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Physica B 301 (2001) 286–291

PHYSICA B

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# The effect of interelectron interactions on thermal fluctuations of a persistent current in a single one-dimensional ballistic ring

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Received 30 August 2000; received in revised form 29 November 2000

**Abstract**

The fluctuations of a persistent current in a one-dimensional system of correlated spinless electrons at finite temperatures are considered. The magnitude of such fluctuations is found to depend on the magnetic flux, the temperature and the coupling to a reservoir. 

*PACS:* 73.20.Dx; 73.50.Td

*Keywords:* Persistent current; Fluctuations; Luttinger liquid

At low temperatures the phase coherence length $L_\phi(T)$ of an electron wave function is large compared to the size $L$ of small samples (mesoscopic samples) [1] that leads to the existence of a thermodynamic equilibrium current in normal-metal rings pierced by a magnetic flux $\Phi$ [2–5]. This is a manifestation of the Aharonov–Bohm effect [6] in solids. Such a current was predicted for the ballistic [7] as well as for the disordered [8] rings and it is an additional thermodynamic quantity (like the temperature $T$, the chemical potential $\mu$, etc.) which characterizes the ground state of two-connected mesoscopic samples.

Like any thermodynamic quantity [9] the persistent current fluctuates when the ring is coupled to a reservoir. In the present paper, we consider persistent current fluctuations (PCFs) due to a non-zero temperature of the reservoir. They are fluctuations in a single ring at fixed (given) temperature and magnetic flux. The interaction with the reservoir broadens the quantum levels in a ring. We assume that the coupling to the reservoir is weak and the broadening $\Gamma$ of the energy levels is small compared with the level spacing $\Delta_F$ (near the Fermi level $\mu$): $\Gamma \ll \Delta_F$. Such a broadening is a consequence of the energy exchange between the ring and the reservoir that is necessary for establishing thermodynamic equilibrium in the ring at a given temperature $T$. The inelastic time $\tau_{in} \sim \hbar/\Gamma$ is a characteristic time scale for such a process (because the inelastic processes in the ring are absent $L_\phi \gg L$) [10–16]. Thus, when we observe a ring during a time $\tau \gg \tau_{in}$ the current $j(t)$ in the ring fluctuates as a function of time. Of course, within the time period $\tau$ all the significant parameters (e.g., the temperature, the magnetic flux, etc.) must be stable. Accounting for this, we further put $\tau \to \infty$

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0921-4526/01/$ - see front matter 
PII: S0921-4526(01)00273-3

and $\Gamma = 0$. Note that in an experiment such fluctuations may be observed as low-frequency fluctuations of the current in the ring.

The persistent current fluctuates in the canonical case (CC) (when the ring exchanges only energy with a reservoir, and the number $N_0$ of electrons in the ring is constant) as well as in the grand canonical case (GCC) (when in addition the exchange (tunneling) of particles is allowed and there is the common chemical potential $\mu$). However, at low temperatures, the physical reasons for such fluctuations are quite different for both cases. In the CC, the current fluctuations are due to transitions of the electron system between closely spaced many-electron energy levels ($\delta E \leqslant T$). Thus, at low temperatures the fluctuations are significant for such values of $\Phi$ which correspond to a degenerate ground state of the electron system in the ring. Whereas in the GCC the additional origin of current fluctuations arises from fluctuations of the number of electrons in the ring. If the ring is coupled to a reservoir (or another system which can emit and absorb electrons) the persistent current fluctuates even at $T = 0$ near transfer charge resonances [17,18]. As we will show below, the difference between PCFs in the CC and in the GCC persists up to the temperatures $T \sim T^*$, where $T^* = \Delta_F/(2\pi^2)$ is the crossover temperature for the persistent current problem (at $T > T^*$ the persistent current exponentially decreases) [19,20]. Note that the dependence of PCFs on the statistical ensemble is a particular example of the significant role of the statistical ensemble in mesoscopics [19,21–28].

In the present paper, we study the change of thermal PCFs when the system continuously transfers from the CC to the GCC. This is possible if we take into account the Coulomb energy $E_c$ associated with the small capacitance $C$ between the ring and the reservoir [29]. At low temperatures $T < E_c = e^2/(2C)$, the charge transfer between parts of the system is suppressed (the Coulomb blockade effect [30–32]) that effectively isolates the ring from the reservoir and it affects the persistent current [17,18,29,33–36]. At the same time, at some values of the potential difference $V_g = V_{g0}$ between the ring and the reservoir the charging energy is degenerate and the Coulomb blockade is lifted [37]. So, within such a model by varying $V_g$ or $E_c$ we can obtain either CC ($E_c \gg T$, $V_g \neq V_{g0}$) or GCC ($E_c \ll T$ or $V_g = V_{g0}$).

We consider a one-dimensional ballistic ring of length $L$. We assume that the system of interacting spinless electrons in the ring may be described within the framework of a Luttinger liquid model [38]. This model allows to obtain an analytical expression for the low-energy spectrum of a many-electron system accounting exactly for a charging energy $E_c$.

The Lagrangian $L_{LL}$ of a Luttinger liquid in bosonic form is [39]

$$L_{LL} = \frac{\hbar v}{2g}\left\{\frac{1}{v^2}\left(\frac{\partial\theta}{\partial t}\right)^2 - \left(\frac{\partial\theta}{\partial x}\right)^2\right\}, \tag{1}$$

where $v$, $g$ are Haldane's parameters [38] which depend on the inter-electron interactions in the ring. In the continuous limit, they satisfy $vg = v_F$, where $v_F = \pi\hbar\rho_0/m^*$ is the Fermi velocity ($\rho_0$ is the mean electron density in the ground state; $m^*$ is the effective electron mass). For non-interacting electrons $g = 1$.

The spatial derivative of a bosonic field $\theta$ determines the deviation of the electron density from that in the ground state $\rho(x,t) = \rho_0 + \pi^{-1/2}\partial\theta/\partial x$. Thus, we can write down the Lagrangian $L_{ex}$ accounting for particle exchange with the reservoir as follows [35]:

$$L_{ex} = -\frac{E_c}{L}\left(\int_0^L dx\rho(x,t) - N(V_g)\right)^2 + \mu\rho(x,t), \tag{2}$$

where $N(V_g) = CV_g/e$.

The Aharonov–Bohm interaction is described by the Lagrangian $L_{AB}$ [39]

$$L_{AB} = \frac{2\hbar}{L}\pi^{1/2}\frac{\partial\theta}{\partial t}\left(\frac{k_j}{2} + \frac{\Phi}{\Phi_0}\right), \tag{3}$$

where $\Phi_0 = h/e$ is the magnetic flux quantum; $k_j$ is the topological number dependent on the parity of the number $N_e$ of electrons in the ring.

In the ballistic case, the dependence of the partition function $Z$ on the magnetic flux $\Phi$ is determined only by the zero modes [39] which are completely decoupled from the non-zero ones. In such a case, the free energy for the model Eqs. (1)–(3) may be expressed through the Jacobi theta-functions [35,36]. Using the Poisson summation formula, we can easily obtain the spectrum $E_k(\Phi)$ of the system of interacting ballistic electrons as follows:

(a) the canonical case:

$$Z_{\mathrm{CC}}(\Phi) = \sum_{k=-\infty}^{\infty} \mathrm{e}^{-E_k/T},$$

$$E_k(\Phi) = \Delta_{\mathrm{F}}\left(k + \frac{\Phi}{\Phi_0} + \frac{N_0 - 1}{2} \bmod 1\right)^2. \tag{4}$$

(b) the grand canonical case:

$$Z_{\mathrm{GCC}}(\Phi) = \sum_{N_{\mathrm{e}}} \mathrm{e}^{\mu N_{\mathrm{e}}/T} \sum_{k=-\infty}^{\infty} \mathrm{e}^{-E_{lk}/T},$$

$$E_{lk}(\Phi) = l\mu + \Delta_{\mathrm{c}}(l/2 + \delta_{\mathrm{c}})^2 + \Delta_{\mathrm{F}}\left(k + \frac{\Phi}{\Phi_0} + \frac{N_{\mathrm{e}} - 1}{2} \bmod 1\right)^2. \tag{5}$$

Here $\Delta_{\mathrm{F}} = 2\pi\hbar v_{\mathrm{F}}/L$; $\Delta_{\mathrm{c}} = \Delta_{\mathrm{F}}/g^2 + 4E_{\mathrm{c}}$; $l = N_{\mathrm{e}} - N_0$; $\delta_{\mathrm{c}} = (4(eV_g - \mu)/\Delta_{\mathrm{c}}) \bmod 1$. Note, when we perform calculations in the CC we omit $L_{\mathrm{ex}}$ Eq. (2).

Using Eqs. (4) and (5), we can obtain the Gibbs distribution function [9] and calculate the persistent current and its fluctuations. As was pointed out in Ref. [17,18], the fluctuations of the persistent current are large and must be characterized by their entire distribution. To this end, we calculate the average value of the powers of both the current $I^n$ and the deviation $\delta I^n$ of the current from its average value

$$I_x^n = Z_x^{-1} \sum_\alpha j_\alpha^n \mathrm{e}^{\sigma\mu N_{\mathrm{e}} - E_\alpha/T}, \tag{6}$$

$$\delta I_x^n = Z_x^{-1} \sum_\alpha (j_\alpha - I)^n \mathrm{e}^{\sigma\mu N_{\mathrm{e}} - E_\alpha/T}. \tag{7}$$

Here, $j_\alpha = -\partial E_\alpha/\partial\Phi$ is the current carried by the many-electron level $E_\alpha$, where $\alpha$ numbers the energy levels and is equal to $k$ in the CC and to the set of $\{l, k\}$ in the GCC; $\sigma = 0(1)$ for $x = \mathrm{CC}$ ($x = \mathrm{GCC}$).

The above expressions have the following meaning. Under influence of the reservoir, the system passes from the quantum state, say $E_{\alpha_1}$ to another one with an energy $E_{\alpha_2}$. (Note that such a transition is accompanied by a change of the total momentum of the system.) The system stays in such a quantum state during a time $\sim \tau_{\mathrm{in}}$ when the current $j$ in the ring is $j = j_{\alpha_2}$. So, we can consider the current as a function of time $j = j(t)$ if the period of observation $\tau$ is large compared with $\tau_{\mathrm{in}}$ (in an experiment, to cancel out the contribution from high-frequency quantum oscillations, the current $j$ must be averaged over a time period $\hbar/\Delta_{\mathrm{F}} \ll \Delta t \ll \tau_{\mathrm{in}}$). According to the basic principles of statistical physics [9], averaging over the time $\langle j(t)\rangle$ is equivalent to averaging over the Gibbs distribution that was performed in the above expressions. Note that averaging over the distribution at a given temperature is possible if $T \gg \hbar/\tau_{\mathrm{in}}$ [9] that restricts our consideration to ultralow temperatures. For high temperatures, the region of validity of our consideration is limited by the condition $L_\phi(T) \gg L$.

It is easy to check that the average current $\langle j(t)\rangle = I$ (persistent current) is equal to that obtained in Refs. [35,39]. Note that in the CC the fluctuations of the persistent current do not depend on the strength of interelectron interactions (the parameter $g$) like the persistent current itself [39,40]. But in the GCC the fluctuations at $0 < T < T^*$ depend on $g$ (and on the charging energy $E_{\mathrm{c}}$) except for the particular values of $V_g$ (correspondent to $\delta_{\mathrm{c}}(V_g) = \pm 1/4$) at which the current fluctuations coincide with the ones in a free-electron ($g = 1, E_{\mathrm{c}} = 0$) ring coupled to a reservoir (at the same value of the parameter $\delta_{\mathrm{c}}$).

The numerical calculations show that the averaged odd powers of current $I^{2n+1} = \langle j^{2n+1}\rangle$ vanish with increasing temperature. However, the higher powers of current persist up to higher temperatures (see Fig. 1). In contrast, the even powers of the current increase with temperature and at $T \gg T^*$ they are $I^{2n} = \Gamma(n + \frac{1}{2})\sqrt{\pi}2^n(I^2)^n$ (we note that $I^{2n} = \langle j^{2n}(t)\rangle \neq (I^2)^n = \langle j^2(t)\rangle^n$). Thus, at high temperatures the current $j$ in a ring fluctuates near zero. Such fluctuations do not depend on the magnetic flux and interelectron interactions, and they are described by the Gauss distribution

$$W(j) = \frac{1}{\sqrt{2\pi I^2}} \exp\left(-\frac{j^2}{2I^2}\right), \quad T \gg T^* \tag{8}$$

with the mean square current

$$I^2 = 2I_0^2 T/\Delta_{\mathrm{F}}, \tag{9}$$

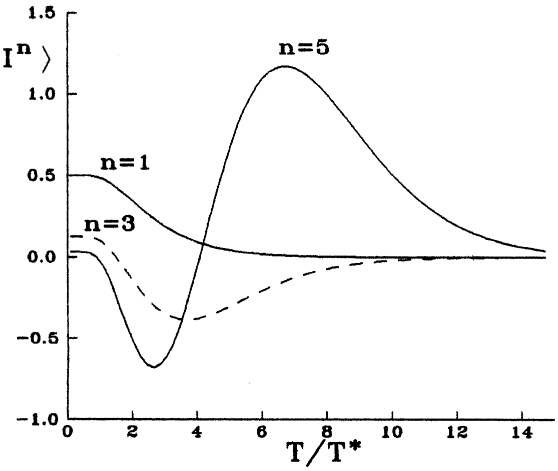


Fig. 1. Dependence of the averaged powers of a current in units of $I_0^n$ on temperature at $\Phi = \Phi_0/4$. The number $N_0$ of electrons in a ring is even.

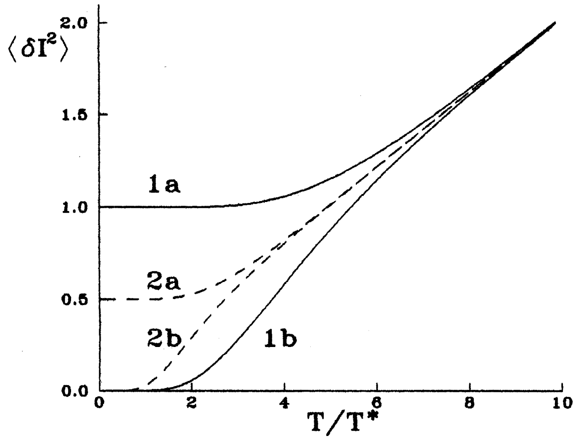


Fig. 2. Dependence of the mean square fluctuations $\delta I^2$ of a current in units of $I_0^2$ on temperature at even values of $N_0 = \text{const}$ (1) and at $\mu = \text{const}$ (2) for $\Phi = 0$ and $\Phi_0/2$. The parameters are $\delta_c = 0$; $g = 1$; $E_c = 0$.

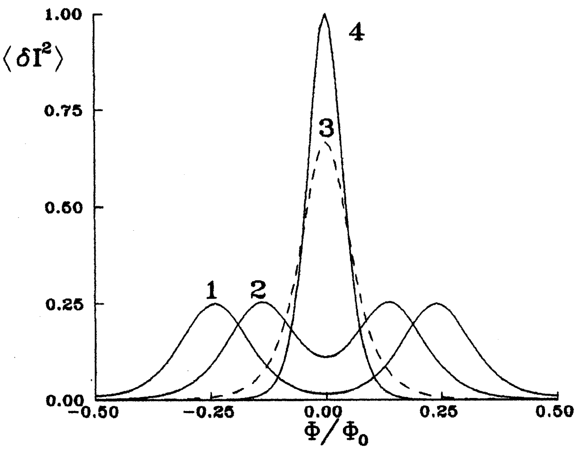


Fig. 3. Dependence of the mean square fluctuations $\delta I^2$ on the magnetic flux for $\sqrt{\Delta_F/\Delta_c} = 1(1); 0.3(2); 0.2(3)$ and for fixed even values of $N_0(4)$. The parameters are $T = T^*/2$; $\delta_c = 0.24$.

where $I_0 = ev_F/L$ is the persistent-current amplitude at $T = 0$. We emphasize that the existence of such thermal fluctuations is essentially a mesoscopic effect. Though these fluctuations grow with temperature, they vanish in the macroscopic limit as $\sqrt[2n]{I^{2n}} \sim L^{-1/2}$ (at a fixed particle density $\rho_0$). Note that the averaged current (at $T < T^*$) scales as $I \sim L^{-1}$. In addition, these fluctuations are non-dissipative and they should not be confused with the ordinary thermal fluctuations of a current (Nyquist noise) which are dissipative by nature and exist in the macroscopic limit.

At low temperatures ($T < T^*$), the current fluctuations are more complicated. They depend on the magnetic flux and are different in the canonical and grand canonical cases. Below we consider the mean square fluctuations of a current (i.e., $\delta I^2$). After some straightforward manipulations, we obtain

$$\delta I^2 = T\left(\frac{\partial I}{\partial \Phi} + \gamma\right), \tag{10}$$

where $\gamma = Z_x^{-1}\sum_\alpha(\partial^2 E_\alpha/\partial\Phi^2)\,e^{\sigma\mu N_c - E_\alpha/T}$. From the above expression it immediately follows that the mean square fluctuations depend on the statistical case under consideration because the persistent current $I$ depends on the regime of coupling to the reservoir ($N_0 = \text{const}$ or $\mu = \text{const}$) [19,35,39]. For the spectrum Eqs. (4) and (5) we have $\gamma = 2\Delta_F/\Phi_0^2$. At $T \gg T^*$ the persistent current vanishes and from Eq. (10) we obtain Eq. (9). Note that for the free-electron gas model ($g = 1$; $E_c = 0$) in the regimes $\mu = \text{const}$ Eq. (10) leads to $\delta I^2 = I_0^2\delta N^2$, where $\delta N^2$ is the mean square fluctuation of the number of electrons in the ring. The dependence $\delta I^2(T)$ is depicted in Fig. 2 in both the regimes $N_0 = \text{const}(1)$ and $\mu = \text{const}(2)$. We see that at $T < T^*$ the mean

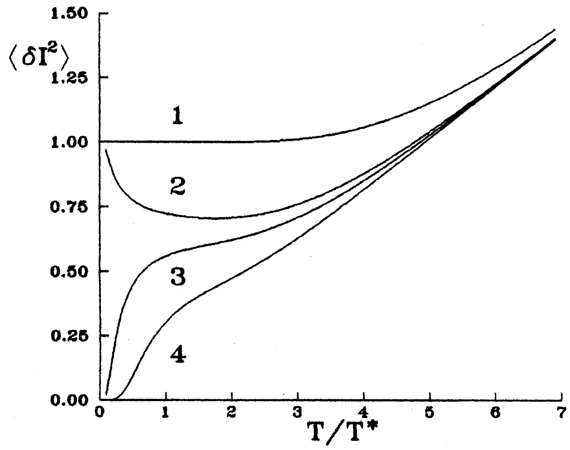


Fig. 4. Dependence of the mean square fluctuations $\delta I^2$ on temperature for fixed even values of $N_0$(1) and for $\sqrt{\Delta_F/\Delta_c} =$ 0.6(2); 0.7(3); 1(4). The parameters are $\Phi = 0$; $\delta_c = 0.15$.

square fluctuations of the current oscillate as a function of the magnetic flux with a period of $\Phi_0$. The amplitude of such oscillations is two times larger in the regime $N_0 = \text{const}$ in comparison with the regime $\mu = \text{const}$ (for non-interacting electrons). Using Eqs. (5), and (7) we may calculate $\delta I^2$ for interacting electrons. Some results of numerical calculations are depicted in Figs. 3 and 4. Thus, the increase of repulsive interactions ($g < 1$) in a ring and/or a charging energy ($E_c > T$), effectively isolating a ring from a reservoir, strengthen PCFs in a ring. Note that at points of degeneracy of the charging energy ($\delta_c = \pm 1/4$), the period of the dependence $\delta I^2(\Phi)$ halves.

In summary, we have considered the effect of interelectron interactions on thermal fluctuations in a one-dimensional ballistic ring containing spinless electrons. We have shown that the suppression of particle exchange between the ring and the reservoir enhances persistent-current fluctuations at low temperatures. Also, we predict the existence of high temperature PCFs which grow with the temperature but vanish in the macroscopic limit as $L^{-1/2}$.

## Acknowledgements

It is a pleasure to thank P.S. Deo for fruitful discussions.

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