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Oscillations of the electrochemical capacitance of a one-dimensional ring of correlated electrons

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Abstract

The effect of the interelectron interaction and the temperature on the dependence of the electrochemical capacitance C_μ of a one-dimensional ballistic ring on both the magnetic flux Φ and the reservoir electrochemical potential μ is considered. It is shown that the effect of the temperature on the dependence $C_\mu(\Phi,\mu)$ can be used to detect a non-Fermi-liquid behaviour of an electron system.

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1. Introduction

The Luttinger liquid (LL) model [1–6] is widely used for describing the low-energy physics of one-dimensional systems of interacting electrons. The properties of a LL as kinetic (transport) [8,9] as thermodynamic [10–14] qualitatively differ from the properties of a Fermi liquid (FL) which describes electrons in normal metals [7]. However, the transport measurements give ambiguous results. For instance, the measurement of the tunneling current between edge states in the fractional quantum Hall effect regime (described as a chiral LL [5]) [15,16] is consistent with a chiral LL theory, while the tunneling current through an additional edge state circling an antidot [17,18] shows a Fermi-liquid behaviour. Probably such a discrepancy is caused by the Fermi liquid leads which always present in the transport measurements. Note that the conductance of a finite ballistic LL wire coupled to FL leads does not depend on the interelectron interaction and coincides with the conductance of a ballistic FL wire [19–21]. Therefore, the contactless methods are more suitable for the detection of a non-Fermi-liquid behaviour of ballistic electron systems.

One such a quantity that may be measured by the contactless method is the capacitance of a sample. As it was shown in Refs. [22–24] the experimentally relevant capacitance is an electrochemical capacitance

$$C_\mu = -e^2\partial^2\Omega/\partial\mu^2, \tag{1}$$

where Ω is the thermodynamic potential of a mesoscopic sample and μ is the chemical (electrochemical) potential of an electron reservoir. The capacitance C_μ

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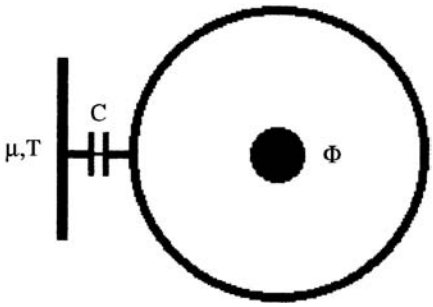

Fig. 1. One-dimensional ring pierced by a magnetic flux Φ and weakly coupled to an electron reservoir with the chemical potential μ and the temperature T. C is the geometrical capacitance between a ring and a reservoir.

depends on both the geometrical capacitance and the density of states (near the Fermi energy) of a sample. The last circumstance is a base for the capacitance spectroscopy of mesoscopic systems [25–32].

The purpose of the present paper is to consider how the Luttinger liquid behaviour appears in C_μ.

As a model we consider a one-dimensional ballistic ring weakly coupled to an electron reservoir with the electrochemcial potential μ and the temperature T. The ring is pierced by the magnetic flux Φ (see Fig. 1). Such a model is often considered in the literature. Note, that the availability of two controllable parameters (μ and Φ) is important for our purposes. Because, the response of a ring to the change in the magnetic flux Φ is connected with the energy Δ_j of current excitations [1] (i.e., the energy necessary to change the current, which is quantized in a finite-size ring), while the response to the change in μ is connected with the energy Δ_ρ of charge excitations [1] (i.e., the energy necessary to change the particle density). These parameters (see explicit definitions after Eq. (10)) are different in the LL model ($\Delta_\rho > \Delta_j$ for repulsive interelectron interactions and $\Delta_\rho < \Delta_j$ for attractive interactions) and are equal in the FL model $\Delta_j = \Delta_\rho = \Delta_F$ (where Δ_F is the level spacing near the Fermi energy at $\Phi = 0$).

At $T \to 0$ the dependence $C_\mu(\mu)$ consists of a set of peaks with a flux dependent position. These peaks correspond to the charge transfer from the ring to the reservoir (or vice versa). We will consider a weak coupling between a ring and a reservoir when the intrinsic width of energy levels Γ in a ring is much smaller than the temperature $\Gamma \ll T$. In such a case at low temperatures the width of a peak is proportional to T.

At $T \to 0$ the dependence $C_\mu(\Phi)$ is periodic in Φ with a period of Φ_o (where $\Phi_o = h/e$ is the magnetic flux quantum) and the dependence $C_\mu(\mu)$ is periodic in μ with a period of Δ_ρ. With increasing temperature, as it will be shown below, the dependences $C_\mu(\Phi)$ and $C_\mu(\mu)$ have different crossover temperatures: $T_j^* \simeq \Delta_j$ and $T_\rho^* \simeq \Delta_\rho$, respectively. Namely, at $T \gg T_j^*$ the oscillations in $C_\mu(\Phi)$ are washed out, while the oscillations in $C_\mu(\mu)$ vanish only at $T \gg T_\rho^*$. In the Fermi liquid model there is a one crossover temperature.

Note, that the charge transfer in the phase-coherent system (a ballistic ring and a quantum dot) was considered in Refs. [33,34] and the charge transfer between a LL ring and a reservoir was discussed in Refs. [35–37].

2. Noninteracting electrons

Let us briefly consider the electrochemical capacitance of a Fermi liquid ring. We first assume spinless electrons. The thermodynamic potential of free electrons is $\Omega = -T\sum_n \mathrm{Ln}(1+\exp((\mu-\varepsilon_n)/T))$ [38]. In an ideal one-dimensional ring with a magnetic flux Φ the single electron spectrum is $\varepsilon_n = 4\pi^2\hbar^2/(2m^*L^2)(n+\Phi/\Phi_o)^2$ [39], where m^* is the effective electron mass; L is the ring circumference; n is an integer. Thus, in the mesoscopic limit $N_o \gg 1$ (where N_o is a number of electrons in a ring in the ground state, and N_o depends on μ), Eq. (1) gives

$$C_\mu = \frac{e^2}{4T}\sum_{k=-\infty}^{\infty}\left\{\cosh^{-2}\left(\frac{\Delta\mu - \Delta_F(\varphi+k)}{2T}\right) + \cosh^{-2}\left(\frac{\Delta\mu + \Delta_F(\varphi+k)}{2T}\right)\right\} \tag{2a}$$

$$= \frac{2e^2}{\Delta_F}\left(1 + 4\pi^2 T/\Delta_F \times \sum_{m=1}^{\infty} m\frac{\cos(2\pi m\varphi)\cos(2\pi m\Delta\mu/\Delta_F)}{\sinh(mT/T_F^*)}\right). \tag{2b}$$

Here $\Delta\mu = \mu - \mu_o$, where $\mu_o = \mu(N_o)$; $T_F^* = \Delta_F/(2\pi^2)$ is the crossover temperature; $\varphi = (\Phi/\Phi_o + N_o/2) \bmod 1$; k and m are integers. We see, that C_μ depends on the number of electrons N_o in the ground state modulo 2 that is the well-known parity effect [10,40].

The crossover temperature T_F^* divides the low- temperature region from the high temperature region for the dependence $C_\mu(\mu,\Phi)$. Note, that this temperature is characteristic of the persistent current $I=-\partial\Omega/\partial\Phi$ in rings of noninteracting electrons [40].

At $T\ll T_F^*$ the dependence $C_\mu(\mu)$ (see Eq. (2a)) is a set of peaks corresponding to the transfer of one (at $\Phi/\Phi_o\neq 0,\ \frac{1}{2}\bmod 1$) spinless electron between a ring and a reservoir. The position of peaks depends on the magnetic flux: $\Delta\mu_{max}=\Delta_F(k\pm\varphi)$. The maximum peak spacing is Δ_F. The peak height $C_{max}=e^2/(4T)$ does not depend on the electron spectrum characteristics and increases at $T\to 0$. So, at $T=0.1$ K we have $C_{max}\simeq 4$ fF. With decreasing temperature, the peak width reduces because we neglect the width Γ of energy levels. At $\Phi/\Phi_o=0,\frac{1}{2}\bmod 1$ the peaks merge in pairs that is due to the dynamic degeneracy of energy levels (relatively to the clockwise and counter-clockwise movement of electrons in a ring). In such a case the peak height doubles but the peak width does not change.

At $T\gg T_F^*$ the electrochemical capacitance does not depend on T,μ and Φ (see Eq. (2b)) and is $C_\mu=2e^2/\Delta_F$.

Now we include an electron spin. In the case of noninteracting electrons spin ($\frac{1}{2}$) simply leads to an additional double degeneracy of energy levels. As a result the properties of an isolated ring depend on the number of electrons $N_o=N_{o\uparrow}+N_{o\downarrow}$ (where N_{os} is the number of electrons with spin $s=_{\uparrow,\downarrow}$ in the ground state) modulo 4 [41]. However, if the electrochemical potential μ of an electron reservoir does not depend on s then in an open (i.e., connected to a reservoir) system the number N_o is always even ($N_{o\uparrow}=N_{o\downarrow}$) and the properties of such a system depend on $N_o/2$ modulo 2. Therefore, the parity effect in an open system with spin like the one without spin (with respect to the doubling of the number of particles). Note, if the magnetic field is present and μ still is independent of s the numbers $N_{o\uparrow}$ and $N_{o\downarrow}$ may be different that is due to Zeeman effect. In such a case the parity effect is determined by N_o modulo 4. However, in the present paper we consider the Aharonov–Bohm flux [42] (i.e., without a magnetic field) and assume for electrons with spin only the case of $N_o=4n,\ 4n+2$. So, the electrochemical capacitance C_μ of noninteracting electrons with spin is given by Eqs. (2) multiplied by 2 with respect to $\varphi=(\Phi/\Phi_o+N_o/4)\bmod 1$.

3. Interacting electrons

Furthermore, we consider how the electron–electron interaction changes results stated in the previous section. The interelectron interaction in a ring we consider in the Luttinger liquid model [1] and the Coulomb interaction with the reservoir we take into account in the geometrical capacitance C approach (Fig. 1) [14].

3.1. Spinless electrons

The Lagrangian of a spinless Luttinger liquid in a bosonic form is [1,8,10].

$$L_{LL}(x,t)=\frac{\hbar v}{2g}\left\{\frac{1}{v^2}\left(\frac{\partial\theta}{\partial t}\right)^2-\left(\frac{\partial\theta}{\partial x}\right)^2\right\}. \tag{3}$$

Here g and v are Haldane's parameters, which depend on the interelectron interaction in a ring. For repulsive interactions $g<1$ and for attractive ones $g>1$. In the case of noninteracting electrons $g=1$ and $v=v_F$, where $v_F=\pi\hbar N_o/(Lm^*)$ is the Fermi velocity. Moreover, for the rotationally invariant ring-like systems $gv=v_F$.

The scalar boson field $\theta(x,t)$ describes the low-energy excitations around the Fermi surface of the spinless interacting electron system. The spatial derivative of the field θ determines the deviation of the particle density ρ from the mean density in the ground state

$$\rho(x,t)=N_o/L+\pi^{-1/2}\partial\theta/\partial x. \tag{4}$$

The field θ obeys twisted boundary conditions on a torus [10]

$$\theta(x+k_1L,\tau+k_2\beta)=\theta(x,\tau)+k_1\pi^{1/2}(2m+k_M)+k_2\pi^{1/2}n. \tag{5}$$

Here $\tau=$ it is the imaginary time; $\beta=\hbar/T$; k_1,k_2,n, and m are integers; and k_M is the topological number, describing the parity of the additional number (over the number in the ground state) of particles in a ring.

The Aharonov–Bohm interaction of electrons in a ring with the magnetic flux Φ is described by the Lagrangian L_{AB} [10]

$$L_{AB}(x,t)=\frac{\hbar}{L}\pi^{1/2}\frac{\partial\theta}{\partial t}\left(k_j+\frac{2\Phi}{\Phi_o}\right), \tag{6}$$

where the topological number k_j depends on the parity of the number of particles in a ring $N_e=\int\rho\,dx$. Note,

that the topological numbers k_j and k_M depend on the parity of the number of electrons N_o in the ground state [10] and are

$$k_j = k_M \quad \text{if } N_o \text{ is odd}$$
$$k_j = 1,\ k_M = 0;\ \text{and } k_j = 0, \tag{7}$$
$$k_M = 1 \quad \text{if } N_o \text{ is even.}$$

The Coulomb interaction with the reservoir in the geometrical capacitance approach may be described by the Lagrangian [14]

$$L_C(t) = -\frac{E_C}{L}\left(\int_0^L dx\,\rho(x,t) - N_o\right)^2, \tag{8}$$

where $E_C = e^2/(2C)$. We assume, that at $T = 0$, $\Phi = 0$ and $\mu = \mu_o \equiv \mu(N_o)$ the ring is neutral and the electrostatic energy vanishes.

The thermodynamic potential $\Omega = -T\,\mathrm{Ln}(Z)$ of a ring is expressed through the grand partition function $Z = \int D\theta \exp(-S_E/\hbar)$, where the Euclidean action is

$$S_E = \int_0^L dx \int_0^\beta d\tau (L_{LL}(x,\tau) + L_{AB}(x,\tau) + L_C(\tau) - \mu\rho(x,\tau)). \tag{9}$$

The zero modes of the boson field θ determine the μ-dependent part $\Delta\Omega(\mu)$ of the thermodynamic potential of a ballistic ring and with respect to the boundary condition (5) are $\theta_{mn}(x,\tau) = \pi^{1/2}((2m + k_M)x/L + n\tau/\beta)$. Note, that the measure $D\theta$ contains the sum over n, m, k_j, and k_M. Performing such a summation with respect to constraints (7) we obtain

$$\Delta\Omega(\mu) = -\mu^2/\Delta_\rho - T\,\mathrm{Ln}(\theta_3(\varphi,q_j)\theta_3(\delta,q_\rho) + \theta_3(1/2+\varphi,q_j)\theta_3(1/2+\delta,q_\rho)). \tag{10}$$

Here $\theta_3(v,q)$ is the Jacobi theta function [43]; $\delta = (\mu - \mu_o)/\Delta_\rho$; $\varphi = (\Phi/\Phi_o + (N_o - 1)/2)\mathrm{mod}\,1$; $q_j = \exp(-\pi^2 T/\Delta_j)$; $q_\rho = \exp(-\pi^2 T/\Delta_\rho)$; $\Delta_j = 2\pi\hbar v g/L$; $\Delta_\rho = \Delta_j/g^2 + 4E_C$. The charging energy E_C increases the energy Δ_ρ of charge excitations, however, does not affect the energy Δ_j of current excitations. Effectively, the charging energy E_C strengthens the electron–electron repulsion ($g \to 0$) and leads to the renormalization of Haldane's parameters: $v_C = vg/g_C$; $g_C = g/(1 + 4E_C g^2/\Delta_j)^{1/2}$.

By substituting Eq. (10) into Eq. (1) we obtain the electro-chemical capacitance of a ring within the present model. In the Coulomb blockade regime (i.e., at the strong electron–electron repulsion and/or the large charging energy E_C) $\Delta_\rho \gg \Delta_j$ and we have

$$C_\mu = \frac{e^2}{4T}\sum_{k=-\infty}^{\infty}\left\{\cosh^{-2}\left(\frac{\Delta\mu - \Delta_\rho(\delta_o + k)}{2T}\right) + \cosh^{-2}\left(\frac{\Delta\mu + \Delta_\rho(\delta_o + k)}{2T}\right)\right\} \tag{11a}$$

$$= \frac{2e^2}{\Delta_\rho}\left(1 + 4\pi^2 T/\Delta_\rho \sum_{m=1}^{\infty} m \times \frac{\cos(2\pi m\delta_o)\cos(2\pi m\Delta\mu/\Delta_\rho)}{\sinh(mT/T_\rho^*)}\right), \tag{11b}$$

where $\Delta\mu = \mu - \mu_o$; $T_\rho^* = \Delta_\rho/(2\pi^2)$; $\delta_o = 1/4 + T\Delta_\rho^{-1}\,\mathrm{Ln}(\theta_3(1/2+\varphi,q_j)/\theta_3(\varphi,q_j))$.

Thus, the electron–electron repulsion removes the degeneracy of electron levels in a ring and leads to the following. At low temperatures ($T \ll T_\rho^*$), the shape of a peak is like to the one in the Fermi liquid model. However, the peak spacing and the dependence of the peak position on the magnetic flux are changed. So, at $T \ll T_j^* = \Delta_j/\pi^2$ the ratio α of the minimum distance between adjacent peaks to the maximum one is

$$\alpha = \frac{1 - g_C^2}{1 + g_C^2}. \tag{12}$$

Note that in Ref. [37] a similar expression was obtained for the ratio of the width of paramagnetic response to the width of diamagnetic response of a LL ring.

With increasing temperature ($T \gg T_j^*$), the magnetic flux Φ does not affect the dependence $C_\mu(\mu)$ which now have a period of $\Delta_\rho/2$.

As it follows from Eq. (11b), at higher temperatures ($T \gg T_\rho^*$) the oscillations in $C_\mu(\mu)$ are washed out (Fig. 2) and the electrochemical capacitance is $C_\mu^{-1} = \Delta_\rho/(2e^2) = (2e^2 g^2/\Delta_j)^{-1} + C^{-1}$ in accordance with Ref. [22].

For generality, we consider the case of strong interelectron attraction also. In such a case $\Delta_j \gg \Delta_\rho$ and at $T \to 0$ the dependence $C_\mu(\mu)$ consists of a set of peaks corresponding to the transfer of two electrons between a ring and a reservoir

$$C_\mu = \frac{e^2}{T}\sum_{k=-\infty}^{\infty}\cosh^{-2}\left(\frac{\Delta\mu - \Delta_\rho(\delta_e + k)}{T}\right), \tag{13}$$

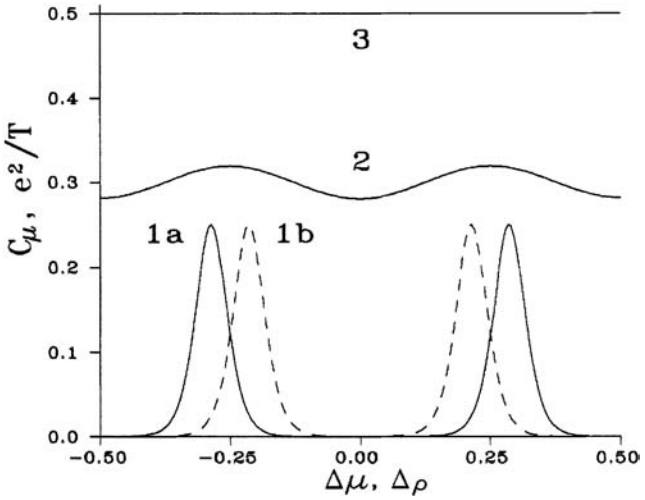

Fig. 2. Electrochemical capacitance C_μ of a ring of spinless repulsively interacting electrons ($g_c^2 = 0.2$) as a function of the electrochemical potential μ of an electron reservoir for $T/\Delta_\rho = 0.02$ (1) ($\Phi = 0$ (a) and $\Phi/\Phi_o = 0.5$ (b)), for $T/\Delta_\rho = 0.15$ (2) (Φ is arbitrary), and for $T/\Delta\rho = 0.25$ (3) (Φ is arbitrary). The number N_o of electrons in the ground state is odd.

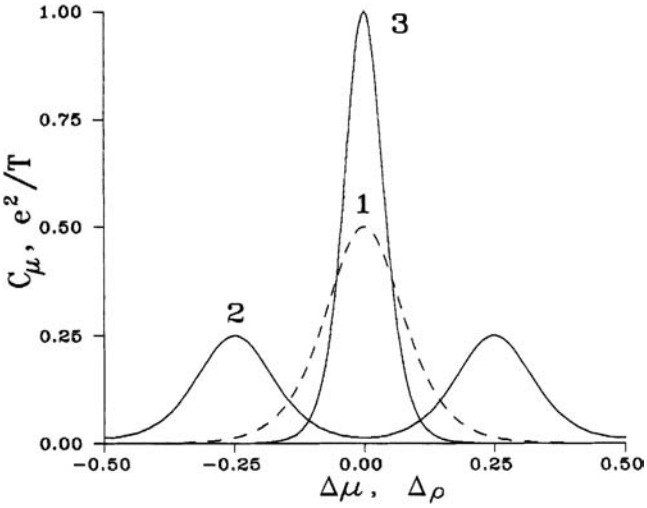

Fig. 3. Electrochemical capacitance C_μ of a ring of spinless electrons as a function of the electrochemical potential μ of an electron reservoir for noninteracting electrons (1), for repulsively interacting electrons (2), and for electrons with attractive interactions (3); $T/\Delta_\rho = 0.05$, $\varphi = 0$.

where $\delta_e = 0(1/2)$ for $-1/4 < \varphi < 1/4$ ($1/4 < \varphi < 3/4$). At $\varphi \simeq \frac{1}{4}, \frac{3}{4}$ the degeneracy is removed and dependence $C_\mu(\mu)$ is given by Eq. (11a).

It is worth emphasizing that, the peak width in Eq. (13) is reduced two times compared to the case of either Luttinger liquid with repulsion (11a) or Fermi liquid (2a) (Fig. 3). Note, that in Ref. [30] the peaks in $C_\mu(\mu)$ corresponding to the transfer of two electrons were observed, however, the comparison of the width of such peaks with the width of other peaks was not reported.

3.2. Electrons with spin

In one-dimensional systems of electrons with a repulsive interaction the spin and charge subsystems are completely separated [4]. However, with respect to the parity effect [10,13,40,41] the spin subsystem affects the capacitance as well as the persistent current [13,44,45].

Now we introduce two boson field $\theta_\uparrow$ and $\theta_\downarrow$ for electrons with spin 'up' ($\uparrow$) and with spin 'down' ($\downarrow$), respectively. Then, the Lagrangian of a LL with spin may be expressed in terms of charge $\theta_\rho = \theta_\uparrow + \theta_\downarrow$ and spin $\theta_\sigma = \theta_\uparrow - \theta_\downarrow$ boson fields as follows [4,8]

$$L_{\mathrm{LL}}(x,t) = \frac{\hbar v_\rho}{2g_\rho}\left\{\frac{1}{v_\rho^2}\left(\frac{\partial\theta_\rho}{\partial t}\right)^2 - \left(\frac{\partial\theta_\rho}{\partial x}\right)^2\right\} + \frac{\hbar v_\sigma}{2g_\sigma}\left\{\frac{1}{v_\sigma^2}\left(\frac{\partial\theta_\sigma}{\partial t}\right)^2 - \left(\frac{\partial\theta_\sigma}{\partial x}\right)^2\right\}. \quad (14)$$

For noninteracting electrons with spin Haldane's parameters are $g_\rho = g_\sigma = 2$ and $v_\rho = v_\sigma = v_F$. Moreover, in the absence of a magnetic field (or any spin-dependent interactions) we have $g_\sigma = 2$ [4,8].

The Aharonov–Bohm interaction of electrons with the magnetic flux Φ through the ring is described by the Lagrangian [44]

$$L_{\mathrm{AB}}(x,t) = \frac{2\hbar}{L}\pi^{1/2}\left\{\frac{\partial\theta_\rho}{\partial t}\left(\frac{k_{j\rho}}{4} + \frac{\Phi}{\Phi_o}\right) + \frac{\partial\theta_\sigma}{\partial t}\frac{k_{j\sigma}}{4}\right\}. \quad (15)$$

The topological numbers $k_{j\rho}$ and $k_{j\sigma}$ are defined by $k_{j\rho} = k_{j\uparrow} + k_{j\downarrow}$; $k_{j\sigma} = k_{j\uparrow} - k_{j\downarrow}$.

The charging energy due to the geometrical capacitance C is described by the Lagrangian L_C (see Eq. (8)), where $N_o = N_{o\uparrow} + N_{o\downarrow}$ and $\rho(x,t) = \pi^{-1/2}\partial\theta_\rho/\partial x$.

The partition function is $Z = \int \mathrm{D}\theta_\rho \mathrm{D}\theta_\sigma \exp(-S_E/\hbar)$, where the Euclidean action S_E is defined by Eq. (9) with respect to Eqs. (8), (14) and (15).

The fields θ_ρ and θ_σ obey twisted boundary conditions (5) with the topological numbers $k_{M\rho} = k_{M\uparrow} + k_{M\downarrow}$ and $k_{M\sigma} = k_{M\uparrow} - k_{M\downarrow}$. Note, that the numbers k_{js} and k_{Ms} (where $s = \uparrow,\downarrow$) depend on N_{os} similar to the case of spinless electrons (see Eq. (7)). The extremal trajectories (zero modes) of these fields are

$$\theta_\rho(x,\tau) = \pi^{1/2}\left((2m_\rho + k_{M\rho})\frac{x}{L} + n_\rho\frac{\tau}{\beta}\right),$$

$$\theta_\sigma(x,\tau) = \pi^{1/2}\left((2m_\sigma + k_{M\sigma})\frac{x}{L} + n_\sigma\frac{\tau}{\beta}\right). \tag{16}$$

Because the fields θ_ρ and θ_σ is expressed in terms of fields $\theta_\uparrow$ and $\theta_\downarrow$ the numbers n_ρ and n_σ (and accordingly m_ρ and m_σ) have the same parity.

The dependence $\Delta\Omega(\mu)$ is defined by the zero modes only. Therefore, calculating the action S_E for the trajectories (16) and performing the summation over n_ρ, n_σ, m_ρ, m_σ, $k_{j\rho}$, $k_{j\sigma}$, $k_{M\rho}$, and $k_{M\sigma}$ with respect to above restrictions we can express $\Delta\Omega$ in terms of Jacobi theta functions.

The thermodynamic potential depends on the parity of the number of particles N_o in the ground state modulo 4 [45]. However, as it was pointed out in Section 2, we consider only $N_o = 4n$, $4n+2$. In such a case we have

$$\begin{aligned}\Delta\Omega(\mu) = -2\mu^2/\Delta_\rho - T\,\mathrm{Ln}\{&\theta_3(1/2,q_\sigma^2)\theta_3(1/2,q_s^2)\theta_3\\ &(2\varphi+1/2,q_j^2)\theta_3(2\delta+1/2,q_\rho^2)\\ &+\theta_3(0,q_\sigma^2)\theta_3(0,q_s^2)\theta_3(2\varphi,q_j^2)\theta_3(2\delta,q_\rho^2)\\ &+\theta_2(0,q_\sigma^2)\theta_2(0,q_s^2)\theta_2(2\varphi,q_j^2)\theta_2(2\delta,q_\rho^2)\}.\end{aligned} \tag{17}$$

Here $\theta_3(v,q)$, $\theta_2(v,q)$ are the Jacobi theta functions [43]; $\delta = (\mu-\mu_o)/\Delta_\rho$; $\varphi = (\Phi/\Phi_o + (N_o/2-1)/2) \bmod 1$; $q_\sigma = \exp(-\pi^2 T/\Delta_\sigma)$; $q_s = \exp(-\pi^2 T/\Delta_s)$; $q_j = \exp(-\pi^2 T/\Delta_j)$; $q_\rho = \exp(-\pi^2 T/\Delta_\rho)$; $\Delta_\sigma = \pi\hbar v_\sigma g_\sigma/L$; $\Delta_s = 4\Delta_\sigma/g_\sigma^2$; $\Delta_j = \pi\hbar v_\rho g_\rho/L$; $\Delta_\rho = 4\Delta_j/g_\rho^2 + 8E_C$. Note, that $\Delta\Omega$ is periodic in Φ with a period of Φ_o and periodic in μ with a period of Δ_ρ.

In the general case the expression for the electrochemical capacitance C_μ (see Eq. (1)) is very complicated. Therefore, we only consider the Coulomb blockade regime at low temperatures. Thus, we assume $\Delta_\rho \gg T$, Δ_j, Δ_s, Δ_σ. In such a case the dependence $C_\mu(\mu)$ is

$$\begin{aligned}C_\mu = \frac{e^2}{4T}\sum_{\pm,k=-\infty}^{\infty}\Bigg\{&\cosh^{-2}\left(\frac{\Delta\mu \pm \Delta_\rho(\delta_1+k)}{2T}\right)\\ &+\cosh^{-2}\left(\frac{\Delta\mu \pm \Delta_\rho(\delta_2+k)}{2T}\right)\Bigg\},\end{aligned} \tag{18}$$

where $\Delta\mu = \mu - \mu_o$; $\delta_1 = 1/8 + T/\Delta_\rho\,\mathrm{Ln}((B+C)/A)$; $\delta_2 = 3/8 - T/\Delta_\rho\,\mathrm{Ln}((B-C)/A)$; $A = \theta_3(1/2,q_\sigma^2)\times\theta_3(1/2,q_s^2)\theta_3(2\varphi+1/2,q_j^2)$; $B = \theta_3(0,q_\sigma^2)\theta_3(0,q_s^2)\times\theta_3(2\varphi,q_j^2)$; $C = \theta_2(0,q_\sigma^2)\theta_2(0,q_s^2)\theta_2(2\varphi,q_j^2)$.

Each peak in $C_\mu(\mu)$ corresponds to the transfer of the charge $1e$ between a ring and a reservoir. The relative position of adjacent peaks depends on both the temperature and the magnetic flux that allows to measure Haldane's parameters. So, at $T \to 0$ we have

$$\delta_1 = \frac{1}{8} + \frac{\Delta_s + \Delta_\sigma - \Delta_j}{8\Delta_\rho} + \frac{\Delta_j}{\Delta_\rho}\left(\frac{1}{4} - |\varphi|\right),$$

$$-1/2 < \varphi < 1/2\,, \tag{19a}$$

$$\delta_2 = \frac{3}{8} - \frac{\Delta_s + \Delta_\sigma - \Delta_j}{8\Delta_\rho} + \frac{\Delta_j}{\Delta_\rho}\left(\frac{1}{4} - |\varphi|\right),$$

$$-1/2 < \varphi < 1/2. \tag{19b}$$

with increasing temperature, the various cases can occur. So, if $\Delta_j \ll \Delta_s, \Delta_\sigma$, then at $T_j^* \ll T \ll \Delta_s/\pi^2, \Delta_\sigma/\pi^2$ we obtain

$$\delta_1 = \frac{1}{8} + \frac{\Delta_s + \Delta_\sigma}{8\Delta_\rho}, \tag{20a}$$

$$\delta_2 = \frac{3}{8} - \frac{\Delta_s + \Delta_\sigma}{8\Delta_\rho}. \tag{20b}$$

At the same time, if $\Delta_j \gg \Delta_s, \Delta_\sigma$, then at temperatures $\Delta_s/\pi^2, \Delta_\sigma/\pi^2 \ll T \ll T_j^*$ the period of the dependence $C_\mu(\Phi)$ halves and becomes equal to $\Phi_o/2$. The period of the dependence $C_\mu(\mu)$ halves also and equals to $\Delta_\rho/2$. In such a case

$$\delta_1 = \frac{1}{8} + \frac{\Delta_j}{\Delta_\rho}\left(\frac{1}{8} - |\varphi|\right), \quad -1/4 < \varphi < 1/4, \tag{21a}$$

$$\delta_2 = \frac{3}{8} - \frac{\Delta_j}{\Delta_\rho}\left(\frac{1}{8} - |\varphi|\right), \quad -1/4 < \varphi < 1/4. \tag{21b}$$

At more high temperatures $\Delta_s/\pi^2, \Delta_\sigma/\pi^2, T_j^* \ll T \ll T_\rho^*$ the peaks are symmetrically located: $\delta_1 = 1/8$; $\delta_2 =$

3/8 and the electrochemical capacitance is flux-independent and periodic in μ with a period of $\Delta_\rho/2$.

4. Conclusion

In the present paper, the expression for the electrochemical capacitance C_μ of a one-dimensional ballistic ring of interacting electrons coupled to a reservoir is obtained. The short-range electron–electron interaction in a ring is considered in the Luttinger liquid (LL) model. The long-range Coulomb interaction with an environment (with a reservior) is taken into account in the geometrical capacitance approach. The last interaction effectively enhances an interelectron repulsion and renormalizes Haldane's parameters for the LL in a ring. We have considered as spinless fermions as electrons with spin.

It is shown that there are two crossover temperatures: T_j^* for the dependence $C_\mu(\Phi)$ and T_ρ^* for the dependence $C_\mu(\mu)$ in the Luttinger liquid model, while in the Fermi-liquid (FL) model these temperatures coincide: $T_j^* = T_\rho^* = \Delta_F/(2\pi^2)$. Thus, the effect of the temperature on the dependence $C_\mu(\Phi, \mu)$ can be used to detect the LL-like behaviour.

The electron–electron interaction (repulsion) removes the degeneracy (as spin as dynamical) of electron levels in a ring. Therefore at low temperatures ($T \ll T_\rho^*$) the dependence $C_\mu(\mu)$ consists of a set of peaks each of which corresponds to the transfer of a charge $1e$ between a ring and a reservoir. The peak spacing depends on the strength of an electron–electron interaction, while the peak shape is independent of the model (either LL or FL). At the same time, in the case of LL with attraction peaks corresponding to the transfer of a charge $2e$ ($4e$ for electrons with spin) are possible as well as in the case of FL. However, the width of such peaks is two times less than the one for the FL model.

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