Interference phenomena and ballistic transport in a one-dimensional ring

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The dependence of the conductance of a one-dimensional ballistic ring on a potential barrier at one of the branches of the ring is considered at a nonzero temperature. The case of a small potential barrier (Aharonov–Bohm electrostatic effect) as well as a tunnel barrier is considered. The possibility of direct measurement of the electron wave function phase variation upon tunneling is discussed. [S1063-777X(97)01110-9]

INTRODUCTION

The preservation of phase coherence during propagation of electrons in mesoscopic samples[1] at low temperatures makes the transport properties of such objects sensitive to the phase of the electron wave function. This makes it possible to observe the Aharonov–Bohm (AB) effect in solids.[2,3]

It was shown earlier[4,5] that the physical properties of doubly connected systems containing an AB magnetic flux Φ are periodic in Φ with a period $\Phi_0 = h/e$. Such a periodicity was indeed observed in Refs. 6–10 during measurement of conductance oscillations in isolated conducting rings in a magnetic field. Calculations of conductance for one-dimensional[11–14] as well as multichannel[15] rings also show that the dependence $G(\Phi)$ must have a period Φ_0. However, experiments on chains formed by many rings[16,17] display a $\Phi_0/2$ periodicity which is associated with the ensemble averaging of macroscopically identical but microscopically different characteristics of the rings (length, impurity distribution, etc.).[1,13,15,18] It can be stated that the $\Phi_0/2$ periodicity may be caused by interference of electrons moving along various trajectories as well as along the same trajectory but in opposite directions. The contribution of the latter processes, which was first considered in the weak localization theory by Al'tshuler, Aronov, and Spivak (AAS),[19] is independent of microscopic characteristics of the sample and is therefore preserved upon averaging. According to the prevailing concepts, averaging over an ensemble of rings is equivalent to averaging over electron energy (see Ref. 1).

In analogy with optical phenomena, attempts were made to control the interference pattern in an AB magnetic interferometer (a mesoscopic ring containing the AB magnetic flux) by changing the electron wave function phase with the help of a variable potential barrier created on one of the branches of the ring. Thus, the phase change in Refs. 20 and 21 was caused by the Aharonov–Bohm electrostatic effect. A potential barrier with resonance levels (quantum dot) was used in Ref. 22. It was assumed in these works that the electron wave function phase variation by a quantity θ caused by a potential barrier with transmission coefficient $t = t_0 \exp(i\theta)$ (where i is the imaginary unit) leads to an identical phase shift in the dependence $G(\Phi)$. However, no such phase shift was observed in the dependence $G(\Phi)$. Moreover, the existence of such a phase shift would contradict the requirement of parity of the kinetic coefficients of doubly connected two-terminal mesoscopic samples upon a reversal of the magnetic field,[23] which is confirmed in experiments.[24] Hence Yacoby *et al.*[25] concluded that a two-terminal interferometer cannot be used in principle for a direct measurement of the phase of the transmission coefficient of an electron passing through a potential barrier.

The present paper aims at determining the effect of a potential barrier of height $e\varphi$ on the conductance of an interferometer with an AB magnetic flux, and at illustrating the possibility of using a two-terminal ballistic interferometer to study the coefficient of transmission of an electron through a potential barrier. Such a formulation of the problem is justified by the following arguments. According to Landauer's formula,[1,26] the conductance G of a sample with two terminals connected with the banks is proportional to the square of the amplitude τ of transmission of a Fermi electron through the sample:

$$G = G_0 |\tau(k_F)|^2. \quad (1)$$

Here, $G_0 = 2e^2/h$. In other words, if the contacts are mounted directly on the potential barrier, it is not possible to measure the phase of the transmission coefficient during conductance measurements. The situation becomes quite different if we locate the potential barrier at one of the branches of the ballistic ring. In this case, $\tau \simeq A_{L1R} + A_{L2R}$, where A_{L1R} is the amplitude of transition through the branch containing the potential barrier, and A_{L2R} is the amplitude of transition through the other (ballistic) branch. Writing the dependence on t explicitly ($A_{L1R} = A_{01} t$), we obtain

$$G \simeq G_0 (|A_{01}|^2 t_0^2 + |A_{L2R}|^2 + 2\,\mathrm{Re}(A_{01} A_{L2R}^* t)).$$

The third term in this expression is proportional to the first power of the coefficient t of electron transmission across the potential barrier and depends on its phase θ.

We shall consider the case of a small potential barrier (potential step) $e\varphi \ll \varepsilon_F$ (where ε_F is the Fermi energy of electrons at the banks), as well as a potential barrier of arbitrary height (including a tunnel barrier with $e\varphi \geqslant \varepsilon_F$).

In the former case, the Aharonov–Bohm electrostatic effect is realized. It will be shown below that, in spite of the additive nature of the contributions from vector and scalar potentials to the electron wave function phase, the magnetic and electrostatic AB effects in a one-dimensional ballistic ring are independent of each other as far as the phase is

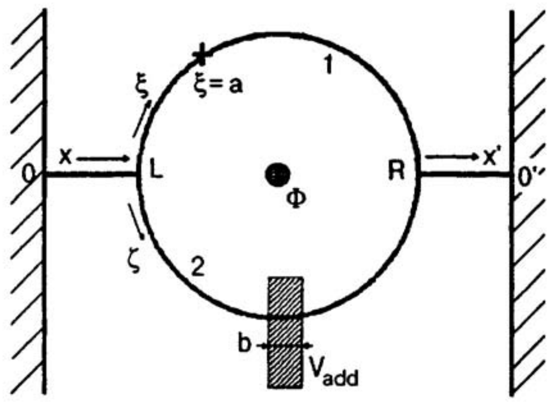

FIG. 1. Model of a one-dimensional ring connected with the banks. The arrows show the positive direction of the coordinate axes. The numbers correspond to the branches of the ring. $0,0'$ are points of contact with the banks, L and R are contact points for terminals, Φ is the magnetic flux, and $V_{\rm add}$ is the potential of the metallic shutter creating a potential step of length b. The cross shows the impurity position, and a is the impurity coordinate.

concerned. In other words, a change in the phase of the electron wave function during the passage across a potential barrier does not change continuously the phase of the dependence $G(\Phi)$. This is due to the different ways in which the vector and scalar potentials affect the spectrum of electron states in the ring.[27] Because of this effect, the interference phenomena in doubly connected systems differ from analogous effects in ballistic flows of particles[28] when the electromagnetic potentials affect only the amplitude of electron transition from the initial to the final point. This is confirmed by the results obtained in Ref. 29 where a phase variation in the dependence $G(\Phi)$ was observed under a scalar potential acting on an electron beam in a two-slit interferometer formed in a two-dimensional electron gas. Such a phase shift corresponds to the additive contribution from scalar and vector potentials to the electron wave function phase. No such phase shift was observed in Refs. 20 and 21 where measurements were made on conducting rings with two terminals.

1. FORMULATION OF THE PROBLEM AND BASIC EQUATIONS

Our aim is to find the conductance G of a one-dimensional ballistic ring with a potential barrier connected to macroscopic banks through one-dimensional conductors (Fig. 1). Following Büttiker *et al.*,[12] we shall solve this problem by using the transfer matrix technique in the approximation of the quantum waveguide theory.[14]

For this purpose, we consider the passage of a plane wave $\exp(ikx)$ through the ring from left to right. According to formula (1), the square of the transmission amplitude $\tau(k_F)$ for an electron with Fermi energy defines the conductance of the system at zero temperature. We single out four one-dimensional segments $0L$, $L1R$, $L2R$, and $R0'$. In each segment, we introduce a coordinate axis with positive direction as indicated by arrows in Fig. 1. We denote the solution of the one-dimensional Schrödinger equation for the wave function of noninteracting electrons at each segment by Ψ_L, Ψ_1, Ψ_2 and Ψ_R and present it in the form

$$\begin{cases} \Psi_L(x)=\exp(ikx)+r(k)\exp(-ikx), \\ \Psi_1(\xi)=(A_1\exp(ik\xi)+B_1\exp(-ik\xi))\exp\left(i2\pi\dfrac{\xi\Phi}{L\Phi_0}\right), \\ \Psi_2(\zeta)=(A_2\exp(ik\zeta)+B_2\exp(-ik\zeta))\exp\left(-i2\pi\dfrac{\zeta\Phi}{L\Phi_0}\right), \\ \Psi_R(x')=\tau(k)\exp(ikx'). \end{cases} \tag{2}$$

Here k is the electron wave vector, Φ the magnetic flux in the ring, and L the length of the ring. In order to find the six unknowns r, A_1, B_1, A_2, B_2, and τ, we use the conditions of continuity of the wave function at the points of intersection of one-dimensional conductors L (coordinates $x=0$, $\xi=0$, $\zeta=0$) and R (coordinates $\xi=L/2$, $\zeta=L/2$, $x'=0$):

$$\begin{cases} \Psi_L(0)=\Psi_1(0)=\Psi_2(0), \\ \Psi_R(0)=\Psi_1(L/2)=\Psi_2(L/2), \end{cases} \tag{3}$$

as well as the conditions of conservation of current at these points:[14]

$$\begin{cases} \dfrac{d\Psi_L}{dx}(x=0)=\dfrac{d\Psi_1}{d\xi}(\xi=0)+\dfrac{d\Psi_2}{d\zeta}(\zeta=0), \\ \dfrac{d\Psi_R}{dx}(x'=0)=\dfrac{d\Psi_1}{d\xi}(\xi=L/2)+\dfrac{d\Psi_2}{d\zeta}(\zeta=L/2). \end{cases} \tag{4}$$

It is convenient to use the matrix approach for solving these equations. For this purpose, we put the column vector $\hat{\Psi}(x)$ in correspondence with the wave function $\Psi(x)=A\exp(ikx)+B\exp(-ikx)$:

$$\hat{\Psi}(x)=\begin{pmatrix}\Psi_+(x)\\ \Psi_-(x)\end{pmatrix}, \tag{5}$$

where $\Psi_+(x)=A\exp(ikx)$ and $\Psi_-(x)=B\exp(-ikx)$. In this case, the wave function $\Psi(x)$ is equal to $\hat{I}\hat{\Psi}(x)$, where $\hat{I}=(1,1)$ is the unit row vector. The boundary conditions (3) and (4) can be represented in matrix form as follows:

$$\begin{cases} \hat{\Psi}_2(0)=\begin{pmatrix}1\\ -1\end{pmatrix}+\hat{T}_L\hat{\Psi}_1(0), \\ \hat{\Psi}_2(L/2)=\hat{T}_R\hat{\Psi}_1(L/2), \end{cases} \tag{6}$$

where

$$\hat{T}_L=\begin{pmatrix}-1 & 1\\ 3 & 1\end{pmatrix},\quad \hat{T}_R=\begin{pmatrix}1 & 3\\ 1 & -1\end{pmatrix}. \tag{7}$$

Here

$$r=-1+\hat{I}\hat{\Psi}_1(0), \tag{8}$$

$$\tau=\hat{I}\hat{\Psi}_2(L/2). \tag{9}$$

Equations (6) must be supplemented by relations connecting the wave function values $\hat{\Psi}_l(0)$ at the beginning and $\hat{\Psi}_l(L/2)$ at the end of each branch $(l=1,2)$. In each particular case, we can obtain such a relation with the help of the transfer matrix.[12,30]

It is well known[12,14] that at a temperature $T=0$, the value of G depends significantly on the product k_FL. In order to study the effect of potential barriers, we also consider the conductance G_T of the ring at a nonzero temperature:[1]

$$G_T = G_0 \int \frac{d\varepsilon}{4T} |\tau(k)|^2 \cosh^{-2}\left(\frac{\varepsilon - \varepsilon_F}{2T}\right), \tag{10}$$

where $\varepsilon = (\hbar k)^2/(2m)$.

2. ELECTROSTATIC AHARONOV–BOHM EFFECT IN THE CONDUCTIVITY OF A ONE-DIMENSIONAL BALLISTIC RING

Let us place a metallic gate over one of the branches of the ring (Fig. 1) with a potential V_{add} which forms a potential step of length b and height $e\varphi$ in the ring. If the condition $|e\varphi| \ll \varepsilon_F$ is satisfied, the coefficient r_φ of reflection of a Fermi electron by the step can be put equal to zero. In this case, the transmission coefficient will be defined as[27]

$$t_\varphi = \exp[i(\delta_F + k_F b)], \tag{11}$$

$\delta_F = 2\pi\varphi/\varphi_0$; $e\varphi_0 = \Delta_F L/b$. The transfer matrix $\hat{T}_\varphi$ for the potential step can be represented in the following form: $\hat{T}_\varphi = \hat{T}_0(\delta_F + k_F b)$, where the ballistic transfer matrix $\hat{T}_0(x)$ is defined as

$$\hat{T}_0(x) = \begin{pmatrix} \exp(ix) & 0 \\ 0 & \exp(-ix) \end{pmatrix}. \tag{12}$$

In this case, the complete transfer matrices for the corresponding branches of the ring can be represented in the form

$$\hat{T}_{L1R} = \hat{T}_0(k_F L/2); \quad \hat{T}_{L2R} = \hat{T}_0(\delta_F + k_F L/2). \tag{13}$$

Note that in, view of the diagonality of matrices $\hat{T}_\varphi$ and $\hat{T}_0$, the matrix $\hat{T}_{L2R}$ does not depend on the position of the potential step.

The relations between the values of wave functions required for solving Eqs. (6) have the following form

$$\begin{cases} \hat{\Psi}_1(L/2) = \hat{T}_{L1R}\hat{\Psi}_1(0)\exp\left(i\pi\dfrac{\Phi}{\Phi_0}\right), \\ \hat{\Psi}_2(L/2) = \hat{T}_{L2R}\hat{\Psi}_2(0)\exp\left(-i\pi\dfrac{\Phi}{\Phi_0}\right). \end{cases} \tag{14}$$

Substituting the solution of Eq. (6) and taking (13) and (14) into account in (9) and then in (1), we obtain

$$G = G_0 \frac{(1-\cos\Delta)(1+\cos\delta_F) + (\cos\delta_F - \cos\Delta)(\cos(2\pi\Phi/\Phi_0) - 1)}{[\cos(2\pi\Phi/\Phi_0) - \cos\Delta + (\cos\delta_F - \cos\Delta)/4]^2 + \sin^2\Delta}. \tag{15}$$

Here $\Delta = k_F L + \delta_F$. Similar expressions were obtained earlier for $\delta_F = 0$ in Refs. 12 and 14.

It follows from formula (15) that G is periodic in φ with a period φ_0. Thus, a long potential barrier of small height ($e\varphi \ll \varepsilon_F$) has a considerable effect on the conductance of a ballistic ring.

For $T=0$, the shape of the curve $G(\varphi)$ depends on the product $k_F L$ (see Fig. 2), and is not symmetric relative to the reversal of the sign of the potential step. However, such a dependence vanishes for $T > T_0 = 0.5\Delta_F$, where $\Delta_F = 2\varepsilon_F \lambda_F/L$ is the separation between the electron energy levels in the ring near the Fermi energy (Fig. 2, curve *3*), and the curve becomes symmetric:

$$G_T(\varphi) = G_T(-\varphi) \quad \text{for } T > T_0. \tag{16}$$

From a formal point of view, the presence of a potential step is equivalent to the nonsymmetric connection of terminals to the ring considered by Xia.[14] Here, $L + \delta_F/k_F \to L_1 + L_2$ and $\delta_F \to k_F \Delta L$, where $\Delta L = L_1 - L_2$, L_1 and L_2 being the lengths of the branches constituting the ring. However, such a correspondence is valid only for $T=0$. For a nonzero temperature, the dependence of G on $k_F \Delta L$ vanishes for $T \geqslant \Delta_F L/\Delta L$, while the dependence on φ is preserved.

The main conclusion that can be drawn from formula (15) is that vector and scalar potentials exert different influence on the conductance of the ring, which is in accord with the results of measurements.[20,21]

Presenting the dependence (15) in the form of a series

$$G(\Phi) = \sum_n C_n(\varphi)\cos\left(2\pi n\frac{\Phi}{\Phi_0}\right), \tag{17}$$

we find that the quantity φ affects only the amplitudes of harmonics of the conductance $G(\Phi)$, and does not affect their phase. Figure 3 shows the dependence of C_1 (curve *1*)

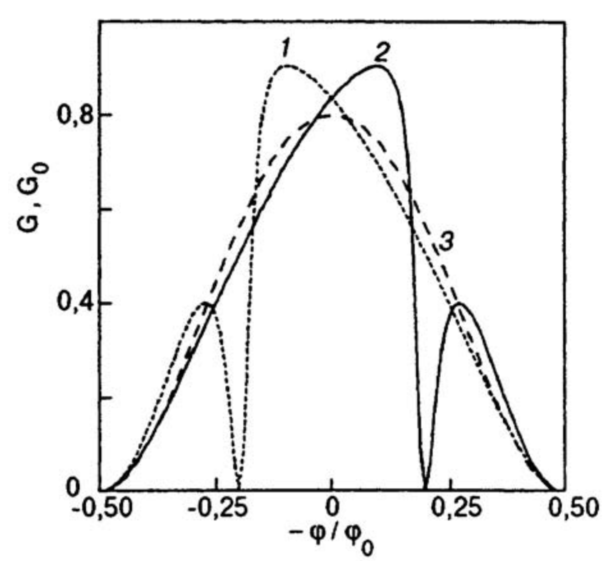

FIG. 2. Dependence of the conductance G of the ring (in units of G_0) on the height φ of the potential step for $T=0$, $\Phi=0$ for $L/\lambda_F = 200.2$ (curve *1*), 200.8 (curve *2*), and at $T = \Delta_F/2$ (curve *3*).

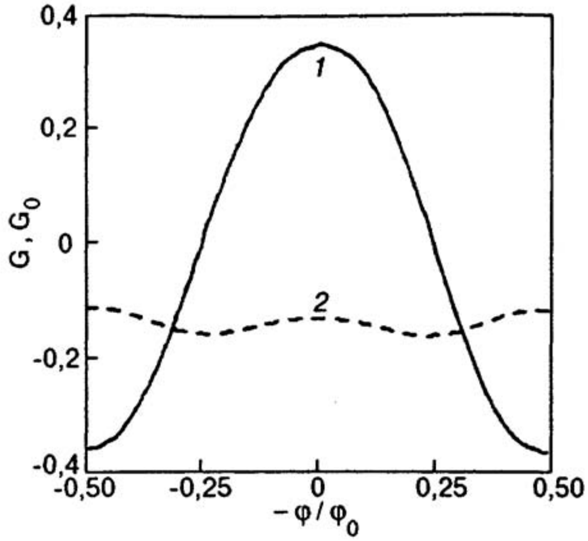

FIG. 3. Dependence of the amplitude of the first (curve *1*) and second (curve *2*) harmonics $G(\Phi)$ (in units of G_0) on the height φ of the potential step for $T=\Delta_F/2$.

and C_2 (curve *2*) on φ/φ_0 at $T=T_0$. For $\varphi=\varphi_1=\pm\varphi_0/4$, C_1 becomes equal to zero, while $C_2(\varphi_1)\neq 0$ and the dependence $G(\Phi)$ near $\varphi=\varphi_1$ is periodic in magnetic flux with a period $\Phi_0/2$. Such a transition was observed experimentally by Yacoby *et al.*[25] Note that the presence of a φ-independent component in the amplitude of the second harmonic C_2 is associated with the contribution from the AAS effect.

For $T>T_0$, the value of G_T remains unchanged upon a sign reversal of Φ or φ.[16] Separating the first harmonic, we can write

$$G_T=0.35\cos\left(2\pi\frac{\Phi}{\Phi_0}\right)\cos\left(2\pi\frac{\varphi}{\varphi_0}\right)+g(\Phi,\varphi). \tag{18}$$

The second term, which contains higher harmonics, is usually smaller than the first term except for the values

$$\Phi=\pm\Phi_0/4, \tag{19a}$$

$$\varphi=\pm\varphi_0/4, \tag{19b}$$

near which G_T is periodic in φ with a period $\varphi_0/2$ (Eq. (19a)) and in Φ with a period $\Phi_0/2$ [Eq. (19b)].

3. BALLISTIC RING WITH A SINGLE IMPURITY

The effect of a single impurity on the conductance of a ballistic ring at $T=0$ was considered earlier in Refs. 12 and 31. We shall consider the case $T\neq 0$. In addition, we shall also consider the effect of the tunneling coefficient phase on the conductance of a ring.

Suppose that an impurity of length d is introduced in one of the branches of the ring at a distance a from the starting point. We denote the transmission and reflection coefficients of a Fermi electron by $t_i=t_0\exp(i\theta)$ and $r_i=r_0\exp(i\rho)$. The other branch of the ring contains a potential step with a transfer matrix $\hat{T}_\varphi$ (see Fig. 1).

Calculations made in a similar manner as above give

$$G=G_0|2X/(Y+Z)|^2,$$

$$X=r_0\exp(i\rho)\cos\left[k_F\left(\frac{L}{2}-d-2a\right)\right]-i\exp(i\theta)\left[\sin\Sigma'+t_0\sin\Delta'\exp\left(2\pi i\frac{\Phi}{\Phi_0}\right)\right],$$

$$Y=r_0\exp(i\rho)\cos\left[k_F\left(\frac{L}{2}-d-2a\right)\right][\cos\Delta'+\exp(-i\Delta')]$$

$$Z=2\exp(-i\Delta)-\exp(i\theta)\left[\sin\Sigma'\sin\Delta'+2t_0\cos\left(2\pi\frac{\Phi}{\Phi_0}\right)\right]. \tag{20}$$

Here $\Delta=k_F(L-d)+\delta_F$; $\Delta'=k_FL/2+\delta_F$; $\Sigma'=k_F(L/2-d)+\theta$.

Formula (20) is symmetric relative to the sign reversal of the magnetic flux: $G(\Phi)=G(-\Phi)$. This can be verified through appropriate transformations. Also, the above expression is also symmetric relative to change in the impurity coordinate a: $G(a)=G(L/2-a-d)$, which indicates that the magnitude of conductance is independent of the direction of the measuring current I: $G(I)=G(-I)$.

For numerical computations, we consider a point impurity $(d=0)$ with a potential U:

$$U(\xi)=\Omega\hbar^2/m\,\delta(\xi-a). \tag{21}$$

The presence of an impurity affects the dependence $G(k_FL)$. This is due to the emergence of new channels of transition from L to R (involving reflection at the barrier) for $r_0>0$. Among other things, this causes a ''large-scale'' modulation (with a period larger than 2π) of the dependence $G(k_FL)$. The amplitude of such a modulation increases with r_0. As the temperature increases, oscillations associated with the interference of waves with the shortest path difference are preserved for the longest period of time. Such oscillations have a period $\Delta(L/\lambda_F)\simeq\max[L/(2a),L/(L-2a)]$ and vanish at $T\geqslant T_{0i}=T_0\Delta(L/\lambda_F)$ (see Fig. 4). The following circumstance is worth noting. If we consider a ring with an overlapping branch $(t_0=0)$, the separation between electron energy levels near ε_F in such a ring will be half the value of Δ_F for a pure ring. Hence it would be natural to expect that a decrease in t_0 causes a decrease in temperature at which oscillations on the dependence $G(k_FL)$ vanish. However, this is not true. This is due to the fact that the conductivity of the ring depends not only on the position of the Fermi level relative to the electron energy levels in the ring, but also on the amplitude of the electron transition through the ring.

A decrease in the barrier transparency causes a decrease in the amplitude of oscillations of the dependence $G(\Phi)$.[31] In this case, the amplitude of the second harmonic increases (Fig. 5) due to an increase in the contribution from the AAS processes to the field dependence of the conductance.

It follows from formula (20) that the dependence $G(\varphi)$ is sensitive to the amplitude and phase of the transmission coefficient t_i. In order to determine the dependence on θ, we write

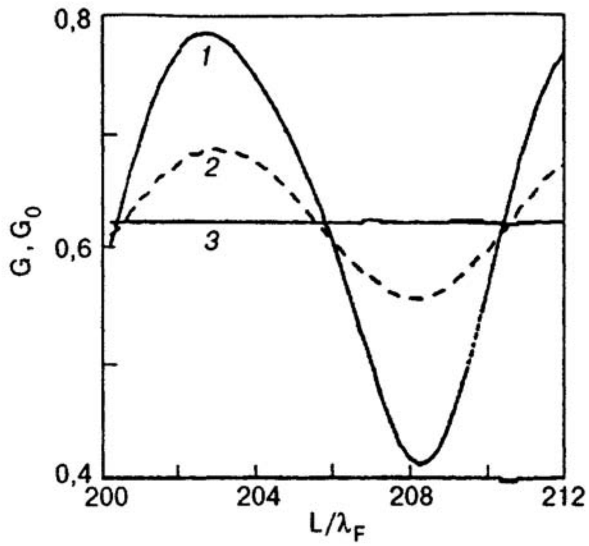

FIG. 4. Dependence of the conductance G of a ring with a single impurity (in units of G_0) on the parameter L/λ_F for $T=\Delta_F/2$ (curve *1*), 1.5 Δ_F (curve *2*) and 5 Δ_F (curve *3*). The values of the parameters are $\Phi=0$, $\varphi=0$, $\Omega/k_F=1$, and $2a/L=0.1$.

$$G(\varphi)=\sum_n D_n \cos\left(2\pi n \frac{\varphi}{\varphi_0}-\gamma_n\right), \tag{22}$$

where D_n and γ_n are the amplitude and phase of the nth harmonic.

Calculations show that the following relation holds for $T \geqslant T_0$:

$$\gamma_1=\theta. \tag{23}$$

In other words, a change in the phase of the transmission coefficient causes a corresponding variation in the phase of

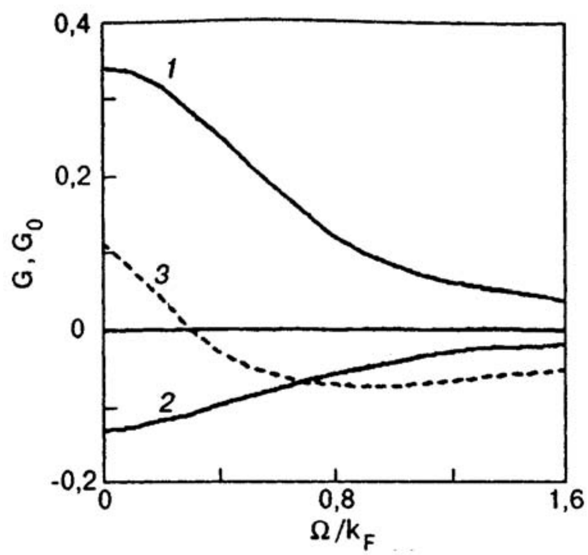

FIG. 5. Dependence of the amplitude of the first (curve *1*) and second (curve *2*) harmonics $G(\Phi)$ (in units of G_0) for $\varphi=0$, and amplitude of the first harmonic for $\varphi=0.2$ (curve *3*) on the ratio Ω/k_F. The values of the parameters are $2a/L=0.5$, $T=\Delta_F$.

the first harmonic in the dependence $G(\varphi)$. Note that the parity condition (16) was obtained for a symmetric ring (for $\varphi=0$), and is not valid in the present case.

The presence of a magnetic flux $\Phi\neq 0$ does not change the relation (23). The only exception are the values $\Phi=\pm\Phi_0/4$ [see Eq. (19a)] near which D_1 vanishes.

One more circumstance deserves attention. A change in the barrier transparency may cause a sign reversal for the first harmonic in the dependence $G(\Phi)$ (Fig. 5, curve *3*). Calculations show that the amplitude of the first harmonic for $T\geqslant T_0$ vanishes if the following condition is satisfied:

$$\delta_F-\theta=\pi/2. \tag{24}$$

Thus, if we keep the amplitude of the first harmonic in the dependence $G(\Phi)$ equal to zero, the change in the height of the potential barrier is proportional to the variation in the phase of the coefficient of transmission of a Fermi electron through the potential barrier.

Yacoby *et al.*[22] found a variation π in the phase of the dependence $G(\Phi)$ in an AB interferometer with a quantum dot (QD) (a potential barrier with a resonance level) upon the passage of a QD level E_n through a resonance: $\varepsilon_F=E_n+V_p$, where V_p is the QD potential relative to the ring. Theoretical studies reveal[25,32,33] that this is due to the vanishing of the first harmonic amplitude near the resonance. It is shown in this section that such an effect may also be observed for a nonresonance potential barrier.

CONCLUSION

In this work, we have studied the effect of a potential barrier of arbitrary height on the conductivity of an AB magnetic interferometer. Results obtained for nonzero temperatures are presented. In this case, the effect of uncontrollable geometrical size (parameter k_FL) is ruled out, but controllable parameters like the magnetic flux Φ and the height $e\varphi$ of the potential barrier continue to exert an influence.

For a small potential step ($e\varphi\ll\varepsilon_F$), the interference of electron waves propagating along different branches of the ring (Aharonov–Bohm electrostatic effect) causes an oscillatory dependence of the conductance G of the ring on φ. It is shown that the quantity δ_F (phase change of the electron wave function due to the presence of a step) does not affect the phase of the dependence $G(\Phi)$ in accordance with the experimental results.[20,21] It is also shown that a change in δ_F may cause a reversal of the sign of the amplitude of the first harmonic C_1 in the dependence $G(\Phi)$. Near the value $C_1(\delta_F)=0$, the period of oscillations changes from Φ_0 to $\Phi_0/2$, which was indeed observed by Yacoby *et al.*[25]

The effect of a tunnel barrier on the conductance of the ring is studied. It is shown that for certain values of parameters, the sign of the amplitude of C_1 may be reversed upon a change in the value of the tunneling coefficient. This effect was observed earlier in the case of resonance tunneling.[22] However, it is shown in the present work that such an effect may also occur during ordinary (nonresonance) tunneling.

It is found that an increase in the coefficient of electron reflection at the potential barrier causes a relative increase in the amplitude of the second harmonic in the dependence $G(\Phi)$ (with a period $\Phi_0/2$) due to an increase in the contribution from the AAS processes.

It is also shown that the use of a ballistic ring with a small potential barrier (electrostatic AB interferometer) allows us to determine directly the change in the phase of the electron wave function upon tunneling through the barrier formed in one of the branches of the ring.

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