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Coulomb blockade of the persistent current in a one-dimensional system of electrons with spin

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Abstract

The effect of long-range Coulomb interaction (in the geometrical capacitance C approach) on a persistent current in a one-dimensional ballistic ring of correlated electrons with spin coupled to a reservoir at nonzero temperatures is considered. It is shown that in the limit of $C \to 0$ a ring and a reservoir are not completely decoupled which is due to a spin degree of freedom and affects considerably the persistent current. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

One of the features of mesoscopic systems [1] at low temperatures is maintenance of an electron wave function coherence across the entire sample. Therefore the physical properties [2,3] of such systems are sensitive to a change of an electron wave function phase that leads, in particular, to the manifestation of the Aharonov–Bohm (AB) effect [4] in solids.

The free energy F of doubly connected systems (rings) pierced by an AB magnetic flux Φ is periodic in Φ with a period of $\Phi_0 = h/e$ [5,6]. The derivative of the free energy over the magnetic flux determines the magnitude of a thermodynamic equilibrium (persistent) current $I = -\partial F/\partial\Phi$ which exists in normal (nonsuperconducting) rings at low temperatures. Such a current was predicted in Refs. [7,8] and was observed experimentally in Refs. [9–11].

The properties of persistent currents [12–31] (the period over the magnetic flux, the crossover temperature T^*, the type of the ground state (either diamagnetic or paramagnetic) etc.) are determined by the properties of an electron system in a ring as well as by the interaction with an environment (with a reservoir). If a ring is coupled to a reservoir which fixes the chemical potential of a ring $\mu = \text{const.}$ (an open system), then the transfer of charge between a ring and a reservoir is allowed, which usually leads to a reduction of the persistent current. This is true for the current amplitude at $T = 0$ [12] as well as for the dependence of a current on the temperature. For instance, the crossover temperature T^* (at $T > T^*$ the amplitude of the persistent current is exponentially suppressed)

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for an isolated ring [25,28] is two times higher than the one for a ring coupled to an electron reservoir [12,13]. The charge transfer is especially important because the persistent current shows the parity effect [5,13,25,32–35]. For spinless fermions the current depends on the parity of the number of electrons N_e in a ring, while for electrons with spin the current depends on N_e modulo 4.

The charging energy $E_C = e^2/(2C)$ (where e is an electron charge and C the geometrical capacitance of the system) associated with the transfer of the elementary charge between different regions of a mesoscopic sample (or between a mesoscopic sample and a reservoir) strongly suppresses the charge transfer (the Coulomb blockade regime) [36–38] at low temperatures ($T \leqslant 1$ K at $C \leqslant 10^{-15}$ F). As a result in the limit $C \to 0$ an open system must be considered as an isolated system, i.e. at a fixed number of particles $N_e =$ const. Therefore this effect is important for the persistent current problem [39–44].

At the same time it is known that at certain values of the potential difference V_g between a mesoscopic sample and a reservoir the charging energy of a system is degenerate in N_e ($N_e \leftrightarrow N_e + 1$) and the Coulomb blockade is lifted [45,46]. In such a case the connection between a mesoscopic sample and a reservoir is restored that affects the persistent current. In particular, for spinless fermions the period of the dependence $I(\Phi)$ halves [40,44] and the crossover temperature is reduced four times [44]. Moreover, the persistent current equals the one at an appropriate fixed chemical potential $\mu = \mu^* =$ const. ($E_C = 0$). Thus, in the limit $C \to 0$ the persistent current for an open ring with spinless fermions shows the features inherent either for the regime $N_e =$ const. or for the regime $\mu =$ const. depending on the potential V_g.

In the present paper we consider the persistent current in a one-dimensional ballistic ring with interacting electrons with spin coupled to an electron reservoir in the limit of large charging energy E_C. The interplay of spin and electron–electron interaction qualitatively changes the effect of charging energy on the persistent current. So, in the general case, the current in the limit $C \to 0$ differs from the one in an isolated ring. This conclusion is justified by the following arguments. Though the magnetic flux affects only the charge degrees of freedom the spin subsystem influences on the persistent current also, that is a consequence of the parity effect [30,34,35]. The charging energy isolates the charge subsystem of a ring from the charge subsystem of a reservoir but with respect to the spin-charge separation in the system of electrons with a repulsive interaction [47] the charging energy does not affect the spin subsystem of a ring which still is coupled to a reservoir.

The closely related system at $T = 0$ with respect to the charging energy was considered in Ref. [40] with a reservoir replaced by a quantum dot. In the present paper we take into account as the long-range Coulomb interaction (in the geometrical capacitance approach) as the shot-range electron–electron interaction in the Luttinger liquid approach [48] and consider the dependence of the persistent current on the temperature. The paper is organized as follows. In Section 2 the expressions for the flux-dependent part of the free energy with respect to the charging energy are obtained. In Section 3 we consider the properties of a persistent current for rings with a different number of electrons in the ground state. The discussion is presented in Section 4.

2. Calculation of the free energy

Let us consider a one-dimensional ballistic ring of length L coupled via a tunnel junction to an electron reservoir (Fig. 1). We assume that the system of correlated electrons with spin in a ring may be described as a Luttinger liquid [48]. For electrons with spin the Lagrangian of a Luttinger liquid L_{LL} in a bosonic form is [47,49]

$$L_{LL}(x,t) = \frac{\hbar v_\rho}{2g_\rho}\left\{\frac{1}{v_\rho^2}\left[\frac{\partial\varphi_\rho}{\partial t}\right]^2 - \left[\frac{\partial\varphi_\rho}{\partial x}\right]^2\right\} + \frac{\hbar v_\sigma}{2g_\sigma}\left\{\frac{1}{v_\sigma^2}\left[\frac{\partial\varphi_\sigma}{\partial t}\right]^2 - \left[\frac{\partial\varphi_\sigma}{\partial x}\right]^2\right\}. \quad (1)$$

The subscripts ρ and σ denote quantities describing the charge and spin degrees of freedom, respectively. The boson fields φ_ρ and φ_σ are defined as follows:

$$\varphi_\rho = \varphi_\uparrow + \varphi_\downarrow, \qquad \varphi_\sigma = \varphi_\uparrow - \varphi_\downarrow, \quad (2)$$

where the boson fields $\varphi_\uparrow$ and $\varphi_\downarrow$ describe electrons with spin “up” ($\uparrow$) and spin “down” ($\downarrow$), respectively.

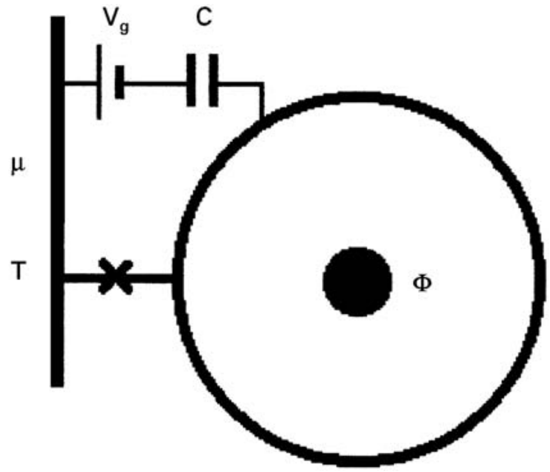

Fig. 1. One-dimensional ring pierced by a magnetic flux Φ and weakly coupled to an electron reservoir with the chemical potential μ and the temperature T. V_g and C are the potential difference and the geometrical capacitance between a ring and a reservoir, respectively.

For noninteracting electrons with spin Haldane's parameters [47,49] are: $g_\rho = g_\sigma = 2$ and $v_\rho = v_\sigma = v_F$, where $v_F = \hbar k_F/m^*$ is the Fermi velocity, k_F is the Fermi wave number, m^* is the effective electron mass. Moreover, in the absence of a magnetic field (or any spin-dependent interactions) we have $g_\sigma = 2$ [47,49].

The Aharonov–Bohm interaction of electrons with the magnetic flux Φ through the ring is described by the Lagrangian [25,35]

$$L_{AB}(x,t) = \frac{2\hbar}{L}\pi^{1/2}\left\{\frac{\partial\varphi_\rho}{\partial t}\left[\frac{k_{j\rho}}{4} + \frac{\Phi}{\Phi_0}\right] + \frac{\partial\varphi_\sigma}{\partial t}\frac{k_{j\sigma}}{4}\right\}. \quad (3)$$

The topological numbers $k_{j\rho}$ and $k_{j\sigma}$ are defined by $k_{j\rho} = k_{j\uparrow} + k_{j\downarrow}$; $k_{j\sigma} = k_{j\uparrow} - k_{j\downarrow}$. Here the topological numbers k_{js} ($s = \uparrow, \downarrow$) subject to the parity-dependent constraints [48,25]: $k_{js} = 0$ (1), if N_{es} is odd (even), where N_{es} is the number of electrons with spin s in the ring.

We take into account a charging energy which is due to a small capacitance C between a ring and a reservoir [44] (Fig. 1). The corresponding Lagrangian is

$$L_C(t) = -\frac{E_C}{L}\left[\int_0^L dx\,\rho(x,t) - N(V_g) + N_0\right]^2. \quad (4)$$

Here $N(V_g) = CV_g/e$ is a parameter depending on the potential difference V_g between a ring and an electron reservoir and characterizing the effective charge of a positive background in the ring; $N_0 = N_{0\uparrow} + N_{0\downarrow}$ is the number of electrons in the ground state. The deviation of the particle density $\rho(x,t)$ from the mean density in the ground state is

$$\rho(x,t) = \pi^{-1/2}\partial\varphi_\rho/\partial x. \quad (5)$$

It is seen from Eqs. (4) and (5) that the charging energy affects the charge subsystem and does not influence the spin subsystem. As a consequence the considered case differs sufficiently from the case of spinless electrons [44].

The partition function Z may be presented in the form of a path integral over the fields φ_ρ and φ_σ

$$Z = \int D\varphi_\rho\, D\varphi_\sigma\, \exp[-S_E/\hbar]. \quad (6)$$

The Euclidean action S_E is

$$S_E = \int_0^L dx \int_0^\beta d\tau\,[L_{LL}(x,\tau) + L_C(\tau) + L_{AB}(x,\tau)], \quad (7)$$

where $\beta = \hbar/T$; $\tau = it$ is an imaginary time.

The fields φ_ρ and φ_σ obey twisted boundary conditions [25]

$$\begin{aligned} &\varphi_\rho(x + k_1L, \tau + k_2\beta) \\ &\quad = \varphi_\rho(x,\tau) + k_1\pi^{1/2}(2m_\rho + k_{M\rho}) + k_2\pi^{1/2}n_\rho, \\ &\varphi_\sigma(x + k_1L, \tau + k_2\beta) \\ &\quad = \varphi_\sigma(x,\tau) + k_1\pi^{1/2}(2m_\sigma + k_{M\sigma}) + k_2\pi^{1/2}n_\sigma, \end{aligned} \quad (8)$$

where $k_1, k_2, n_\rho, n_\sigma, m_\rho, m_\sigma$ are integers; $k_{M\rho}, k_{M\sigma}$ are topological numbers characterizing the parity of additional numbers (over the number in the ground state) of charge N_ρ and of spin N_σ excitations in a ring. From Eq. (2) it follows that both n_ρ and n_σ (and accordingly m_ρ and m_σ) have the same parity. Moreover, we can write $k_{M\rho} = k_{M\uparrow} + k_{M\downarrow}$; $k_{M\sigma} = k_{M\uparrow} - k_{M\downarrow}$, where the topological number k_{Ms} ($s = \uparrow, \downarrow$) characterizes the parity of the additional number of electrons with spin s. The numbers k_{js} and k_{Ms} depend on the parity of the number of electrons with spin s N_{0s} in the ground state [35] as well as in the case of spinless fermions [25]

$$k_{js} = k_{Ms} \quad \text{if } N_{0s} \text{ is odd}, \quad (9)$$

$k_{js}=1;\quad k_{Ms}=0$ and $k_{js}=0;$

$k_{Ms}=1$ if N_{0s} is even.

Note that for an isolated ring we have $m_\rho=m_\sigma=k_{M\rho}=k_{M\sigma}=0$.

The present Lagrangian $L=L_{\rm LL}+L_{\rm C}+L_{\rm AB}$ is quadratic in fields φ_ρ and φ_σ, therefore the extremal trajectories obeying the boundary conditions (8) and determining the flux-dependent part of the free energy $\Delta F(\Phi)$ are linear functions of both x and τ

$$\varphi_\rho(x,\tau)=\pi^{1/2}\left[(2m_\rho+k_{M\rho})\frac{x}{L}+n_\rho\frac{\tau}{\beta}\right],$$
$$\varphi_\sigma(x,\tau)=\pi^{1/2}\left[(2m_\sigma+k_{M\sigma})\frac{x}{L}+n_\sigma\frac{\tau}{\beta}\right]. \qquad (10)$$

By using extremal trajectories (10) for calculating the Euclidean action $S_{\rm E}$ (7) and performing the summation over n_ρ,n_σ,m_ρ and m_σ with respect to above restrictions one can express $\Delta F(\Phi)$ in terms of Jacobi theta functions [50] $\theta_2(v,q)$ and $\theta_3(v,q)$.

We consider two cases:

(a) $N_{0\uparrow}$ is even; $N_{0\downarrow}$ is odd (or vice versa)

$$\Delta F(\Phi)=-T\,{\rm Ln}\{\theta_3[0,q_0^4]\theta_3[1/2,q_\sigma^4]\times\theta_3[2\varphi+1/2,q_\rho^4]\theta_3[2\delta,q_{\rm C}^4]+\theta_3[1/2,q_0^4]\theta_3[0,q_\sigma^4]\times\theta_3[2\varphi,q_\rho^4]\theta_3[2\delta+1/2,q_{\rm C}^4]\}. \qquad (11a)$$

(b) both $N_{0\uparrow}$ and $N_{0\downarrow}$ are odd

$$\Delta F(\Phi)=-T\,{\rm Ln}\{\theta_3[1/2,q_0^4]\theta_3[1/2,q_\sigma^4]\times\theta_3[2\varphi+1/2,q_\rho^4]\theta_3[2\delta+1/2,q_{\rm C}^4]+\theta_3[0,q_0^4]\theta_3[0,q_\sigma^4]\theta_3[2\varphi,q_\rho^4]\theta_3[2\delta,q_{\rm C}^4]+\theta_2[0,q_0^4]\theta_2[0,q_\sigma^4]\theta_2[2\varphi,q_\rho^4]\theta_2[2\delta,q_{\rm C}^4]\}. \qquad (11b)$$

Note that if both $N_{0\uparrow}$ and $N_{0\downarrow}$ are even the expression for $\Delta F(\Phi)$ can be deduced from Eq. (11b) by changing $\varphi\to\varphi+1/2$ (or $\delta\to\delta+1/2$).

In Eqs. (11) we introduce the following designations: $\varphi=\Phi/\Phi_0$; $q_0=\exp[-T/T_{0\sigma}]$; $q_\sigma=\exp[-T/T_\sigma^*]$; $q_{\rm C}=\exp[-\pi^2T/(2T_{\rm C})]$; $q_\rho=\exp[-T/T_\rho^*]$; $T_\rho^*=g_\rho\Delta_{\rm F}/\pi^2$; $T_\sigma^*=v_\sigma g_\sigma\Delta_{\rm F}/(\pi^2 v_\rho)$; $T_{0\sigma}=4v_\sigma\Delta_{\rm F}/(\pi^2 v_\rho g_\sigma)$ and

$$T_{\rm C}=2\Delta_{\rm F}/g_\rho+8E_{\rm C}, \qquad (12)$$

where $\Delta_{\rm F}=hv_\rho/L$. The above introduced quantities characterize the energy necessary for exciting a charge ($\approx T_{\rm C}$) or a spin ($\approx T_{0\sigma}$) excitation in a ring and the level spacing for the charge ($\approx T_\rho^*$) and for the spin ($\approx T_\sigma^*$) subsystems in an isolated ring.

The quantity δ is

$$\delta=e(V_{\rm g}-V_{\rm g0})/T_{\rm C}. \qquad (13)$$

We define $N(V_{\rm g0})=N_0$ therefore the charging energy does not affect the ground state ($T=0$) at $\delta=0$ (see Eq. (4)). At the same time at $T\neq0$ and/or $V_{\rm g}\neq V_{\rm g0}$ the effect of the charging energy on the system of electrons with spin is important.

Note that for noninteracting electrons $T_\rho^*=T_\sigma^*=T_{0\sigma}=2T_N^*$, where T_N^* is the crossover temperature for an isolated ring with spinless fermions [25], and $\Delta_{\rm F}$ is the level spacing at the Fermi surface for $\Phi=0$.

It should be noted that if both the energy and the particle exchange with a reservoir are to be allowed we must calculate the thermodynamic potential Ω instead of the free energy F. The flux-dependent part of Ω is determined by Eq. (11) with respect to the replacement of $\delta\to\delta+\mu/T_{\rm C}$, where μ is the chemical potential of an electron reservoir. Such a modification does not change the resulting expressions and leads to a redefinition of a dependence $\delta(V_{\rm g})$ only. Note also that from these expressions follows the well known fact [51] that the properties of a mesoscopic system are periodic in μ with a period $T_{\rm C}$ which for the Coulomb blockade regime ($E_{\rm C}\gg\Delta_{\rm F}/(4g_\rho)$) far exceeds the one characteristic for a free electron gas $\Delta_{\rm F}$ [13].

Eq. (11) defines the dependence of the free energy (and the persistent current) on both the magnetic flux Φ and the potential $V_{\rm g}$ (the parameter δ). Considering that $\theta_3(v+1,q)=\theta_3(v,q)$ and $\theta_2(v+2,q)=\theta_2(v,q)$ we conclude that, in the general case, the free energy is periodic in Φ with a period of $\Phi_0/2$ if N_0 is odd and with a period of Φ_0 if N_0 is even, as well as in the absence of the charging energy [33,30].

The free energy as a function of δ is periodic with a period of 1/2 if N_0 is odd and with a period of 1 if N_0 is even. When δ changes by 1 (the value $eV_{\rm g}$ changes $2\Delta_{\rm F}/g_\rho+8E_{\rm C}$) then N_ρ changes by 4.

At some values of the parameters the period of oscillations may be changed. In the next section we consider such a correlation.

3. Calculation of the persistent current

The purpose of the present paper is to consider the effect of the charging energy on the properties of a persistent current $I=-\partial F/\partial\Phi$. By using Eq. (11) we can calculate the current for arbitrary values of E_C. Below we consider the limit of a small capacitance C when the charging energy exceeds characteristic scales of energy excepting the Fermi energy (the Coulomb blockade regime).

From Eq. (12) it follows that the charging energy E_C renormalizes (increases) the energy necessary for exciting a charge excitation $\Delta N_\rho = 1$ in an electron system. Note that the increase of a repulsive electron–electron interaction ($g_\rho \ll 2$) leads to a similar effect. Thus, we will assume

$$\mu \gg (2/\pi^2)T_C \gg T^*_\rho, T^*_\sigma, T_{0\sigma}. \tag{14}$$

In such a case at $T \ll T_C$ the number of charge excitations in a ring is conserved $N_\rho = \text{const}$. However, the properties of persistent currents for an open system of electrons with spin in the Coulomb blockade regime differ from the ones for an isolated system that is due to an interaction with a spin subsystem.

Following Eq. (11) we calculate the persistent current in two cases according to the parity of the electrons number N_0.

3.1. The odd number of electrons in the ground state

As it follows from Eq. (11a), the persistent current is periodic in Φ with a period of $\Phi_0/2$. Let us find the dependence of a current amplitude on the potential V_g. We calculate the current at $\Phi=\Phi_0/8$. Note that at $T \gg T^*_\rho$ we obtain the second harmonic amplitude I_2 while at $T \ll T^*_\rho$ we obtain the sum $\sum_{k=1}^{\infty} I_{2(2k-1)}$

$$I(\varphi=1/8,\delta) = \frac{2T}{\Phi_0}F(3/4, 4T/T^*_\rho)A_2(\delta), \tag{15}$$

where

$$F(\varphi,p) = 2\pi\sum_{m=1}^{\infty}(-1)^m \sin[2\pi m\varphi]/\sinh[mp]. \tag{16}$$

The even function $A_2(\delta)$ has a period of 1/2.

Consider first the case of $g_\sigma = 2$. After a little manipulation we get

$$A_2(\delta) = \frac{\theta_2[4\delta, q_C^{16}]}{\theta_3[4\delta, q_C^{16}]} = \begin{cases} \dfrac{1-\exp[-(1/8-\delta)T_C/T]}{1+\exp[-(1/8-\delta)T_C/T]}, \\ \quad 0<\delta<1/4,\ T \ll T_C, \\ 2\exp[-2\pi^2 T/T_C]\cos(4\pi\delta), \quad T \gg T_C. \end{cases} \tag{17}$$

The dependence $A_2(\delta)$ is depicted in Fig. 2a. It is seen, when the potential V_g (the parameter δ) is varied the current amplitude changes sign (i.e. the phase of the dependence $I(\delta)$ changes by π) that is due to the change of the number of charge excitations $\Delta N_\rho = 1$ in a ring. Such a change takes place at $\delta_0 = \pm 1/8$ (that corresponds to a half-integer values of $N(V_g) - N_0$ in Eq. (4)) when the Coulomb blockade is lifted and when $A_2(\delta_0) = 0$. As a result the period of a dependence $I(\Phi)$ halves and becomes equal to $\Phi_0/4$. In such a case the persistent current is

$$I(\varphi,\delta_0) = \frac{4T}{\Phi_0}F(4\varphi, 16T/T^*_\rho). \tag{18}$$

Note that the crossover temperature $T^* = T^*_\rho/16$ is reduced four times compared to the Coulomb blockade regime.

In the case of $g_\sigma \neq 2$ the zeros δ_0 of the function $A_2(\delta)$ depend on the parameters characterizing the spin subsystem. So, at $T \to 0$ within the interval $0<\delta<1/4$ the zero is

$$\delta_0 = \left(\frac{1}{8} + \frac{\pi^2(T_{0\sigma} - T^*_\sigma)}{16T_C}\right). \tag{19}$$

While away from the points $\delta = \delta_0$ (namely at $\delta = 0$ and $\delta = 1/4$) the persistent current does not depend on the spin subsystem parameters at $T \ll T_C$

$$I(\varphi,0) = \frac{2T}{\Phi_0}F(2\varphi + 1/2, 4T/T^*_\rho). \tag{20}$$

$$I(\varphi,1/4) = \frac{2T}{\Phi_0}F(2\varphi, 4T/T^*_\rho). \tag{21}$$

The current Eq. (20) is paramagnetic and coincides with the one in the isolated ring containing an odd number of electrons with spin [33,30]. While the current equations (21) and (18) are diamagnetic around $\Phi = 0$.

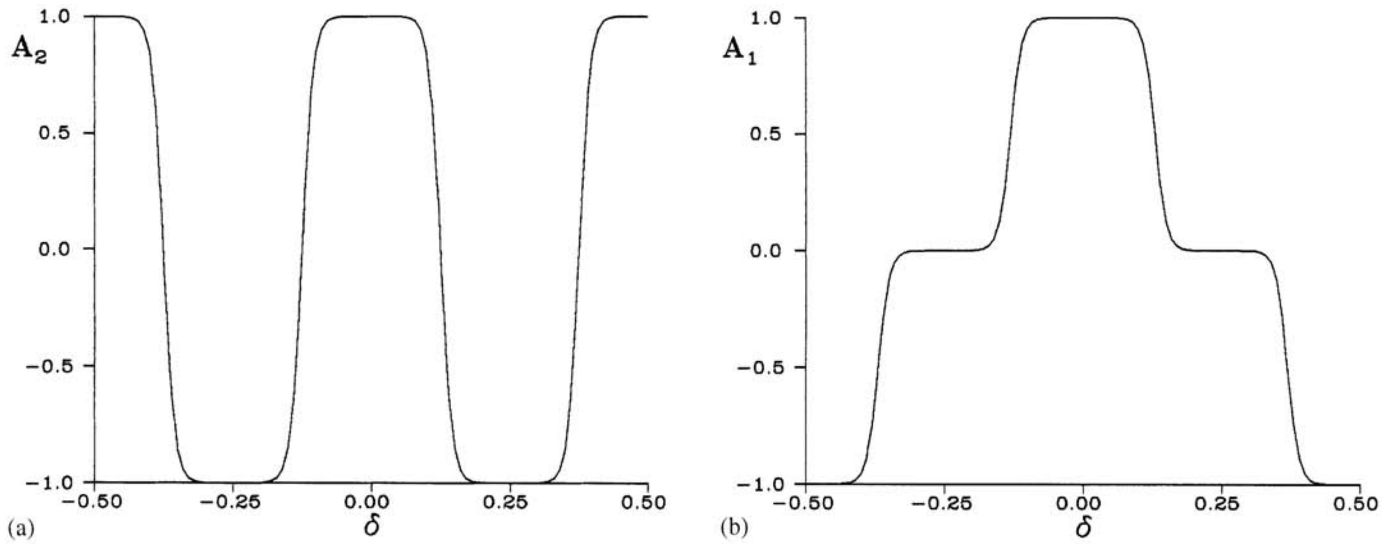

Fig. 2. (a) The dimensionless current amplitude A_2 (Eq. (17)) as a function of the dimensionless potential difference δ for a ring with an odd number of electrons with spin; $T_C/T=100$. (b) The dimensionless current amplitude A_1 (Eq. (23)) as a function of the dimensionless potential difference δ for a ring with an even number of electrons with spin; $T_C/T=100$; $\alpha=0.1$.

3.2. The even number of electrons in the ground state

As it follows from Eq. (11b), in the general case, the persistent current $I=-\partial F/\partial\Phi$ is periodic in Φ with a period of Φ_0. However, at some values of V_g the period may be reduced to two ($\Phi_0/2$) or four ($\Phi_0/4$) times depending on the electrons system parameters. The crossover temperature T^* is also reduced.

We consider the case when both $N_{0\uparrow}$ and $N_{0\downarrow}$ are odd. Note that after the replacement $\varphi\to\varphi+1/2$ (or $\delta\to\delta+1/2$) the expressions obtained below are valid for the case when both $N_{0\uparrow}$ and $N_{0\downarrow}$ are even.

The dependence of the current amplitude (more precisely of the current magnitude at $\Phi=\Phi_0/4$) on the potential V_g is

$$I(\varphi=1/4,\delta)=\frac{T}{\Phi_0}F(1/4,T/T_\rho^*)\,A_1(\delta). \tag{22}$$

The even dependence $A_1(\delta)$ has a period of 1. At $T\to 0$ it is

$$A_1(\delta)=\frac{\sinh[(1/4-|\delta|)T_C/T]}{\cosh((1/4-|\delta|)T_C/T)+\exp((1-\alpha)T_C/(8T))},\quad -1/2<\delta<1/2, \tag{23}$$

where $\alpha=\pi^2(T_{0\sigma}+T_\sigma^*-T_\rho^*)/(2T_C)$. The dependence $A_1(\delta)$ is shown in Fig. 2b. The similar dependence was obtained in Ref. [40] for the ring coupled to a side branch quantum dot.

At $\delta=\pm 1/8$ and $\delta=\pm 3/8$ the Coulomb blockade is lifted and the number of charge excitations N_ρ in a ring changes by 1. Note that in the considered case the current changes sign when the number of charge excitations N_ρ changes by $2n$ (where n is an integer) over the ground state number N_0. If $N_\rho-N_0=\xi+2n$ (where $\xi=1,3$) the odd harmonics vanish and period of the current halves ($\Phi_0/2$). At both $\delta=\pm 1/4$ and $T\ll T_C$ the current is similar to the one for odd N_0 at $\delta=0$ (Eq. (20)). At $T\to 0$ such a current is characteristic for whole plateau of length $\Delta\delta\simeq(1-\alpha)/4$.

At both $\delta=0$ and $T\ll T_C$ the current in the Coulomb blockade regime is similar to the one for an isolated ring [30]; however, it differs from the earlier one which is due to the effect of the reservoir spin subsystem. Note that at $\delta=1/2$ the current is $I(\varphi,1/2)=I(\varphi+1/2,0)$. In the case of noninteracting electrons ($g_\rho=g_\sigma=2$; $v_\rho=v_\sigma=v_F$) the current has a period of Φ_0 and the crossover temperature is $T^*\simeq T_\rho^*/2=T_N^*$. In the case of correlated electrons the current depends on the electron–electron interaction.

So, in the case of $T_\rho^*\ll T_\sigma^*, T_{0\sigma}$ the spin subsystem is frozen in all the temperature range $T\leqslant T_\rho^*$ where the persistent current exists. This leads to a doubling of the crossover temperature $T^* = T^*_\rho = 2T^*_{\rm N}$. In such a case the current is

$$I(\varphi, 0) = \frac{T}{\Phi_0} F(\varphi, T/T^*_\rho). \qquad (24)$$

On the other hand, at $T_{\rm C} \gg T \simeq T^*_\rho \gg T^*_\sigma, T_{0\sigma}$, when the discreteness of the energy spectrum of a spin subsystem is irrelevant, both the period of a current and the crossover temperature are reduced two times compared to the case of noninteracting electrons and at $\delta = 0$ the current is given by the same expression as Eq. (21). Thereto the dependence of a current amplitude on the potential $V_{\rm g}$ (on the parameter δ), i.e. $I(1/8, \delta)$, can be deduced from that for odd N_0 Eqs. (15), (17) by changing $\delta \to \delta + 1/4$. Such a dependence has a period of 1/2 and its zeros are $\delta_0 = \pm 1/8$. At $\delta = \delta_0$ the current is given by Eq. (18), i.e. the current has a period of $\Phi_0/4$ and the crossover temperature is $T^* = T^*_\rho/16$.

4. Discussion and conclusion

In the present paper the effect of the charging energy due to the charge transfer between a mesoscopic ring and a reservoir on the persistent current is considered. In the Luttinger liquid approach [48] for electrons with spin Eq. (1) the analytical expression for the flux-dependent part of the free energy with respect to the charging energy is obtained. The charging energy is taken into account in the geometrical capacitance C approach. The obtained expressions allow us to analyse the effect of model parameters on the persistent current properties (the period and the sign of the current, the dependence of the current on the temperature, etc.) which characterize the ground state (and low-lying excited states) of an electron system.

The charging energy $E_{\rm C}$ renormalizes the energy $E_{0\rho} = \Delta_{\rm F}/(2g_\rho)$ necessary for exciting a charge excitation in a ring coupled to a reservoir (see Eq. (12)). In the limit $E_{\rm C} \gg E_{0\rho}$ (the Coulomb blockade regime) the number of charge excitations N_ρ in a ring is conserved (when both the magnetic flux Φ and the temperature $T \ll E_{\rm C}$ are varied). In the spinless case [44] an open mesoscopic system in the Coulomb blockade regime is equivalent to an isolated system ($N_{\rm e} = $ const.). In the case of electrons with spin this is not true, because the spin subsystem of a ring is not decoupled from a reservoir and affects the persistent current pursuant to the parity effect [30,34,35].

At certain values of the potential difference $V_{\rm g}$ (the parameter δ, see Eq. (13)) between a ring and a reservoir (see Fig. 1), the Coulomb blockade is lifted and the number of charge excitations N_ρ in a ring changes by 1. Since the potential $V_{\rm g}$ does not affect the spin subsystem the condition $\Delta N_\rho = 1$ is not equivalent to the change of the number of electrons with spin $N_{\rm e}$ in a ring by 1. At the same time the change $\Delta N_\rho = 2$, in fact, is equivalent to the change of the number of electrons with spin "up" $\Delta N_{{\rm e}\uparrow} = 1$ and with spin "down" $\Delta N_{{\rm e}_\downarrow} = 1$. Therefore, the dependence of the current amplitude I on $V_{\rm g}$ for the ring with an odd number of electrons N_0 in the ground state (see Fig. 2a), in the general case, is quite different from the one for the ring with even N_0 (see Fig. 2b). However, as it was pointed out in Ref. [40] both the dependences have a similar feature. Namely, in the case of a large charging energy the dependence $I(V_{\rm g})$ has a sequence of plateaus of diamagnetic and paramagnetic states.

The current for the isolated ring with an odd number N_0 of electrons with spin is periodic in Φ with a period of $\Phi_0/2$ and is paramagnetic (at $\Phi \simeq 0$) [30,33]. In the Coulomb blockade regime such a current is characteristic for the plateau near $\delta = 0$ (see Eq. (20)). While for $\delta = 1/4$ (Eq. (21)) the current is diamagnetic. In the vicinity of the charging energy degeneracy point $\delta = 1/8$ the number N_ρ fluctuates with $\delta N_\rho = 1$, which leads to period halving ($\Phi_0/4$). However, such a conclusion is true for noninteracting (between themselves) electrons only. For interacting electrons the behaviour of a current is more complicated. From Eq. (11a) it follows that for $\delta = 1/8 + n/2$ (where n is an integer) the persistent current does not depend on the charging energy (the parameter $q_{\rm C}$) and coincides with the one for a ring coupled to a reservoir (i.e. for the regime $\mu = $ const. at $E_{\rm C} = 0$) when both the parity of N_0 and the value of δ are same (as it is pointed out above, the parameter δ depends on both $E_{\rm C}$ and μ). At $g_\sigma = 2$ or at $T_{0\sigma}, T^*_\sigma \ll T \simeq T^*_\rho$ the current is defined by Eq. (18) and the period halves. On the other hand, the current is defined by Eq. (21) at $T_{0\sigma} \ll T \simeq T^*_\rho \ll T^*_\sigma$ and by Eq. (20) at $T^*_\sigma \ll T \simeq T^*_\rho \ll T_{0\sigma}$ and the period does not halve. In the last cases the period halves at other values of the parameter $\delta = \delta_0$ (Eq. (19)) which depend on both the spin subsystem parameters and the charging energy $E_{\rm C}$.

For the ring with an even number N_0 of electrons with spin, the effect of a spin subsystem on the persistent current is more strong. In particular, for the case of noninteracting electrons such an effect reduces the current in the Coulomb blockade regime $I(1/4,0) \simeq \exp[-3T/T_\rho^*]$ compared to the current in an isolated ring ($N_e = \text{const.}$) $I(\varphi = 1/4) \simeq \exp[-2T/T_\rho^*]$ at $T > T_\rho^*$. On the other hand, if the parameters of spin and charge subsystems are quite different, $T_\rho^* \ll T_\sigma^*, T_{0\sigma}$ or $T \simeq T_\rho^* \gg T_\sigma^*, T_{0\sigma}$, the persistent current in the Coulomb blockade regime at $\delta = 0$ coincides with the one in an isolated ring at the same parity of N_0. At $\delta = 1/8 + n/2$ when the Coulomb blockade is lifted the current does not depend on the charging energy; however, it does not coincide with the one for a ring coupled to a reservoir ($\mu = \text{const.}$, $E_C = 0$).

In conclusion, in the present paper we have discussed the effect of the charging energy on the persistent current in an open mesoscopic ring containing correlated electrons with spin and have shown that the Coulomb blockade regime for the persistent current is quite different from both the regime $\mu = \text{const.}$ and the regime $N_e = \text{const.}$

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