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The influence of the capacity upon the persistent current in a one-dimensional ballistic ring

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Abstract

In the present paper within a phenomenological approach a simple expression for the persistent current in a one-dimensional ballistic ring is proposed in view of a geometrical capacity C between a ring and an electron reservoir. It is shown that the current averaged within the grand-canonical ensemble is other than zero and is proportional to C^{-1} at a large magnitude of the capacity.

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One of the manifestations of the Aharonov–Bohm effect [1] in the physics of mesoscopic systems [2] is an existence of a thermodynamic equilibrium (persistent) current in double-connected nonsuperconducting samples threaded by a magnetic flux Φ at low temperatures [3–5]. The persistent current I is periodic in Φ with the period $\Phi_0 = h/e$ and its magnitude is determined by the thermodynamic potential Ω of the system [6,7]:

$$I = -\frac{\partial \Omega}{\partial \Phi}. \tag{1}$$

For the first time the magnetic moment oscillated with the period Φ_0 of a thin-walled ballistic cylinder was considered in Ref.[8]. Existence of a persistent current in normal-metal one-dimensional rings with a disorder was predicted in Ref.[9]. The persistent current was observed experimentally in the ensemble of mesoscopic rings [10] as well as in solitary rings in diffusive [11] and ballistic [12] regimes. The current amplitude measured in the ballistic low-channel ring which was formed in the $A1_xGa_{1-x}As/GaAs$ semiconductor heterostructure [12], is in good agreement with the predictions of the theory, based on the model of the noninteraction spinless particles [13]. In Ref.[10] current $\langle I \rangle$, averaged over an ensemble of the macroscopically identical rings in a diffusive regime, was measured. This current is periodic in Φ with a period of $\Phi_0/2$ and its size is two orders of magnitude greater than those obtained from theoretical results [3].

The theoretical consideration has shown, that the magnitude of the averaged current considerably depends on the nature of a statistical ensemble. Usually we have two situations: the canonical

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ensemble (CE) and the grand canonical ensemble (GCE). In the first case each element of the ensemble has a fixed, flux-independent number of electrons N, which can either vary from one element to another (weak canonical ensemble) or be identical to all the elements of the ensemble (strong canonical ensemble) [14]. This situation corresponds to the system of isolated rings. In the second case all the elements of the ensemble coupled to an electron reservoir, which fixes the chemical potential ε_F (Fermi energy) common for all the elements of the ensemble. As it was shown, the magnitude $\langle I \rangle$ is exponentially small in the case of GCE [13,15–17] and has a finite size for CE [13,18–25].

Therefore studying continuous transitions from the situation of CE to the situation of GCE within one model is of specific interest.

For this purpose let us consider the mesoscopic sample weakly coupled to an electron reservoir taking into account the capacity C between the sample and the reservoir. The closely related model was considered in the Coulomb blockade regime in Ref.[26] for the incoherent charge transfer and in Refs.[27,28] for the coherent charge transfer.

Obviously, the ratio of the Coulomb energy $E_C = e^2/(2C)$ to the electron level spacing Δ_F near the Fermi energy in the sample for $\Phi = 0$ determines an appropriate statistical ensemble [29]. In the case of $E_C \gg \Delta_F$ the number of electrons in the sample is flux-independent: $N(\Phi) = \text{const}$. For the other limit ($E_C \ll \Delta_F$) the Fermi energy of electrons in the sample is equal to the chemical potential of the electron reservoir μ and does not depend on Φ: $\varepsilon_F(\Phi) = \mu = \text{const}$. These two cases correspond to the CE and GCE, respectively. In the general case the reservoir fixes the size of the electrochemical potential of electrons in the sample while the number of electrons and chemical potential oscillate with the magnetic flux.

In the present paper we obtain the expression for the persistent current true for the arbitrary ratio between E_C and Δ_F for the one-dimensional ballistic ring with spinless electrons noninteracting among themselves. We also show, that the current, averaged over an ensemble of macroscopically identical rings, decreases at $C \to \infty$ only as power: $\langle I \rangle \simeq E_C/\Delta_F$. It should be noted that increasing of the current with reduction of the capacity is in agreement with Ref.[21], where it was emphasized that for the diffusive regime the suppression of fluctuations in the electron density causes an increase of the current. In our case increasing of the Coulomb energy E_C (i.e. decreasing of the capacity C) causes reduction in the number of electron fluctuations in the sample: $\delta N(\Phi) \simeq 1 - E_C/\Delta_F$ as well as an increase of the current $\langle I \rangle$.

Let us consider the one-dimensional ballistic ring with a length L threaded by a magnetic flux Φ and weakly coupled to the electron reservoir with the temperature T and the chemical potential μ (Fig. 1). In the present paper we assume that the ring locates near the reservoir and we do not consider the influence of the states in the lead between the reservoir and the ring on the persistent current [30,31].

The connection of the ring with the reservoir, on one hand, should be weak enough in order not to destroy the electron spectrum in the ring $\varepsilon_l^{\pm}(\Phi) = h^2/(2mL^2)(l \pm \Phi/\Phi_0)^2$. Otherwise the effect of a reservoir on the persistent current is significant [32]. On the other hand, an existence of the connection is necessary for the establishment of the thermodynamic equilibrium between the ring and the reservoir. As a result the temperature of electrons in the ring coincides with the temperature of the reservoir. Besides the electrochemical potentials of the ring and the reservoir coincide. Assuming that the electrostatic potential of the reservoir is equal to zero,

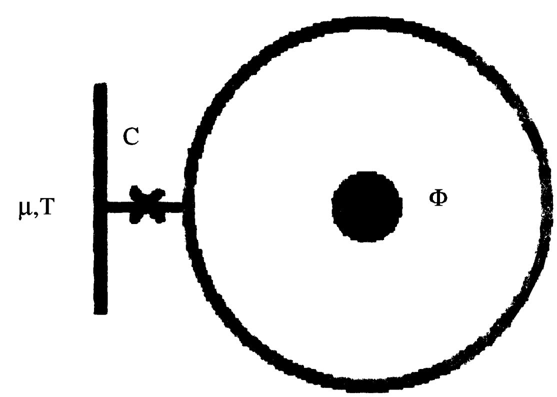

Fig. 1. One-dimensional ring threaded by a magnetic flux Φ and weakly connected to an electron reservoir with the temperature T and the chemical potential μ. C is the geometrical capacity.

we have

$$\mu = \varepsilon_F + e\varphi. \tag{2}$$

Here φ is an electrostatic potential of the ring relative to the reservoir [30]. Let us assume that the potential φ is a constant along the ring and may be determined self-consistently. In the local approximation we can write

$$Q_+ + eN = C\varphi. \tag{3}$$

Here Q_+ is a charge of the positive continuous background (charge of the ions) in the ring: C is a capacity of a ring-reservoir system. The number N of electrons in the ring depends on φ and equals

$$N = \sum_{l\pm} f_0\left(\frac{\varepsilon_l^{\pm}(\Phi) - \varepsilon_F}{T}\right). \tag{4}$$

Here $f_0(x)$ is the Fermi function. Substituting (Eq. (2)) in (Eq. (4)) and assuming $\varphi \ll \mu$ and $L \gg \lambda_\mu$, we obtain

$$N = N_0(\mu) - K(0,0) + K(\varphi,\Phi) - 2e\varphi/\Delta_F, \tag{5}$$

where $K(\varphi,\Phi) = 4\pi T\Delta_\mu^{-1}\sum_{k=1}^{\infty}\cos(2\pi K\Phi/\Phi_0)\, sh^{-1}(kT/T^*)\sin(2\pi k\{L/\lambda_\mu - e\varphi/\Delta_\mu\})$; $N_0(\mu) = \sum_{l\pm} f_0(\{\varepsilon_l^{\pm}(0) - \mu\}/T)$; $\Delta_\mu = hv_\mu/L$ is the electron level spacing near the Fermi energy in the ring for $\Phi = 0$; $T^* = \Delta_\mu/(2\pi^2)$; $\lambda_\mu = 2\pi/k_\mu$; $v_\mu = \hbar k_\mu/m$; $\hbar k_\mu = (2m\mu)^{1/2}$ is the Fermi momentum. From Eqs. (3) and (5) we obtain the self-consistent condition for the potential φ

$$\varphi(C + C_0) = e(N_0(\mu) + Q_+/e - K(0,0) + K(\varphi,\Phi)). \tag{6}$$

Here $C_0 = 2e^2/\Delta_\mu$ is a capacity due to the density of states of spinless electrons in the ring [33].

Let us consider two limiting cases.

(a) $C \to 0$. From (Eq. (6)) it follows, that the potential φ of the ring and consequently the Fermi energy ε_F of electrons in the ring (see (Eq. (2))) depend on the magnetic flux Φ. The amplitude of the oscillations at $T = 0$ has the same order as in the isolated ring: $e\delta\varphi(\Phi) \simeq \delta\varepsilon_F(\Phi) \simeq \Delta_\mu$. Thus the number N of electrons in the ring does not depend on Φ for $C \ll C_0$. It should be noted that for the more rigid condition $C \ll C_0/(1 + N_0 + Q_+/e)$ the ring is neutral; $N \simeq -Q_+/e$. This regime corresponds to consideration of the ring (more accurately, the ensemble of the rings) in the canonical ensemble approach.

(b) $C \to \infty$. In this case, as it follows from (Eq. (6)), $\varphi = 0$, and the Fermi energy ε_F of electrons in the ring coincides with the chemical potential μ of the electron reservoir (see (Eq. (2))). This regime corresponds to the grand canonical ensemble. The known fact, that the fluctuations of the number of electrons with a magnetic flux in GCE $\delta N(\Phi) \simeq 1$, follows from (Eq. (5)).

In the general case (i.e. for arbitrary ration between C and C_0) both the number N of electrons and the Fermi energy ε_F depend on the magnetic flux Φ.

Further, let us calculate the persistent current I. It is known, that at $N = \text{const.}$, $I = -\partial F/\partial\Phi$, where F is a free energy, and at $\varepsilon_F = \text{const.}$ $I = -\partial\Omega/\partial\Phi$, where Ω is the thermodynamic potential. Using the thermodynamic correlations, it is possible to write down the expression for the persistent current in the form true in CE as well as in GCE [23]:

$$I = -\left(\frac{\partial\Omega}{\partial\Phi}\right)_{\varepsilon_F = \varepsilon_F(\Phi)}. \tag{7}$$

Let us assume that (Eq. (7)) defines the persistent current in a ring as well as for the arbitrary ratio between C and C_0. In other words, in order to obtain the expression for the persistent current for the arbitrary ratio between C and C_0 it is necessary to substitute the dependence $\varepsilon_F = \varepsilon_F(\Phi)$ in the expression for the current $I_0(\Phi, \varepsilon_F)$ obtained for $\varepsilon_F = \text{const}$. The expression $I_0(\Phi, \varepsilon_F)$ for the one-dimensional ballistic ring is obtained in Ref.[13]. Thus,

$$I = \frac{2}{\pi} I_0 \frac{T}{T^*} \sum_{q=1}^{\infty} \sin(2\pi q\Phi/\Phi_0) \times \frac{\cos(2\pi q(L/\lambda_\mu - e\varphi/\Delta_\mu))}{sh(qT/T^*)}. \tag{8}$$

Here $I_0 = ev_\mu/L$. Thus, Eqs. (6) and (8) allow us to calculate the persistent current taking into account the capacity in the considered model.

Fig. 2 shows the dependence $I(\Phi)$ at $T = 0$ for several values of ration C/C_0. It can be seen that the

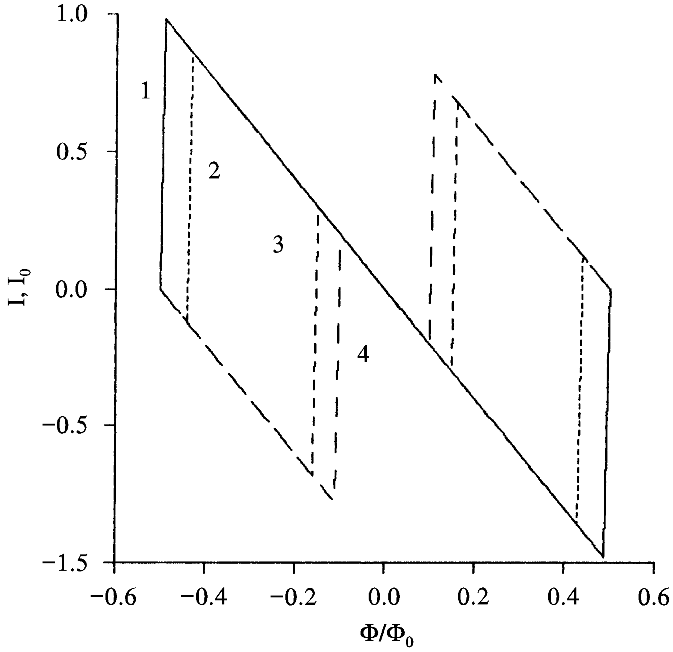

Fig. 2. Persistent current I versus Φ at $T=0$ for $C/C_0 = 0(1)$; 3(2); 20(3) and ∞(4). The parameters: $L/\lambda_\mu = 100.1$; $N_0(\mu) = Q_+$.

form of the curve is continuously transformed from the case corresponding to $N=$ const. (curve 1) to the case corresponding to $\varepsilon_F =$ const. (curve 4) with the increasing capacity C.

It should be noted that for $C \ll C_0$ the current I ((Eq. (8))) at $T=0$ does not depend on the magnitude of the chemical potential μ of the electron reservoir (in the interval $\Delta\mu \ll \mu$), that is due to the following circumstance. At arbitrary magnitude μ (excepting such μ when $L/\lambda_\mu - (N_0 + Q_+/e - K(0,0))/2 = 1$, where 1 is a natural) the number of spinless electrons in the ring is odd and Φ-independent. Consequently, according to the parity effect [5,13], the persistent current in the ring does not depend on the number of electrons but depends on its parity. A similar result ($I(\mu) =$ const. at $T=0$) was obtained in Ref. [29] in the limit of strong ($e^2/\hbar v_\mu) \gg 1$) electron–electron interaction.

With increasing temperature ($T > T^*$), the oscillating dependence $I(\mu)$ with a period Δ_μ appears. The current amplitude in a minimum is exponentially small in comparison with one in a maximum

$$\frac{I_{\min}}{I_{\max}} \simeq \left(\frac{T}{\pi T^*}\frac{C_0}{C+C_0} - 1\right)\exp(-T/T^*). \tag{9}$$

The dependence $I(\Phi)$ has the period Φ_0 in a maximum and the period $\Phi_0/2$ in a minimum.

Now let us calculate the current averaged over an ensemble of the macroscopically identical rings (in particular, having the identical capacity C). We shall average by a small range ΔL of the length L of the ring ($\lambda_\mu \ll \Delta L \ll L$). We note, that the background charge Q_+ is varied along with the length of the ring.

For $C \gg C_0$ the magnitude of the potential φ is small and Eqs. (6) and (8) may be simplified. As a result we obtain

$$\langle I\rangle_{C\gg C_0} = I_0 \frac{C_0}{2\pi^2 C}\left(\frac{T}{T^*}\right)^2 \sum_{q=1}^{\infty} \frac{\sin(4\pi q\Phi/\Phi_0)}{sh^2(qT/T^*)}. \tag{10}$$

Thus, for the finite magnitude of the capacity ($C < \infty$) the averaged current is other than zero and periodic in a magnetic flux with a period $\Phi_0/2$.

The similar result can be obtained in the limit of high temperature $T \gg T^*$ for arbitrary ratio between C and C_0

$$\langle I\rangle_{T\gg T^*} = I_0 \frac{2}{\pi^2}\frac{C_0}{C+C_0}\left(\frac{T}{T^*}\right)^2 \times \exp(-2T/T^*)\sin(4\pi\Phi/\Phi_0). \tag{11}$$

In conclusion, we have considered the influence of capacity C between the mesoscopic ballistic sample and the electron reservoir upon the persistent current within the phenomenological approach. It is shown that the account of the finite size of the capacity ($C < \infty$) results in a nonvanishing magnitude of the current $\langle I\rangle \simeq 1/C$ averaged within the grand canonical ensemble (i.e. when the electrochemical potential in the ring is fixed).

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