron transport ($K \propto T^3$ for $V \gg T$ and $K \propto T$ for $V = 0$).

We shall analyze heat transfer Q across a quantum dot in the linear regime characterized by the thermal conductivity $K = -Q/\Delta T$, where $\Delta T = T_2 - T_1 \ll T$ is the temperature difference between the electron reservoirs to the left ($x < x_1$) and right ($x > x_2$) of the quantum dot ($\mu_1 = \mu_2 = \mu$) [26]. It was shown in [18] that the thermal conductivity is expressed in terms of the electrical conductance $G(\omega)$:

$$K = \frac{\hbar^3}{4e^2T^2}\int_0^{\infty} d\omega \frac{\omega^2 {\rm Re}G(\omega)}{\sinh^2(\hbar\omega/(2T))}, \tag{11}$$

where ${\rm Re}G(\omega)$ is the real part of $G(\omega)$.

In order to calculate the dependence $G(\omega)$ in the model (9), we use the Kubo formula [30, 31]:

$$G(\omega) = \frac{-i}{\hbar\omega} \lim_{\omega_n \to -i\omega} \int_0^\beta d\tau \exp(i\omega_n \tau) \langle I(\tau) I(0) \rangle. \quad (12)$$

Substituting the formula for the current I expressed in terms of $\theta(\tau)$we obtain

$$G(\omega) = \frac{e^2 T}{\pi\hbar^2} \lim_{\omega_n \to -i\omega} \omega_n \Phi_n,$$

where $\Phi_n = \langle \theta_n \theta_{-n} \rangle$. Averaging

$$\Phi_n = \frac{1}{Z} \int \theta_n \theta_{-n} \exp(-S_E/\hbar) D\theta$$

can easily be performed if the following fictitious term is added to the action

$$S_j = \hbar \sum_n j_n \theta_{-n}.$$

Then we have

$$\Phi_n = \frac{1}{Z} \left(\frac{\delta^2 Z}{\partial j_n \delta j_{-n}} \right)_{j=0}.$$

Direct calculations using the effective action (9) give

$$\Phi_n = \frac{\beta}{2} \left[\frac{2\omega_n}{1 + \exp(-\omega_n/\Delta\omega)} - \omega_V \right]^{-1}.$$

Finally the rf conductivity of the quantum dot in this model is given by

$$\mathrm{Re}G(\omega) = G_0 \frac{\omega^2 \left(1 + \cos\frac{\omega}{\Delta\omega}\right)}{\omega_V^2 \left(1 + \cos\frac{\omega}{\Delta\omega}\right) + 2\omega_V \omega \sin\frac{\omega}{\Delta\omega} + 2\omega^2}. \quad (13)$$

Expressions (13) and (11) determine the thermal conductivity of the quantum dot allowing for the Coulomb blockade effect in the self-consistent harmonic approximation. The dependence $K(T, \omega_V)$ will be analyzed in the following section.

It should be stated that the expression obtained for $G(\omega)$ describes the plasmon contribution to the conductivity of a double barrier (a similar expression for a single barrier was given in [22]). In the limit $\omega \longrightarrow 0$ this contribution vanishes since these plasmons (small-amplitude oscillations of the boson field corresponding to electron–hole pairs) are neutral particles and do not carry charge [22]. In the formalism used, topological excitations of the boson (phase) field [16] carry charge [16] which corresponds to an appreciable change in the value of θ (transport of a single electron across the barrier corresponds to a change in θ by $\pi^{1/2}$ [17]). Note that expression (13) was obtained assuming that the characteristic energy scale (T or $\hbar\omega$) is much less than E_c (6).

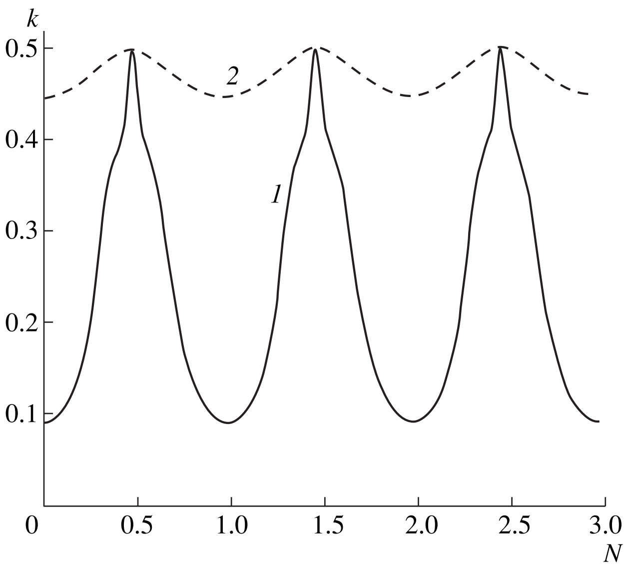

Fig. 1. Dependences of the normalized thermal conductivity $\kappa = K/K_0$ ($K_0 = \pi T/(6\hbar)$) on the quantum dot potential $N = CV_g/e$ for low ($\hbar\omega_{V0}/T = 10$, curve *1*) and high ($\hbar\omega_{V0}/T = 0.1$, curve *2*) temperatures. The curves were plotted for a symmetric quantum dot ($V_1 = V_2$) for $T = \Delta_F$.

3. THERMAL CONDUCTIVITY OF A SYMMETRIC QUANTUM DOT

In this section we consider the case where the potential barriers separating the quantum dot from the supply conductors have the same height: $V_1 = V_2$, $r_i = r$, $t_i = t$ ($i = 1, 2$). In this case we have

$$\omega_V = \omega_{V0} \cos^2(\pi N + k_F d),$$
$$\hbar\omega_{V0} = \frac{16\gamma E_c}{\pi^2} \left(\frac{r}{t}\right)^2. \quad (14)$$

It can be seen from this expression that for certain values of $N(V_g)$ the effective potential barrier disappears ($\omega_V = 0$ as a result of the energy degeneracy of the system with respect to a change in the number of particles in the quantum dot by one ($q \longleftrightarrow q + 1$) [7, 11] and corresponds to destruction of the Coulomb blockade. As a result, the heat flux increases appreciably and this is observed as a series of peaks on the dependence $K(V_g)$ (Fig. 1). This effect is exactly the same as the conductance oscillations [1–4].

For the case of an asymmetric quantum dot ($V_1 \neq V_2$) the value of V (8) does not vanish for any V_g so that the oscillations on the dependence $K(V_g)$ are weak.

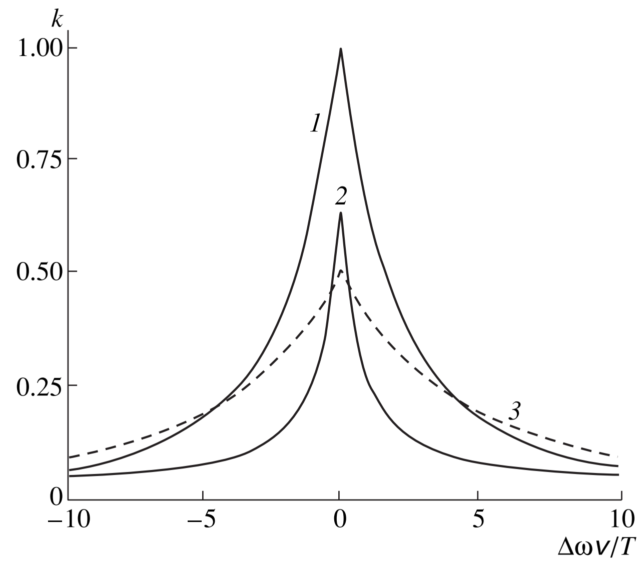

Fig. 2. Dependences of the normalized thermal conductivity κ on the potential barrier height for a symmetric quantum dot. The value of $\Delta\omega_V$ is measured from the value of ω_V corresponding to the position of the maximum. The curves correspond to $T \ll T^*$ (*1*); $T = 2T^*$ (*2*); $T \gg T^*$ (*3*) ($T^* = \Delta_F/\pi^2$).

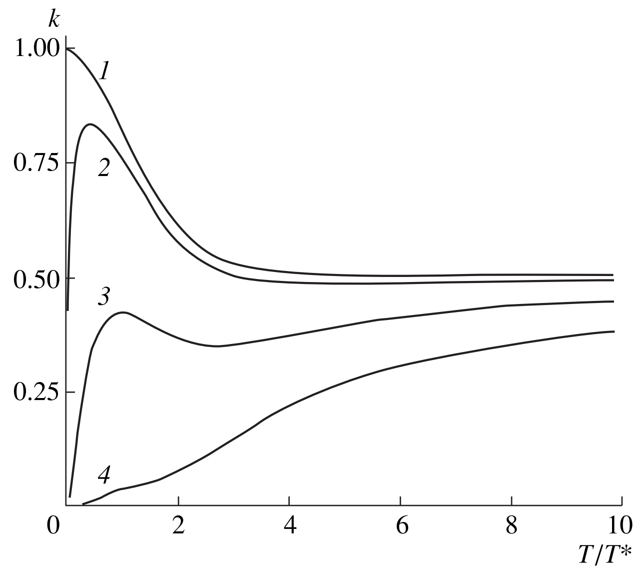

Fig. 3. Temperature dependences of the normalized thermal conductivity for a symmetric quantum dot for $\omega_V = 0$ (*1*); $\hbar\omega_V/T^* = 0.1$ (*2*), 1 (*3*), and 10 (*4*).

3.1. Shape of Coulomb Peak

The shape of the peak on the dependence $K(V_g)$ depends strongly on temperature (see Fig. 2). A crossover takes place at $T \sim T^*$ where $T^* = \Delta_F/\pi^2$; $\Delta_F = \pi\hbar v_F/d$ is the spacing between the spatial quantization levels near the Fermi energy μ in an isolated ($V_{1,2} \longrightarrow \infty$) quantum dot. Near the maximum of the peak ($\hbar\omega_V \ll T$) the thermal conductivity is given by

$$\frac{K}{K_0} = 1 - \frac{3\hbar\omega_V}{2\pi T}, \quad \hbar\omega_V \ll T \ll T^*, \tag{15}$$

$$\frac{K}{K_0} = \frac{1}{2} - \frac{3\hbar\omega_V}{2^{5/2}\pi T}, \quad T \gg T^*, \hbar\omega_V. \tag{16}$$

Here $K_0 = \pi T/(6\hbar)$ is the thermal conductivity of a one-dimensional ballistic channel [32, 33]. Thus, as the temperature increases, the thermal conductivity at the peak maximum is halved. This is attributable to the influence of the electrostatic energy (capacitance C) which is responsible for the frequency dispersion of the conductance $G(\omega)$ (13).[3] At low temperatures ($T \ll T^*$) the main contribution to the thermal conductivity is made by low-frequency (long-wavelength) plasmons which do not "sense" the internal structure of the double barrier. In this case, the thermal conductivity of the system (at the peak maximum) is determined by the thermal conductivity of the one-dimensional ballistic channel. As the temperature increases, as a result of the destructive interference of plasmon contributions at different frequencies the thermal conductivity is halved. This can effectively be considered to be the result of the incoherent (at $T \gg T^*$) propagation of plasmons through two barriers. In this case, at the peak maximum ($\omega_V = 0$) we have two series-connected incoherent (classical) contacts (barriers having the transmission coefficient $t = 1$) each characterized by the thermal conductivity K_0. In this case, the thermal conductivity of the system will be $K = K_0/2$ (for similar reasoning on the electrical conductance see [12, 17]). However, it should be stressed that the halving of the thermal conductivity is caused by averaging over temperature in the phase-coherent system in the absence of real inelastic processes (which take place far from the system in the electron reservoirs). The destructive interference effect with increasing temperature is fairly general for the mesoscopic physics of ballistic structures. The characteristic energy scale $T^* \sim \Delta_F/\pi^2$ was first introduced in the persistent current problem [34]. This energy scale is also important for describing the kinetic properties of ballistic mesoscopic samples [35].

3.2. Temperature Dependence of the Thermal Conductivity

Figure 3 shows temperature dependences of the thermal conductivity for various values of the effective potential barrier. It can be seen that for a small barrier

$$\hbar\omega_V \sim T^*, \tag{17}$$

the dependence $K(T)$ is essentially nonmonotonic for $T \sim T^*$ because of the relative influence of two effects. First the destructive interference of plasmon contribu-

[3] t should be stated that for $E_c = 0$ and $\hbar\omega_V = 0$ we obtained $G(\omega) = G(0)$.

tions at different energies leads to a reduction in K with increasing temperature. Second, an increase in the thermal conductivity is caused by an increase in the number of above-barrier (ballistic) plasmons with increasing temperature. Note that the condition (17) can only be satisfied in the immediate vicinity of the maximum of the $K(V_g)$ peak or for barriers having a low reflection coefficient ($r \longrightarrow 0$; $t \sim 1$) (strong tunneling). In this last case, the dependence $K(V_g)$ only contains weakly defined oscillations (see Fig. 1, curve 2).

Analytic expressions for the dependence $K(T)$ can be obtained for $\omega_V = 0$ and $\hbar\omega_V \gg T$.

3.2.1. Thermal Conductivity at the Maximum of the Coulomb Peak

Assuming that $\omega_V = 0$, we obtain from (13) and (11)

$$\frac{K}{K_0} = \frac{1}{2}\left[1 + 3\frac{(T/T^*)\tanh(T/T^*) - 1}{\sinh^2(T/T^*)}\right], \qquad (18)$$

where $T^* = \Delta_F/\pi^2$. The dependence $K(T)$ is plotted in Fig. 3 (curve *1*). Note that a similar crossover for the conductance at the maximum of the Coulomb peak (at $g = 1$) was obtained in [27].

3.2.2. Thermal Conductivity Far From the Maximum of the Coulomb Peak. We shall now assume that the following condition is satisfied

$$\hbar\omega_V \gg T, T^*. \qquad (19)$$

In this case the main heat transfer mechanism is plasmon tunneling. For $T \ll T^*$ expression (13) yields $\mathrm{Re}\,G(\omega) = G_0(\omega/\omega_V)^2$. Substituting into (11), we obtain

$$K(T) = K_0\frac{4\pi^2}{5}\left(\frac{T}{\hbar\omega_V}\right)^2, \quad T \ll T^*. \qquad (20)$$

A similar expression was first obtained in [18, 21] for a single potential barrier.

At higher temperatures ($T \gg T^*$) the thermal conductivity is strongly influenced by an effect involving the resonant tunneling of plasmons through the quantum dot. The importance of allowing for this effect was emphasized in [20, 21].

Resonant tunneling occurs for plasmons of frequency $\omega \simeq \omega_n^r = \omega_n[1 + 2\Delta_F/(\pi\hbar\omega_V)]$, where $\hbar\omega_n = \Delta_F(2n + 1)$, $n = 0, 1, \ldots$. Under condition (19) expression (13) may be represented as a sum of Breit–Wigner resonances and a quadratic background in terms of frequency:

$$\mathrm{Re}\,G(\omega) = G_0\left\{\sum_{n=0}^{\infty}\frac{\Gamma_n^2}{(\omega - \omega_n^r)^2 + \Gamma_n^2} + \left(\frac{\omega}{\omega_V}\right)^2\right\}, \qquad (21)$$

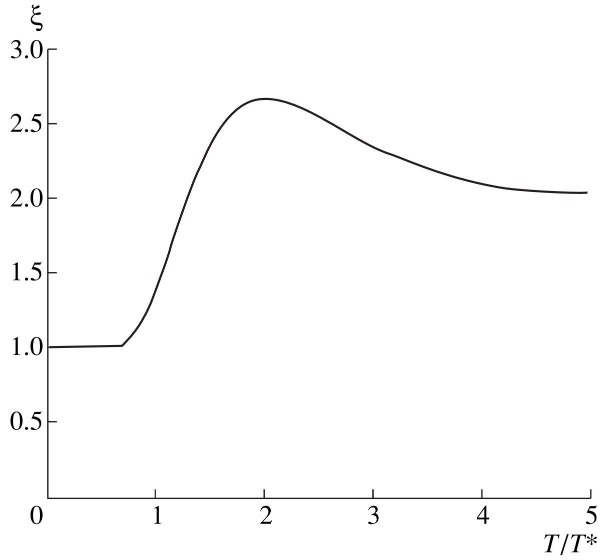

Fig. 4. Temperature dependence of the coefficient ξ in the expression $K/K_0 = (4\pi^2/5)\xi(T)(T/\hbar\omega_V)^2$ for the thermal conductivity of a symmetric quantum dot far from the maximum of the Coulomb peak.

where the resonance width is

$$\Gamma_n = \frac{2\Delta_F}{\pi}\left(\frac{\omega_n}{\omega_V}\right)^2. \qquad (22)$$

Substituting (21) into (11), we obtain

$$K = K_0\left\{\frac{4\pi^2}{5}\left(\frac{T}{\hbar\omega_V}\right)^2 + \frac{24T\Delta_F}{(\pi\hbar\omega_V)^2}\sum_{n=0}^{\infty}\frac{[\omega_n/(2T)]^4}{\sinh^2[\omega_n/(2T)]}\right\} \qquad (23)$$

(here we neglected the negligible difference between ω_n and ω_n^r). In the limit $T \gg T^*$ we can substitute

$$\sum_n \cdots \longrightarrow \int \cdots \frac{d\omega}{2\Delta_F},$$

which gives

$$K(T) = K_0\frac{8\pi^2}{5}\left(\frac{T}{\hbar\omega_V}\right)^2, \quad T \gg T^*. \qquad (24)$$

On comparing expressions (20) and (24) we can see that in the plasmon tunneling regime, $K \sim T^3$ is obtained over the entire temperature range. However, the proportionality factor is doubled for $T \sim T^*$ (see Fig. 4) as a result of the resonant tunneling effect.

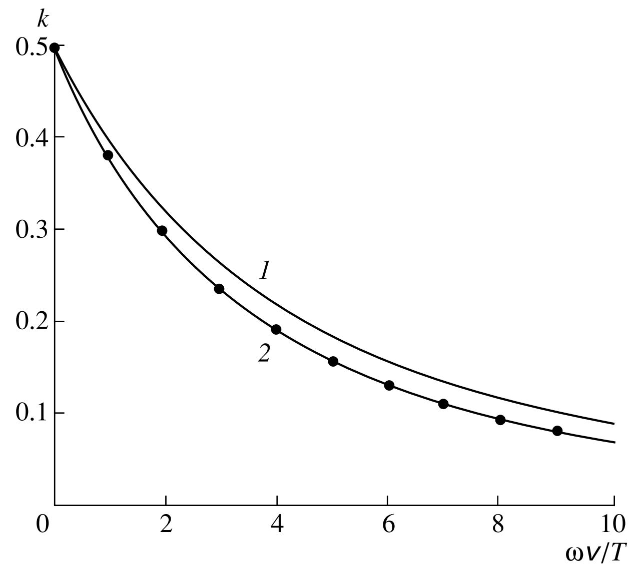

Fig. 5. Dependence of the normalized thermal conductivity κ for a symmetric quantum dot in the strong cotunneling regime ($E_c \gg T \gg \Delta_F$; $r \longrightarrow 0$) on the ratio $\hbar\omega_V/T$. The thermal conductivity was calculated using the self-consistent harmonic approximation (curve *1*), the exact expression for the conductance obtained in [12] (curve *2*), and the self-consistent harmonic approximation using a refined value of the renormalized potential $\omega_V^* = 1.2\omega_V$ (circles on curve *2*).

4. DISCUSSION AND CONCLUSIONS

In the present study we have considered the thermal conductivity of a quantum dot in the Coulomb blockade regime at low temperatures for the case of spin-zero electrons. The thermal conductivity was calculated using a self-consistent harmonic approximation [22, 23] which describes the plasmon heat conduction mechanism. It was shown that the dependence of the thermal conductivity on the potential of the quantum dot contains peaks caused by destruction of the Coulomb blockade.

At the maximum of the Coulomb peak, the thermal conductivity is linear with respect to temperature $K \propto T$ as a result of the ballistic heat transfer regime. However, the proportionality factor is halved for $T > T^*$ (18) because of a transition from coherent plasmon propagation through a two-barrier potential to incoherent propagation through two series-connected barriers. This is consistent with the behavior of the conductance at the maximum of the Coulomb peak (for $g = 1$) [12, 27].

In the plasmon tunneling regime ($\hbar\omega_V \gg T$) the thermal conductivity is $K \propto T^3$ (20), (24). We shall compare the plasmon contribution to the electron tunneling contribution. At low temperatures in the Coulomb blockade regime the dominant electron transport mechanism is elastic ($T \ll \Delta_F$) and inelastic ($T \gg \Delta_F$) cotunneling [8, 9]. Both in the case of a weakly reflecting potential ($r \longrightarrow 0$; $t \longrightarrow 1$) [12] and for a potential having the transmission coefficient $t \longrightarrow 0$ [8, 9] inelastic cotunneling processes lead to a quadratic dependence of the conductance on temperature $G \propto T^2$ (for $T \ll \hbar\omega_V$) which corresponds to the electron contribution to the thermal conductivity $K_e \propto T^3$. This temperature dependence was obtained in our study (24). At lower temperatures ($T \ll T^*$) when the dominant charge transport mechanism is elastic electron cotunneling, the electrical conductance does not depend on temperature $G \propto (\Delta_F/E_c)^2$. In this case the electron contribution to the thermal conductivity exceeds the plasmon contribution $K_p \propto (T/E_c)^2$ (20): $K_e/K_p \sim (\Delta_F/T)^2 \gg 1$. For $T \sim \Delta_F$ (more accurately $T \sim T^*$) these contributions are comparable and the nonmonotonicity caused by the resonant plasmon tunneling (see Fig. 4) may be observed in the total thermal conductivity ($K = K_e + K_p$).

In the strong cotunneling limit ($r \longrightarrow 0$: $E_c \gg T \gg \Delta_F$) the conductance of the quantum dot can be calculated exactly [12]. Substituting this expression into (11), we obtain the thermal conductivity in the strong cotunneling regime (SC):

$$K_{\rm SC} = \frac{\hbar^2}{16\pi T^2}\int_0^\infty d\omega \frac{\omega^2}{\sinh^2(\hbar\omega/(2T))} \times\left\{1 + \frac{1}{\hbar\omega}\int_{-\infty}^{\infty} d\epsilon \frac{\Gamma_0^2}{\epsilon^2+\Gamma_0^2}[f_0(\epsilon+\hbar\omega) - f_0(\epsilon)]\right\}, \tag{25}$$

where $f_0(x) = [\exp(\beta x) + 1]^{-1}$ is the Fermi function,

$$\Gamma_0 = \frac{2\gamma E_c}{\pi^2}[r_1^2 + r_2^2 + 2r_1r_2\cos(2\pi N + 2k_F d)].$$

We compare the coefficient (25) with the thermal conductivity in the self-consistent harmonic approximation (11), (13) (it should be borne in mind that $\hbar\omega_V = 2\Gamma_0$). Figure 5 gives dependences of $K_{\rm SC}$ (curve *1*) and K (curve *2*) on the ratio $\hbar\omega_V/T$ It can be seen that these curves show fairly good agreement. For example, for $T \gg \Gamma_0$ we have

$$K_{\rm SC} = \frac{K_0}{2}\left(1 - \frac{3\pi\Gamma_0}{16T}\right).$$

On comparing with (16) we can see that the deviation from the ballistic value is described by the plasmon approximation with a relative accuracy of around 15%. At low temperatures $\Delta_F \ll T \ll \Gamma_0$, the thermal conductivity is

$$K_{\rm SC} = K_0\frac{3\pi^2}{10}\left(\frac{T}{\Gamma_0}\right)^2,$$

and a comparison with (24) shows that the accuracy is around 30%. The agreement between the plasmon approximation and (25) can be improved by introducing the correction factor $a \approx 1.2$ in the definition of the renormalized potential (10) $\hbar\omega_V^* = 2\pi a V^2/\mu$, which in fact implies a negligible change in the rf cutoff. The resulting dependence $K(\hbar\omega_V^*/T)$ is shown by the circles in Fig. 5.

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