Magnetic and electrostatic Aharonov–Bohm effects in a pure mesoscopic ring

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(Submitted July 10, 1996; revised July 30, 1996)
Fiz. Nizk. Temp. **23**, 425–427 (April 1997)

The effect of weak scalar potential on a persistent current in a one-dimensional ballistic ring is considered. It is shown that the current amplitude oscillates with a change in the potential.

The devising of systems[1,2] whose size L is smaller than the electron phase coherence length L_Φ at low temperatures makes it possible to study phenomena sensitive to the phase of the electron wave function, including the Aharonov–Bohm (AB) effect[3] in solids. This effect lies in the influence of the electromagnetic potential $A^\mu=(\varphi,A)$ on the phase of the electron wave function in the case when the forces acting on the electron can be neglected. This effect is manifested in the kinetic as well as thermodynamic properties of nonsuperconducting, doubly-connected mesoscopic samples.

The AB effect is manifested in kinetics, for example, in the form of oscillations of magnetoresistance of conducting rings in weak magnetic fields[4–6] with the magnetic flux period $\Phi_0=h/e$.

In thermodynamics, the AB effect leads, among other things, to the existence of a persistent (thermodynamically equilibrium) current in mesoscopic rings at low temperatures, which is periodic in the magnetic flux with a period Φ_0.[7] The existence of such a current in real nonsuperconducting metallic one-dimensional rings was predicted theoretically by Buttiker *et al.*[8] The effect was observed experimentally in the ensemble of mesoscopic rings[9] as well as in solitary rings.[10,11] It should be noted that Mailly *et al.*[11] observed a good agreement of the magnitude of the effect with the predictions of the theory for a ballistic multichannel ring.

The influence of the scalar potential (electrostatic AB effect) on the properties of mesoscopic samples has been studied to a smaller extent than the effect of magnetic field (the ''magnetic'' AB effect). The electrostatic AB effect was investigated on individual conducting rings.[12,13] The presence of the scalar potential φ leads to the additional phase lead $\Delta\theta\simeq 2\pi e\varphi\tau_F/h$ of the elecreon wave (τ_F is the time during which a Fermi electron stays in the region with potential φ). It is assumed that a change in the value of φ must lead to a change in the phase of oscillations (e.g., of magnetoresistance) in the magnetic field by $\Delta\theta$. However, no such systematic phase shift was observed in Refs. 12 and 13. It should be noted that the existence of such a phase shift for a ring-shaped sample with two current contacts would contradict the parity requirements in the case of magnetic field sign reversal.[14–16]

This research aims at an analysis of the AB oscillations of thermodynamic parameters of a ballistic one-dimensional ring, associated with a change in the magnitude of the vector and scalar potentials.

By way of a model, we consider a one-dimensional ballistic ring of length $L=2\pi R(L\ll L_\Phi)$ whose segment of length a (Fig. 1) has the potential φ. A solenoid with a magnetic flux Φ is inserted in the ring. We direct the x-axis along the perimeter of the ring. Solving the one-dimensional Schrödinger equation for noninteracting electrons with the potential energy

$$V(x)=\begin{cases} e\varphi, & 0<x<a \\ 0, & a<x<L \end{cases} \tag{1}$$

and with the cyclic boundary conditions

$$\Psi(0)=\Psi(L);\quad \frac{d\Psi}{dx}(0)=\frac{d\Psi}{dx}(L),$$

we obtain the following equation for the eigenvalues of the electron momentum $p=\hbar k$:

$$\cos\left(2\pi\frac{\Phi}{\Phi_0}\right)-\cos(k(L-a)+k'a)+\frac{(k-k')^2}{2kk'}\sin[k(L-a)]\sin(k'a)=0, \tag{2}$$

where $\hbar k=(2mE)^{1/2}$, $\hbar k'=[2m(E-e\varphi)]^{1/2}$. We shall henceforth assume that the potential is weak: $(e\varphi/E)^2\ll 1$. In this case, the reflection of electrons from the potential step can be neglected, and the electron spectrum $E=p^2/2m$ can be determined easily:

$$E_{\pm n}=\frac{h^2}{2mL^2}\left(n\pm\frac{\Phi}{\Phi_0}-\frac{\delta_n}{2\pi}\right)^2,\quad n=0,1,2\ldots \tag{3}$$

Here δ_n is the phase of the ''forward'' scattering for electrons at the nth energy level. For the weak potential (1), we have

$$\delta_n=-2\pi\frac{e\varphi}{\Delta_n}\frac{a}{L}, \tag{4}$$

where $\Delta_n=hv_n/L$ is the separation between the electron energy levels (for $\Phi=0,\varphi=0$) in the vicinity of level $n(n\gg 1)$ and $v_n=\hbar k_n/m$ is the electron velocity.

It should be noted that spectrum (3) is observed not only for a ring with potential (1), but also for a ring with an arbitrary potential scattering only in the ''forward'' direction: the scattering amplitude $t=\exp[i\delta(p)]$. This can be easily verified by the method of transfer matrix (see, for example, Ref. 17).

Using spectrum (3), we can calculate the oscillating component of the thermodynamic potential $\tilde{\Omega}$ of a ring in contact with the electron reservoir (the electron spin is not taken into account; $L\gg\lambda_F$):

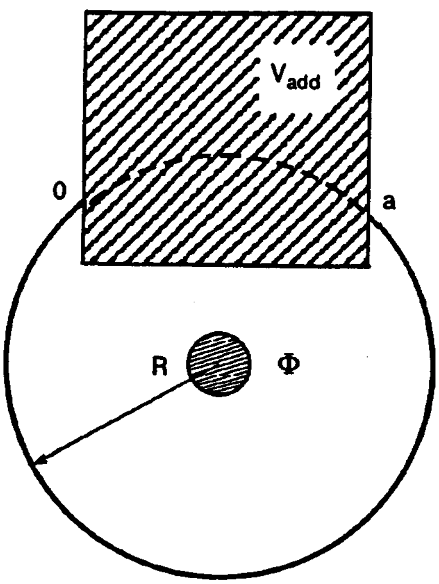

FIG. 1. Model of a one-dimensional ring pierced by the magnetic flux Φ. The metal plate $V_{\rm add}$ induces the local potential φ.

$$\tilde{\Omega}=2T\sum_{q=1}^{\infty}\cos(2\pi q\Phi/\Phi_0)\,\frac{\cos[2\pi q(L/\lambda_F+\delta_F/2\pi)]}{q\,\sinh(qT/T^*)}. \tag{5}$$

Here $T^* = \Delta_F/(2\pi^2)$; $\lambda_F = h/(2m\mu)^{1/2}$, and μ is the chemical potential of the electron reservoir. The subscript F indicates that the value of the quantity is determined on the Fermi level. Differentiating expression (5) with respect to magnetic flux Φ and taking into account expression (4), we obtain the following expression for the experimentally measured quantity, viz., the persistent current $I=-\partial\tilde{\Omega}/\partial\Phi$:

$$I=\frac{2}{\pi}\,I_0\,\frac{T}{T^*}\sum_{q=1}^{\infty}\sin\left(\frac{2\pi q\Phi}{\Phi_0}\right)\frac{\cos[2\pi q(L/\lambda_F-\varphi/\varphi_0)]}{\sinh(qT/T^*)}. \tag{6}$$

Here $I_0 = ev_F/L$ and $e\varphi_0 = \Delta_n L/a$. Expressions similar to (6) for $\varphi=0$ were obtained for a thin-walled cylinder in Ref. 18 and for a one-dimensional ring in Ref. 17.

Thus, the persistent current in the ring oscillates upon a change in magnetic flux as well as electrostatic potential. The existence of these oscillations is associated with a change in the position of quantization levels E_n relative to the level of chemical potential μ.

It should be noted that the presence of the potential $\varphi(x)$ scattering only in the "forward" direction as well as the magnetic flux Φ can be actually taken into account by introducing the corresponding changes in the boundary conditions.[19] This can be clearly seen from the eigenvalue equation for the electron momentum in the ring, i.e.,

$$k_nL=2\pi n+2\pi\Phi/\Phi_0-\delta(k_n).$$

However, in contrast to the magnetic contribution, the electrostatic correction depends on the electron momentum and changes sign upon the sign reversal of k_n. The latter circumstance explains why the oscillating component of thermodynamic potential contains two factors describing the magnetic and electrostatic oscillations, respectively.

It has been shown that the oscillations of thermodynamic parameters of a one-dimensional ballistic ring associated with the electrostatic and magnetic Aharonov–Bohm effects are independent: modulating the amplitudes of each other, they do not affect the phase. Such a behavior is due to a qualitative difference in the effects of the vector and scalar potentials on the electron spectrum (3) in the ring.

[1] Y. Imry, *Physics of Mesoscopic Systems: Directions in Condensed Matter Physics* (ed. by G. Grinstein nd G. Mazenco), World Scientific, Singapore (1986).
[2] S. Washburn and R. A. Webb, Adv. Phys. **35**, 375 (1986).
[3] Y. Aharonov and D. Bohm, Phys. Rev. **115**, 484 (1959).
[4] R. A. Webb, S. Washburn, C. P. Umbach, and R. B. Laibowitz, Phys. Rev. Lett. **54**, 2696 (1985).
[5] V. Chandrasekhar, M. J. Rooks, S. Wind, and D. E. Prober, Phys. Rev. Lett. **55**, 1610 (1985).
[6] S. Datta, M. R. Mellock, S. Bandyopadhyay *et al.*, Phys. Rev. Lett. **55**, 2344 (1985).
[7] A. A. Zvyagin and I. V. Krive, Fiz. Nizk. Temp. **21**, 687 (1995) [Low Temp. Phys. **21**, 533 (1995)].
[8] M. Büttiker, Y. Imry, and R. Landauer, Phys. Lett. **96A**, 365 (1983).
[9] L. P. Levy, G. Dolan, J. Dunsmuir, and H. Bouchiat, Phys. Rev. Lett. **64**, 2074 (1990).
[10] V. Chandrasekhar, R. A. Webb, M. J. Brady *et al.*, Phys. Rev. Lett. **67**, 3578 (1991).
[11] D. Mailly, C. Chapelier, and A. Benoit, Phys. Rev. Lett. **70**, 2020 (1993).
[12] S. Washburn, H. Schmid, D. Kern, and R. A. Webb, Phys. Rev. Lett. **59**, 1791 (1987).
[13] P. G. N. de Vergar, G. Timp, P. M. Mankiewich, *et al.*, Phys. Rev. **B40**, 3491 (1989).
[14] M. Büttiker, IBM J. Res. Dev. **32**, 317 (1988).
[15] C. J. B. Ford, A. B. Fowler, J. M. Hong *et al.*, Sur. Sci. **229**, 307 (1990).
[16] A. Levy Yeyati and M. Büttiker, Phys. Rev. **B52**, R14360 (1995).
[17] H. F. Cheung, Y. Gefen, E. K. Riedel, and W. H. Shih, Phys. Rev. **B37**, 6050 (1988).
[18] I. O. Kulik, Pis'ma Zh. Éksp. Teor. Fiz. **11**, 407 (1970) [JETP Lett. **11**, 275 (1970)].
[19] N. Byers and C. N. Yang, Phys. Rev. Lett. **7**, 46 (1961).

Translated by R. S. Wadhwa