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Persistent current in a one-dimensional ring with a weak link

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Abstract

The effect of a single impurity on the persistent current in a one-dimensional ring of correlated electrons is considered. It is shown that the dependence of the current amplitude on the temperature may be used for detecting a non-Fermi liquid behaviour of an electron system. © 1999 Elsevier Science B.V. All rights reserved.

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1. Introduction

At low temperatures the phase breaking length $L_\phi(T)$ of an electron wave function is large compared to the size L of small samples (mesoscopic samples) [1]. As a result the properties of such systems are sensitive to the change of a wave function phase, which provides the possibility to study the Aharonov–Bohm (AB) effect [2] in solids.

The physical properties of two-connected mesoscopic samples with an AB magnetic flux Φ through its opening are periodic in Φ with a period of $\Phi_0 = 2\pi\hbar/e$ [3,4]. One of the manifestations of this effect is the existence of a thermodynamic equilibrium (persistent) current

$$I(\Phi) = -dF/d\Phi, \quad (1)$$

where F is the free energy (for an isolated sample) or the thermodynamic potential (for a sample connected to an electron reservoir) [3,4]. For the first time the existence of such a current (or a magnetic moment) was predicted in Ref. [5] for ballistic cylinders and in Ref. [6] for one-dimensional rings with a disorder. The persistent current was observed experimentally in the ensemble of many mesoscopic rings [7,8] as well as in single rings [9,10].

The investigation of the persistent current problem [11–48] shows that the properties of such currents depend on both the features of an electron system in a ring and the character of an interaction with an environment (with a reservoir). Note, that the model of noninteracting spinless electrons in a ballistic ring [13] agrees with

the experiment [10]. At the same time in the case of disordered rings the experimentally observed persistent current [7,9] is two orders of magnitude larger than the theoretically predicted one [21,22]. By now such a discrepancy has not a satisfactory explanation and the study of the interplay of interelectron interactions and impurities stays yet an actual problem of persistent currents physics.

In the present paper we consider the effect of a single-point impurity on the persistent current in a one-dimensional ring of correlated electrons. In the limit of a small transmission coefficient through the impurity this problem may be solved analytically that allows to investigate the effect of parameters of an interelectron interaction on the persistent current in the isolated ring ($N = \text{const}$), as well as in the ring coupled to a reservoir ($\mu = \text{const}$). In the case of a high concentration of impurities the same problem was considered in Ref. [48]. The effect of a single impurity in the limit of a strong electron–electron repulsion ($g \ll 1$) at a fixed number of electrons (the model of a pinned Wigner crystal) was investigated in Ref. [45].

On the other hand, recently the interest in the low-dimensional interacting electron systems [49–54] have increased with a view to find a non-Fermi liquid behaviour of the electron systems. This also makes the present paper actual. It is known that the persistent current amplitude is proportional to the transmission coefficient of a potential barrier created by an impurity in a ring [13,36]. At the same time, in the presence of interelectron interactions the effective transparency of a barrier depends on the temperature [55]. Therefore, the measurement of a dependence of a persistent current amplitude on the temperature may be used as contactless methods (in contrast with the measurement of a conductance) for the detection of a non-Fermi liquid behaviour of electron systems.

2. Model and main equations

Let us consider a one-dimensional isolated ring of interacting electrons with a weak link located at $x = 0$. Our aim is to calculate the current due to a magnetic flux Φ to a leading order in the hopping strength t .

We describe interacting electrons in a ring as a Luttinger liquid [52,55]. The Euclidean action S_0 (in the ϕ representation) describing the Luttinger liquid is [55]

$$S_0 = \hbar \int_0^L dx \int_0^\beta d\tau \frac{vg}{2} \left\{ \left(\frac{\partial \phi}{\partial x} \right)^2 + \frac{1}{v^2} \left(\frac{\partial \phi}{\partial \tau} \right)^2 \right\}, \quad (2)$$

where τ is an imaginary time, $\beta = 1/T$, T is the temperature, v and g are the Haldane's parameters. For noninteracting electrons $g = 1$; $v = v_F$, where $v_F = \pi \hbar N / (mL)$ is the Fermi velocity (here L is the length of a ring, N is the number of electrons in a ring, m is an electron mass). The hops of electrons across the weak link we include by adding [55]

$$S_t = -\hbar t \int_0^\beta d\tau \cos(2\sqrt{\pi} \Delta \phi(\tau) + 2\pi(\Phi/\Phi_0 + \alpha_N)). \quad (3)$$

Here $\Delta \phi(\tau) = (\phi(x=0, \tau) - \phi(x=L, \tau))$, the term $\alpha_N \equiv \alpha(N) = (N - 1)/2 \bmod 1$ describes the parity effect for the ring with a fixed number of electrons N [26]. If we integrate out fluctuations in $\phi(x)$ for all $x \neq 0$ we obtain the effective action S_0^{eff} which depends on $\Delta \phi$ only [55]. For this purpose we expand $\Delta \phi(\tau)$ in the Fourier series

$$\Delta \phi(\tau) = \frac{1}{\beta} \sum_{n=-\infty}^{\infty} e^{i\omega_n \tau} \phi_n + \phi_0 L/2, \quad (4)$$

where $\omega_n = 2\pi n/\beta$ is the Matsubara frequency ($n \neq 0$). ϕ_0 is an arbitrary real constant (the zero mode). Finally, the effective action is

$$S_0^{\text{eff}}[\phi, g, v] = \frac{\hbar g}{\beta} \left\{ \sum_{n \neq 0} |\omega_n| |\phi_n|^2 f_n(v) + L\beta^2 v \phi_0^2/2 \right\}. \quad (5)$$

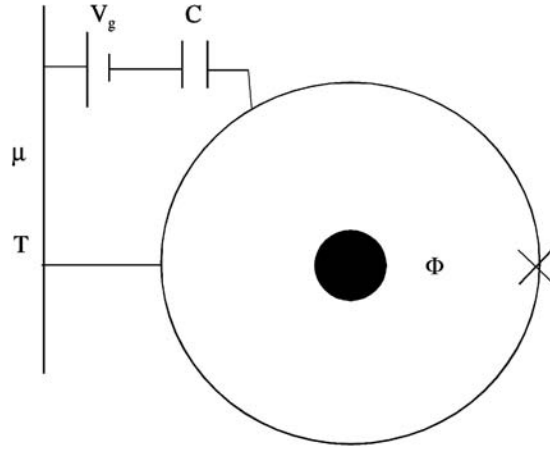


Fig. 1. One-dimensional ring pierced by a magnetic flux Φ and weakly coupled to an electron reservoir with the chemical potential μ and the temperature T . V_g and C are the potential difference and the geometrical capacitance between a ring and a reservoir, respectively. The cross denotes a weak link.

Here $f_n(v) = (1 + \exp(-|\omega_n|L/v))/(1 - \exp(-|\omega_n|L/v))$. Taking into account that $\phi_n = \phi_{-n}^*$ we write down the partition function Z_N (the index N denotes a fixed number of electrons in a ring) which determines the free energy $F = -T \ln(Z_N)$ as follows: $Z_N = \int \mathcal{D}\phi e^{-(S_0^{\text{eff}} + S_t)/\hbar}$, where $\int \mathcal{D}\phi \equiv \int_{-\infty}^{\infty} d\phi_0 \prod_{n=1}^{\infty} \int_{-\infty}^{\infty} d\text{Re}(\phi_n) \times \int_{-\infty}^{\infty} d\text{Im}(\phi_n)$.

In the case of spinfull electrons [55] the effective action is $S_{0s}^{\text{eff}} = S_0^{\text{eff}}[\phi_\rho, g_\rho, v_\rho] + S_0^{\text{eff}}[\phi_\sigma, g_\sigma, v_\sigma]$. The perturbation is

$$S_{ts} = -2\hbar t \int_0^\beta d\tau \{ \cos(2\sqrt{\pi}\Delta\phi_\rho(\tau) + 2\pi\Phi/\Phi_0 + \pi(\alpha_\uparrow + \alpha_\downarrow)) \cos(2\sqrt{\pi}\Delta\phi_\sigma(\tau) + \pi(\alpha_\uparrow - \alpha_\downarrow)) \}, \quad (6)$$

and the partition function is $Z_{N_s} = \int \mathcal{D}\phi_\rho \mathcal{D}\phi_\sigma e^{-(S_{0s}^{\text{eff}} + S_{ts})/\hbar}$. Here the bosonic fields $\phi_\rho = (\phi_\uparrow + \phi_\downarrow)/2$ and $\phi_\sigma = (\phi_\uparrow - \phi_\downarrow)/2$ describe the charge (ρ) and the spin (σ) subsystems. The fields $\phi_\uparrow, \phi_\downarrow$ correspond to electrons with spin up (\uparrow) and down (\downarrow). For noninteracting electrons the Haldane's parameters [53,55] are: $g_\rho = g_\sigma = 2$ and $v_\rho = v_\sigma = v_F$. The quantity $\alpha_s = \alpha(N_s)$, where $s = \uparrow, \downarrow$, and N_s is the number of electrons with spin s in the ring. The quantities $\phi_\rho(\tau)$ and $\phi_\sigma(\tau)$ are expanded in the Fourier series by analogy with $\phi(\tau)$ (4).

Expanding the partition function Z_N in powers of t we may integrate out ϕ (or ϕ_ρ and ϕ_σ in the case with spin) that allows to obtain an analytical expression for the persistent current (1). We consider an isolated ring as well as a ring coupled to a reservoir. In the last case we also take into account the small geometrical capacitance C between a ring and a reservoir (see Fig. 1). At low temperatures the charging energy $E_c = e^2/(2C)$ associated with the transfer of an elementary charge between a ring and a reservoir may exceed the temperature T and strongly suppresses the charge transfer (the Coulomb blockade regime [56–58]). This effect considerably affects the persistent current [59–65].

3. Fixed number of electrons

3.1. Spinless electrons

For spinless electrons in the limit of $t \rightarrow 0$ the persistent current $I = T \partial \ln(Z_N) / \partial \Phi$ is periodic in the magnetic flux with a period of Φ_0 and in the leading order is proportional to t [13]. Using the partition function Z_N we obtain the persistent current I_1 as follows:

$$I_1 = -I_{01} A_1(T) \sin(2\pi\Phi/\Phi_0 + \alpha_N), \quad (7)$$

$$I_{01} = \frac{ev}{2L} t_F \left(\frac{\pi^2 T^*}{\varepsilon_F} \right)^{1/g-1}, \quad (8)$$

$$A_1(T) \equiv A(T, T^*, g) = \left(\frac{T}{2\pi T^*} \theta_2^2 \left(0, \exp \left(-\frac{T}{2T^*} \right) \right) \right)^{1/g}$$

$$= \begin{cases} 1, & T \ll T^*, \\ \left(\frac{2gT}{\pi T_F^*} \right)^{1/g} e^{-T/4T_F^*}, & T \gg T^*. \end{cases} \quad (9)$$

Here $t_F = \pi \hbar t / \varepsilon_F$ is the transmission coefficient of a potential barrier (of a weak link) within the free electron gas model at the Fermi energy; $T^* = \hbar v / (2\pi L)$; $T_F^* = gT^*$; $\theta_2(v, q)$ is the Jacobi theta functions [66].

The obtained expressions describe the dependence of a current of correlated electrons on the temperature and reproduce some features obtained in the papers of other authors [13,36,45]. Note that in the considered model the dependence of a current on the temperature is monotonic, which differs from the nonmonotonic dependence in the model of a pinned Wigner crystal [45].

3.2. Spinfull electrons

The presence of spin considerably affects the persistent current [65,67–69]. In particular, for the ring with a fixed number of electrons the dependence $I(\Phi)$ has a period of Φ_0 if both N_\uparrow and N_\downarrow have the same parity, otherwise the period is $\Phi_0/2$. The last circumstance is important for the ring with a weak link ($t_F \ll 1$). Because, the dependence $I_2 \sim \sin(4\pi\Phi/\Phi_0)$ appears in the second order in t and the current amplitude $I_{2s} \sim t_F^2$ in the ring with an odd number of electrons $N = N_\uparrow + N_\downarrow$ is much less than the one I_{1s} in the ring with an even number of electrons N : $I_{2s}/I_{1s} \sim t_F \ll 1$.

Note, that in the present paper we consider a conventional Aharonov–Bohm effect. At the same time, as it was found in Refs. [68,70–72] the interelectron correlations result in a fractional Aharonov–Bohm effect with a period of Φ_0/N , where N is the number of electrons in the system. However, the amplitude of such oscillations vanishes with an increase in the number of electrons N (at constant parameters of a system) that is numerically shown in Ref. [71]. In the present paper we consider the case of $N \gg 1$ when the spectrum may be linearized near the Fermi points (Luttinger liquid model). In such a case the fractional oscillations do not appear.

3.2.1. Even number N of electrons

In this case the current is periodic in Φ with a period of Φ_0 . Calculating the current I_{1s} in the first order in t with respect to Eq. (6) we get $I_{1s} = -I_{01s} A_{1s}(T) \sin(2\pi\Phi/\Phi_0 + \pi(\alpha_\uparrow + \alpha_\downarrow))$, where

$$I_{01s} = \frac{e(v_\rho v_\sigma)^{1/2}}{L} t_F \left(\frac{\pi^2 T_\rho^*}{\varepsilon_F} \right)^{1/g_\rho-1/2} \left(\frac{\pi^2 T_\sigma^*}{\varepsilon_F} \right)^{1/g_\sigma-1/2}. \quad (10)$$

$$A_{1s}(T) = A(T, T_\rho^*, g_\rho) A(T, T_\sigma^*, g_\sigma). \quad (11)$$

Here $T_r^* = \hbar v_r / (2\pi L)$ ($r = \rho, \sigma$), the quantity $A(T, T^*, g)$ is defined in Eq. (9). As it follows from the above expressions the interplay of spin and interactions leads to an existence of three characteristic temperature intervals which are separated by two crossover temperatures T_ρ^* and T_σ^* . So, in the case of a strong interelectron

repulsion $g_\rho \ll 2$ which does not depend on spin $g_\sigma = 2$ we have $T_\sigma^* \ll T_\rho^*$ that leads to

$$A_{1s}(T) = \begin{cases} 1, & T \ll T_\rho^*, T_\sigma^*, \\ \left(\frac{2T}{\pi T_\sigma^*}\right)^{1/2} \exp\left(-\frac{T}{8T_\sigma^*}\right), & T_\sigma^* < T < T_\rho^*, \\ \left(\frac{2T}{\pi T_\sigma^*}\right)^{1/2} \left(\frac{2T}{\pi T_\rho^*}\right)^{1/g_\rho} \exp\left(-\frac{T}{8T_\sigma^*} - \frac{T}{4g_\rho T_\rho^*}\right), & T_\sigma^*, T_\rho^* \ll T. \end{cases} \quad (12)$$

For noninteracting electrons $T_\rho^* = T_\sigma^*$. Thus, the dependence of a persistent current on the temperature may be used as an indicator of an electron–electron interactions.

3.2.2. Odd number N of electrons

In this case $\alpha_\uparrow \neq \alpha_\downarrow$ and as it follows from Eq. (6) the persistent current of the order of t vanishes that is due to quantum fluctuations in spin subsystem. The calculation of a current up to the order t^2 gives

$$I_{2s} = I_{02s}(-\gamma_1 \cos(2\pi(\alpha_\uparrow - \alpha_\downarrow)) - \gamma_2) A_{2s}(T) \sin(4\pi\Phi/\Phi_0). \quad (13)$$

$$I_{02s} = \frac{e(v_\rho v_\sigma)^{1/2}}{\pi L} t_F^2 \left(\frac{\pi^2 T_\rho^*}{\varepsilon_F}\right)^{2/g_\rho - 1} \left(\frac{\pi^2 T_\sigma^*}{\varepsilon_F}\right)^{2/g_\sigma - 1}. \quad (14)$$

$$A_{2s}(T) = A^2(T, T_\rho^*, g_\rho) A^2(T, T_\sigma^*, g_\sigma) \frac{\pi^2 (T_\rho^* T_\sigma^*)^{1/2}}{2T}. \quad (15)$$

$$\gamma_{1,2} = 2 \int_0^{1/2} dx \left(\frac{\theta_1(x, q_\rho)}{\theta_4(x, q_\rho)}\right)^{2/g_\rho} \left(\frac{\theta_4(x, q_\sigma)}{\theta_1(x, q_\sigma)}\right)^{\pm 2/g_\sigma}, \quad (16)$$

where $q_\rho = \exp(-T/T_\rho^*)$, $q_\sigma = \exp(-T/T_\sigma^*)$. Note, that at $g_\sigma \neq 2$ the quantity γ_1 diverges at some values of parameters. This indicates that the perturbation theory fails. In this paper we consider the case $g_\sigma = 2$ only, when the quantity γ_1 is limited and the perturbation theory may be used. In such a case, the dependence of a current in the ring with an odd number of electrons on the temperature is

$$\gamma A_{2s}(T) = \begin{cases} \zeta((2/g_\rho)^{1/2}), & T \ll T_\rho^*, T_\sigma^*, \\ \frac{\pi^{1/2} 2^{2/g_\rho - 1} \Gamma(1/g_\rho)}{\Gamma(1/g_\rho + 1/2)} \left(\frac{2T}{\pi T_\sigma^*}\right)^{1/2} \left(\frac{2T}{\pi T_\rho^*}\right)^{2/g_\rho - 1/2} \exp\left(-\frac{T}{g_\rho T_\rho^*}\right), & T_\sigma^*, T_\rho^* \ll T. \end{cases} \quad (17)$$

Here $\gamma = \gamma_1 - \gamma_2$,

$$\zeta(\xi) = 4 \int_0^\infty dx \frac{\exp(-2x/\xi)}{1 - \exp(-4x/\xi)} \left(\frac{1 - \exp(-2x/\xi)}{1 + \exp(-2x/\xi)}\right)^{\xi^2}.$$

The dependence $\zeta(\xi)$ is depicted in Fig. 2. $\Gamma(x)$ is the gamma function.

Thus, in the case of an odd number of spinfull electrons in the isolated ring with a weak link the persistent current is paramagnetic (near $\Phi = 0$), has a period $\Phi_0/2$, and does not depend on the temperature at $T \rightarrow 0$. These features coincide with the ones for the ballistic case [47,65,67]. At the same time, in contrast with the ballistic case for a ring with a weak link the current amplitude is considerably modified by the interelectron interaction compared to the free electrons gas model.

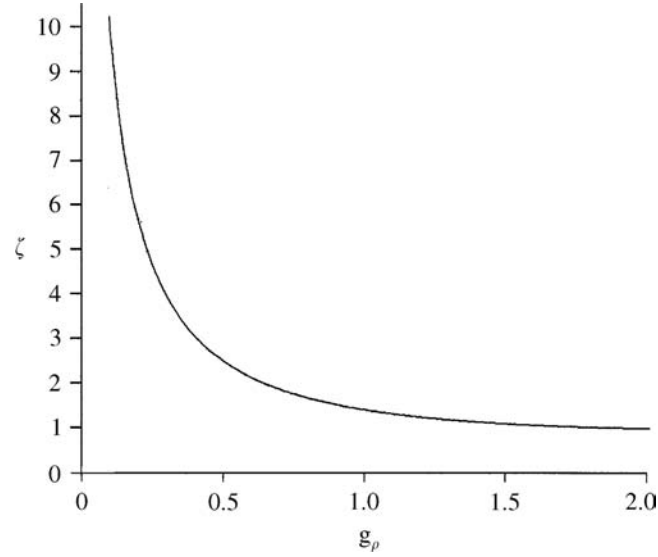


Fig. 2. Plot of ζ versus g_ρ (see Eq. (17)).

4. Fixed chemical potential

In this section we consider a ring coupled to an electron reservoir with a chemical potential μ . We assume that the coupling is weak, i.e. the lifetime of an electron in a ring is much larger than the characteristic time scales of a problem under consideration (the maximum time is an inverse frequency of tunneling through the barrier). In such a case, the reservoir does not affect the energy levels in a ring. Therefore, we can perform the quantum mechanical averaging in two steps. First, we perform an averaging at a fixed number of electrons and then we average over fluctuations of the number of electrons in a ring. Such an approximation corresponds to an incoherent exchange of particles between a ring and a reservoir. The averaging at $N = \text{const}$ was performed in the previous section. The aim of the present section is to perform an averaging over fluctuations of number of electrons at $\mu = \text{const}$.

The change in the number of particles leads to an appearance of zero modes in the τ -sector of the field ϕ [26,47]. Such modes do not affect the perturbation S_I (that is a not quadratic in ϕ part of the action) which depends on the difference $\Delta\phi(\tau) = \phi(x=0, \tau) - \phi(x=L, \tau)$ only. Thus, the averaging over fluctuations in the number of particles can be performed independently. Such an averaging we perform in θ representation [55]. If the number of particles changes we have to add the term in the Euclidean action [64]

$$S_\mu = \hbar \int_0^L dx \int_0^\beta d\tau \left\{ \frac{v}{2g} \left(\frac{\partial\theta}{\partial x} \right)^2 + \frac{1}{L} \mu \delta N + \frac{E_c}{L} (\delta N - N(V_g))^2 \right\}. \quad (18)$$

Here $\delta N = N - N_0$, where N_0 is the number of particles in the ground state ($T=0, N(V_g)=0$). The last term in Eq. (18) is the charging energy due to a small geometrical capacitance C between a ring and a reservoir (see Fig. 1). $E_c = e^2/(2C)$; $N(V_g) = C(V_g - V_{g0})/e$, where e is an electron charge. We assume that at $T=0$ and $V_g = V_{g0}$ the ring is electrically neutral. Taking into account that the zero mode $\theta_m(x) = \sqrt{\pi}mx/L$ corresponds to an additional number (over the number in the ground state) of particles $m \equiv \delta N$, we finally obtain

$$S_\mu(m) = \frac{T_c}{T} (m - 2\delta)^2 + \frac{E_c}{T} N^2(V_g) - \frac{T_c}{T} 4\delta^2, \quad (19)$$

where $T_c = T_0 + E_c$; $T_0 = \pi\hbar v/(2gL)$; $\delta = (e(V_g - V_{g0}) - \mu)/(4T_c)$. The partition function takes the form $Z_\mu = \sum_{m=-\infty}^{\infty} \exp(-S_\mu(m)/\hbar) Z_{N=N_0+m}$.

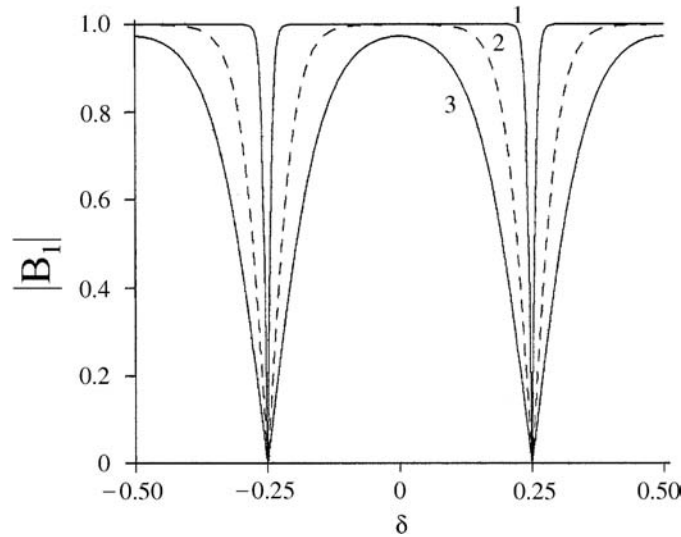


Fig. 3. Spinless electrons. Dependence of B_1 (see Eq. (21)) on the dimensionless potential difference δ for $T/T_c = 0.02$ (1); 0.1 (2); 0.2 (3).

Similarly, in the case of spinfull electrons we have

$$S_{\mu s}(m_\rho, m_\sigma) = \frac{T_{cs}}{T}(m_\rho - 4\delta_s)^2 + \frac{T_{0\sigma}}{T}m_\sigma^2 + \frac{E_c}{T}N^2(V_g) - \frac{T_{cs}}{T}16\delta_s^2. \quad (20)$$

Here m_ρ and m_σ have the same parity and characterize an additional number of charge $m_\rho = N_\rho - N_{0\rho}$ and spin $m_\sigma = N_\sigma - N_{0\sigma}$ excitations. Here $N_{0\rho} = N_{0\uparrow} + N_{0\downarrow}$ and $N_{0\sigma} = N_{0\uparrow} - N_{0\downarrow}$, where N_{0s} is the number of particles with spin $s = \uparrow, \downarrow$ in the ground state. $T_{cs} = T_{0\rho} + E_c$; $T_{0\rho,\sigma} = \pi\hbar v_{\rho,\sigma}/(2g_{\rho,\sigma}L)$; $\delta_s = (e(V_g - V_{g0}) - \mu)/(8T_{cs})$. To calculate the partition function Z_μ we have to perform the summation over both m_ρ and m_σ .

4.1. Spinless electrons

First we consider spinless electrons. Taking into account the results of Section 3.1 and Eq. (19) we obtain the following expression for the persistent current $I_{1\mu} = -I_{01}A_1(T)B_1(\delta)\sin(2\pi\Phi/\Phi_0 + \alpha(N_0))$, where

$$B_1(\delta) = \frac{\theta_2(2\delta, q_c^4)}{\theta_3(2\delta, q_c^4)} = \begin{cases} \frac{1 - \exp(-(T_c/T)(1 - 4|\delta|))}{1 + \exp(-(T_c/T)(1 - 4|\delta|))}, & T \ll T_c; |\delta| < \frac{1}{2}, \\ 2 \exp\left(-\frac{\pi^2 T}{4T_c}\right) \cos(2\pi\delta), & T \gg T_c. \end{cases} \quad (21)$$

Here $q_c = \exp(-\pi^2/4T/T_c)$. The quantities I_{01} and $A_1(T)$ are defined in Eqs. (8) and (9), respectively. The function $B_1(\delta)$ is periodic in δ with a period of 1, that corresponds to a periodicity in V_g (or in μ) with a period of $4T_c$ (this period is the same as for the ballistic case [64]). Such a period corresponds to the change of the number of particles in the ring by 2. The dependence $B_1(\delta)$ is depicted in Fig. 3.

Note that in the case of noninteracting electrons ($g = 1$, $v = v_F$, $E_c = 0$), the calculation of the persistent current using the Fermi distribution function [13] gives the same result as $I_{1\mu}$. In such a case, the persistent current I is

$$I = \sum_{n=1}^{\infty} f_0(\varepsilon_n)I_n. \quad (22)$$

Here $f_0(\varepsilon_n) = (1 + \exp((\varepsilon_n - \mu)/T))^{-1}$ is the Fermi function, $I_n = -\partial\varepsilon_n/\partial\Phi$ is the current carried by level $\varepsilon_n(\Phi) = \hbar^2 k_n^2(\Phi)/(2m)$, where $k_n(\Phi) = (1/L)(\pi n + (-1)^n t_n \cos(2\pi\Phi/\Phi_0))$, $t_n = t_F n/n_F$ (n_F is the topmost occupied level in the ring at $T = 0$).

In the limit of a large charging energy $E_c \gg T$ (the Coulomb blockade regime) the current amplitude $I_{1\mu}$ does not depend on V_g and coincides with the one at a fixed number of electrons excepting of a region near $\delta_0 = \pm 1/4 + n$ with n being an integer, where the number of electrons in a ring fluctuates $N \leftrightarrow (N + 1)$. Near δ_0 the odd harmonics of a current $I(\Phi)$ vanish [60]. In the case of a ring with a weak link this leads to a large reduction in the current amplitude (the Coulomb dip in the dependence $I(V_g)$). Since the second harmonic I_2 is proportional to t^2 and much less than the first harmonic $I_1 \sim t$. Note, that in the ballistic case [64] such a reduction of a current amplitude is predicted at high temperatures $T \gg T^*$ only.

4.1.1. Persistent current in the Coulomb dip: $\delta = \pm \frac{1}{4}$

At $\delta = \delta_0$ the first harmonic of a current $I_{1\mu}$ vanishes: $B_1(\delta_0) = 0$. Expanding the partition function up to the order of t^2 we get

$$I_{2\mu} = -I_{02}A_2(T) \sin(4\pi\Phi/\Phi_0). \quad (23)$$

$$I_{02} = \frac{\pi ev}{4L} t_F^2 \left(\frac{\pi^2 T^*}{\varepsilon_F} \right)^{2/g-2}. \quad (24)$$

$$A_2(T) = \frac{2T^*}{T} A^2(T, T^*, g) \int_0^{1/2} dx \left(\frac{\theta_1(x, q)}{\theta_4(x, q)} \right)^{2/g} \\ = \begin{cases} \frac{T^*}{T}, & T \ll T^*, \\ \frac{2^{2/g+1} \Gamma(1/2 + 1/g)}{\pi^{3/2} \Gamma(1 + 1/g)} \left(\frac{2T}{\pi T^*} \right)^{2/g-1} \exp(-T/T_F^*), & T \gg T^*. \end{cases} \quad (25)$$

Here $q = \exp(-T/T^*)$.

The unusual feature of this regime is an increase of a current $I_{2\mu} \sim 1/T$ at low temperatures $T \ll T^*$. Therefore, strictly speaking, at $T \rightarrow 0$ the perturbation theory fails. Comparing Eqs. (7) and (23) we see that at low temperatures the actual parameter of an expansion is $p = t_F(T^*/T)(\pi^2 T^*/\varepsilon_F)^{1/g-1}$ and the perturbation theory ($p \ll 1$) is valid if

$$T \gg t_F T^* \left(\frac{\pi^2 T^*}{\varepsilon_F} \right)^{1/g-1}. \quad (26)$$

The simplest way to understand both the dependence $1/T$ and restriction (26) is to consider the case of noninteraction electrons (22). If the chemical potential μ coincides with one of the energy levels ε_n of electrons in a ring the first harmonics of a current vanishes. The calculation of the second harmonic of a current I_2 (with a period of $\Phi_0/2$) within the perturbation theory requires the expansion of the Fermi function in Eq. (22) $f_0(\varepsilon_n(\Phi)) = f_0(\varepsilon_n(0)) + \partial f_0/\partial \varepsilon \Delta \varepsilon_n(\Phi)$, where $\Delta \varepsilon_n(\Phi) \approx \Delta_F t_F (-1)^n \cos(2\pi\Phi/\Phi_0)$; Δ_F is the level spacing near the Fermi energy at $\Phi = 0$. Such an expansion is valid if $\Delta \varepsilon_n(\Phi) \ll T$. This leads to restriction (26). The increase of I_2 with a decrease in the temperature is due to a multiplier $\partial f_0/\partial \varepsilon \sim -1/T$, which characterizes the intensity of a change of a level occupation with the magnetic flux.

4.2. Spinfull electrons

We assume that the chemical potential of an electron reservoir does not depend on spin. In such a case, the ground state of electrons in a ring coupled to a reservoir is nonmagnetic $N_{0\sigma} = 0$. Therefore, in contrast with the case of an isolated ring (with a fixed but arbitrary number of electrons) in the regime $\mu = \text{const}$ the states with a different parity in the number of electrons with an opposite spin in the ring do not exist. Thus, in this section we consider the case $N_{0\uparrow} = N_{0\downarrow}$ only.

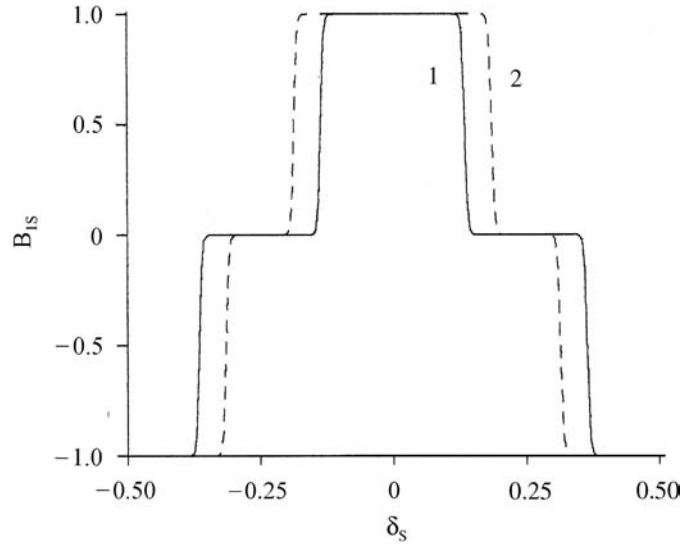


Fig. 4. Spinfull electrons. Dependence of B_{1s} (see Eq. (27)) on the dimensionless potential difference δ_s for $T_{0\sigma}/T_c = 0.1$ (1); 0.5 (2) at $T/T_c = 0.02$.

In the case of an even number of electrons $N_0 = N_{0\uparrow} + N_{0\downarrow}$ in the ground state the persistent current is of the order of t (10). Note that the change of N_{\uparrow} and N_{\downarrow} by 1 changes the sign of a current. Moreover, if N_{\uparrow} changes by 1(0) and N_{\downarrow} changes by 0(1) the current I_{1s} (of the order of t) vanishes. The straightforward calculations give $I_{1s\mu} = I_{1s}(\Phi)B_{1s}(V_g)$, where

$$B_{1s} = \frac{\theta_2(2\delta_s, q_{cs})\theta_2(0, q_{0\sigma})}{\theta_3(2\delta_s, q_{cs})\theta_3(0, q_{0\sigma}) + \theta_3(2\delta_s + \frac{1}{2}, q_{cs})\theta_3(\frac{1}{2}, q_{0\sigma})}. \quad (27)$$

Here $q_{cs} = \exp(-(\pi^2/4)T/T_{cs})$, $q_{0\sigma} = \exp(-(\pi^2/4)T/T_{0\sigma})$, $\delta_s = (eV_g - \mu)/(8T_c)$. The dependence $B_{1s}(\delta_s)$ is depicted in Fig. 4. The current (as well as in the ballistic case [65]) is periodic in V_g with a period of $8T_c$ that corresponds to the change in the number of particles by 4.

Note that in the case of noninteracting electrons ($g_{\rho} = g_{\sigma} = 2, E_c = 0$) the dependence $B_{1s}(V_g)$ is the same as $B_1(V_g)$ (21), i.e. the effect of a reservoir does not depend on the spin. However, in the case of correlated electrons the spin subsystem considerably affects the current. In particular, at a strong interelectron repulsion ($g_{\rho} \ll 2$) the current decreases exponentially at relatively low temperatures $T \geq T_{0\sigma}$ at which the discreteness of the energy spectrum of a spin subsystem is irrelevant.

$$B_{1s} = \begin{cases} \frac{\sinh((2T_{cs}/T)(1 - 4|\delta_s|))}{\cosh((2T_{cs}/T)(1 - 4|\delta_s|)) + \exp(T_{cs} - T_{0\sigma}/T)}, & T \ll T_{0\sigma}, T_{cs}; |\delta_s| < \frac{1}{2}, \\ 2 \exp\left(\frac{-\pi^2 T}{16T_{0\sigma}}\right) \frac{\sinh((2T_{cs}/T)(1 - 4|\delta_s|))}{\cosh((2T_{cs}/T)(1 - 4|\delta_s|)) + \frac{1}{2}\exp(T_{cs}/T)}, & T_{0\sigma} \ll T \ll T_{cs}; |\delta_s| < \frac{1}{2}, \\ 2 \exp\left(-\frac{\pi^2}{16} \left(\frac{T}{T_{0\sigma}} + \frac{T}{T_{cs}}\right)\right) \cos(2\pi\delta_s), & T_{0\sigma}, T_{cs} \ll T. \end{cases} \quad (28)$$

Note that in the case of spinless correlated electrons the current exponentially decreases at higher temperatures ($T \geq T^*$, see Eq. (9)).

At $\delta_{s0} = \pm \frac{1}{4} + n$ the current I_{1s} vanishes as well as in the case of spinless electrons. However, in the case of spinfull electrons if $T_{cs} \gg T_{0\sigma}, T$ the first harmonic of a current I_{1s} vanishes through the entire interval $\Delta\delta_s$ near $\delta_s \approx \delta_{s0}$ [60,61] (see Fig. 4). As it follows from Eq. (28) at low temperatures $T \ll T_{0\sigma}, T_{cs}$ the length of this interval is $\Delta\delta_s = (1 - T_{0\sigma}/T_{cs})/4$. The current through this interval is of the order of t^2 .

4.2.1. The second harmonic of a current

Calculating the current $I_{2s\mu}$ we have to take into account that the corresponding current I_{2s} (13) in the regime $N = \text{const}$ contains two contributions. The first term depends on the parity of the number of electrons and the second term is parity independent. The calculation gives $I_{2s\mu} = I_{02s}(-\gamma_1 B_{2s}(V_g) - \gamma_2) A_{2s}(T) \sin(4\pi\Phi/\Phi_0)$.

The quantities I_{02s} , A_{2s} , γ_1 , and γ_2 are defined in Eqs. (14)–(16) and

$$B_{2s} = \frac{\theta_3(2\delta_s, q_{cs})\theta_3(0, q_{0\sigma}) - \theta_3(2\delta_s + \frac{1}{2}, q_{cs})\theta_3(\frac{1}{2}, q_{0\sigma})}{\theta_3(2\delta_s, q_{cs})\theta_3(0, q_{0\sigma}) + \theta_3(2\delta_s + \frac{1}{2}, q_{cs})\theta_3(\frac{1}{2}, q_{0\sigma})}. \quad (29)$$

At $T \ll T_{cs}$ and $\delta_s \approx \delta_{s0}$ we have $B_{2s} = -1$ and the current $I_{2s\mu}$ in the ring coupled to a reservoir is the same as the current I_{2s} in the ring with a fixed (odd) number of spinfull electrons: $I_{2s\mu}(T \ll T_{cs}, \Phi, \delta \approx \pm \frac{1}{4}) = I_{2s}(T, \Phi)$. The distinction appears at higher temperatures $T \geq T_{cs}$ at which the particle exchange with a reservoir plays a significant role. In such a case, we have $I_{2s\mu}(T \geq T_{cs}, \Phi, \delta_s) = -2I_{2s}(T, \Phi)\exp(-\pi^2 T/4T_{cs})\cos(4\pi\delta_s)$.

5. Discussion and conclusion

In the present paper we consider the effect of an interelectron interaction (repulsion), an electron spin and the temperature on the persistent current in a one-dimensional ring with a weak link. An isolated ring with a fixed number of particles ($N = \text{const}$) as well as a ring coupled to a reservoir ($\mu = \text{const}$) are considered. In the last case the effect of a charging energy due to the small geometrical capacitance C on the charge transfer between a ring and a reservoir is taken into account. Interacting electrons in a ring are described within the framework of the Luttinger liquid model [52]. We consider a weak link as a point potential barrier with a small transparency $t_F \ll 1$.

In the ring with a fixed number of electrons the magnitude of a current decreases if the interelectron repulsion strengthens $g < 1$ (see Eq. (8)). The crossover temperature T^* , which separates the low-temperature region (where the current is temperature independent) from the high-temperature region (where the current decreases exponentially), increases $T^* = T_F^*/g$ with decreasing g in contrast with the ballistic case [26] for which the crossover temperature does not depend on g . At the same time, at high temperatures $T \gg T^*$ the power of exponent Eq. (9) is the same as for the ballistic case. In the case of spinfull electrons the spin subsystem affects considerably the persistent current. In particular, this leads to the existence of two crossover temperatures T_σ^* and T_ρ^* which depend on the parameters of spin (σ) and charge (ρ) subsystems (see Eq. (12)). The current amplitude strongly depends on the parity of the fixed number of electrons in the ring.

The properties of a persistent current in the ring coupled to an electron reservoir are also considered. At the strong interelectron repulsion and/or the large charging energy we have $T \ll T_c$ at the whole temperature region where the persistent current exists. In this regime (the Coulomb blockade regime of a persistent current [64]) the number of electrons (the number of charge excitations in the case of electrons with spin) in the ring is conserved with the exception of values V_g (see Fig. 1) for which the charging energy is degenerate in N and the Coulomb blockade is lifted [73]. In the case of spinless electrons the Coulomb blockade is lifted at certain values of V_{g0} (which correspond to $\delta_0 = \pm \frac{1}{4} + n$, where n is an integer). At the same time in the case of spinfull electrons the Coulomb blockade of a persistent current is not effective within the whole interval near V_{g0} [60,61,65] (see Fig. 4).

In the case of spinless electrons the current in the Coulomb blockade regime is the same as in the ring with a fixed number of electrons. However, the dependence of a current amplitude on V_g (see Fig. 3) consists of

a series of Coulomb dips [64] located at $\delta(V_g) = \delta_0$ and corresponding to a lift of the Coulomb blockade. At the point $\delta = \delta_0$ the period of the dependence $I(\Phi)$ halves (23) and the current amplitude is much less than the one in the regime $N = \text{const}$. The characteristic of such a current is an increase in the current amplitude with decreasing temperature: $I_2 \sim 1/T$ (25).

In the case of an even number of spinfull electrons in the ring with a weak link (as well as in the ballistic ring [65]) the current in the Coulomb blockade regime is not the same as in the isolated ring. This is due to the fact that the charging energy does not isolate the spin subsystem of a ring (which affects the current) from the reservoir. At the same time, at $T < T_c$ and $V_g \approx V_{g0}$ (see Fig. 4) the current is the same as in the isolated ring with an odd number of spinfull electrons. In particular, the current has a period of $\Phi_0/2$ and is paramagnetic (near $\Phi = 0$). The distinction appears at high temperatures ($T > T_c$) that is due to a particle exchange with a reservoir ($\Delta N > 1$). Note, that in the case of the electrons with spin the amplitude of the second harmonic of a current does not increase with decreasing temperature, which is due to quantum fluctuations in the spin subsystem.

In conclusion, in the present paper the effect of the statistical ensemble, interelectron interactions, spin and temperature on the persistent current in a one-dimensional ring with a weak link has been considered.

References

- [1] Y. Imry, in: G. Grinstein, G. Mazenko (Eds.), *Directions in Condensed Matter Physics*, World Scientific, Singapore, 1986.
- [2] Y. Aharonov, D. Bohm, *Phys. Rev.* 115 (1959) 484.
- [3] N. Byers, C.N. Yang, *Phys. Rev. Lett.* 7 (1961) 46.
- [4] F. Bloch, *Phys. Rev. B* 2 (1970) 109.
- [5] I.O. Kulik, *JETP Lett.* 11 (1970) 275.
- [6] M. Büttiker, Y. Imry, R. Landauer, *Phys. Lett.* 96 A (1983) 365.
- [7] L.P. Levy, G. Dolan, J. Dunsmuir, H. Bouchiat, *Phys. Rev. Lett.* 64 (1990) 2074.
- [8] B. Reulet, M. Ramin, H. Bouchiat, D. Mailly, *Phys. Rev. Lett.* 75 (1995) 124.
- [9] V. Chandrasekhar, R.A. Webb, M.J. Brady, M.B. Ketchen, W.J. Gallagher, A. Kleinsasser, *Phys. Rev. Lett.* 67 (1991) 3578.
- [10] D. Mailly, C. Chapelier, A. Benoit, *Phys. Rev. Lett.* 70 (1993) 2020.
- [11] R. Landauer, M. Büttiker, *Phys. Rev. Lett.* 54 (1985) 2049.
- [12] M. Büttiker, *Phys. Rev. B* 32 (1985) 1846.
- [13] H.F. Cheung, Y. Gefen, E.K. Riedel, W.H. Shih, *Phys. Rev. B* 37 (1988) 6050.
- [14] H. Bouchiat, G. Montambaux, *J. Phys. (Paris)* 50 (1989) 2695.
- [15] H.F. Cheung, E.K. Riedel, Y. Gefen, *Phys. Rev. Lett.* 62 (1989) 587.
- [16] O. Entin-Wohlman, Y. Gefen, *Europhys. Lett.* 8 (1989) 477.
- [17] E.K. Riedel, H.F. Cheung, Y. Gefen, *Phys. Scripta* T25 (1989) 357.
- [18] V. Ambegaokar, U. Eckern, *Phys. Rev. Lett.* 65 (1990) 381.
- [19] G. Montambaux, H. Bouchiat, D. Sigeti, R. Friesner, *Phys. Rev. B* 42 (1990) 7647.
- [20] E. Akkermans, *Europhys. Lett.* 15 (1991) 709.
- [21] B.L. Altshuler, Y. Gefen, Y. Imry, *Phys. Rev. Lett.* 66 (1991) 88.
- [22] F. von Oppen, E.K. Riedel, *Phys. Rev. Lett.* 66 (1991) 84.
- [23] A. Schmid, *Phys. Rev. Lett.* 66 (1991) 80.
- [24] S. Oh, A. Yu. Zyuzin, R.A. Serota, *Phys. Rev. B* 44 (1991) 8858.
- [25] T. Chakraborty, *Comments Condens. Matter Phys.* 16 (1992) 35.
- [26] D. Loss, *Phys. Rev. Lett.* 69 (1992) 343.
- [27] R.A. Smith, V. Ambegaokar, *Europhys. Lett.* 20 (1992) 161.
- [28] M. Abraham, R. Berkovits, *Phys. Rev. Lett.* 70 (1993) 1509.
- [29] N. Argaman, Y. Imry, *Physica Scripta* T 49 (1993) 333.
- [30] A. Kamenev, Y. Gefen, *Phys. Rev. Lett.* 70 (1993) 1976.
- [31] P. Kopietz, *Phys. Rev. Lett.* 70 (1993) 3123.
- [32] D. Loss, Th. Martin, *Phys. Rev. B* 47 (1993) 4619.
- [33] P.A. Mello, *Phys. Rev. B* 47 (1993) 16358.
- [34] A. Müller-Groeling, H.A. Weidenmüller, C.H. Lewenkopf, *Europhys. Lett.* 22 (1993) 193.
- [35] G. Bouzerar, D. Poilblanc, G. Montambaux, *Phys. Rev. B* 49 (1994) 8258.

- [36] A.O. Gogolin, N.V. Prokofiev, *Phys. Rev. B* 50 (1994) 4921.
- [37] H. Kato, D. Yoshioka, *Phys. Rev. B* 50 (1994) 4943.
- [38] A. Müller-Groeling, H.A. Weidenmüller, *Phys. Rev. B* 49 (1994) 4752.
- [39] P. Singha Deo, A.M. Jayannavar, *Phys. Rev. B* 49 (1994) 13 685.
- [40] G. Vignale, *Phys. Rev. Lett.* 72 (1994) 433.
- [41] R. Berkovits, Y. Avishai, *Europhys. Lett.* 29 (1995) 475.
- [42] T. Giamarchi, B.S. Shastry, *Phys. Rev. B* 51 (1995) 10 915.
- [43] T. Chakraborty, P. Pietilainen, *Phys. Rev. B* 52 (1995) 1932.
- [44] M. Kamal, Z.H. Musslimani, A. Auerbach, *J. Phys.* 15 (1995) 1487.
- [45] I.V. Krive, P. Sandström, R.I. Shekhter, S.M. Girvin, M. Jonson, *Phys. Rev. B* 52 (1995) 16 451.
- [46] T.P. Pareek, P.S. Deo, A.M. Jayannavar, *Phys. Rev. B* 52 (1995) 14 657.
- [47] A.A. Zvyagin, I.V. Krive, *Fiz. Nizk. Temp.* 21 (1995) 687.
- [48] A.S. Rozhavsky, *J. Phys.: Condens. Matter* 9 (1997) 1521.
- [49] J.M. Luttinger, *J. Math. Phys.* 15 (1963) 609.
- [50] J. Solyom, *Adv. Phys.* 28 (1979) 209.
- [51] V.J. Emery, in: J.T. Devreese, R.P. Evrard, V.E. van Doren (Eds.), *Highly Conducting One-Dimensional Solids*, Plenum Press, New York, 1979, p. 247.
- [52] F.D.M. Haldane, *J. Phys. C* 14 (1981) 2585.
- [53] H.J. Schulz, *Int. J. Mod. Phys. B* 5 (1991) 57.
- [54] J. Voit, *Rep. Prog. Phys.* 58 (1995) 977.
- [55] C.L. Kane, M.P.A. Fisher, *Phys. Rev. B* 46 (1992) 15 233.
- [56] D.V. Averin, K.K. Likharev, in: B. Altshuler, P.A. Lee, R.A. Webb (Eds.), *Mesoscopic Phenomena in Solids*, Elsevier, Amsterdam, 1991.
- [57] H. Grabert, M.H. Devoret (Eds.), *Single Charge Tunneling*, Plenum Press, New York, 1992.
- [58] M.A. Kastner, *Rev. Mod. Phys.* 64 (1992) 849.
- [59] M. Büttiker, *Phys. Scripta T* 54 (1994) 104.
- [60] M. Büttiker, C.A. Stafford, *Phys. Rev. Lett.* 76 (1996) 495.
- [61] M. Büttiker, C.A. Stafford, in: T. Martin, G. Montambaux, J. Tran Thanh Van (Eds.), *Correlated Fermions and Transport in Mesoscopic systems*, Editions Frontieres, Gif-sur-Yvette, 1996, p. 491.
- [62] P. Cedraschi, M. Büttiker, *Suppression of level hybridization due to Coulomb interactions*, UGVA-DPT 1997/12-994, Geneva, 1997.
- [63] M.V. Moskalets, *Physica B* 252 (1998) 244.
- [64] M.V. Moskalets, *Eur. Phys. J. B* 7 (1999) 645.
- [65] M.V. Moskalets, *Physica E* 4 (1999) 17.
- [66] H. Bateman, A. Erdelyi, *Higher Transcendental Function*, Vol. 3, McGraw-Hill, New York, 1955.
- [67] D. Loss, P. Goldbart, *Phys. Rev. B* 43 (1991) 13762.
- [68] N. Yu, M. Fowler, *Phys. Rev. B* 45 (1992) 11795.
- [69] S. Fujimoto, N. Kawakami, *Phys. Rev. B* 48 (1993) 17 406.
- [70] F.V. Kusmartsev, *J. Phys.: Condens. Matter* 3 (1991) 3199.
- [71] F.V. Kusmartsev, J.F. Weisz, R. Kishore, M. Takahashi, *Phys. Rev. B* 49 (1994) 16 234.
- [72] K. Niemela, P. Pietiläinen, P. Hyvönen, T. Chakraborty, *Europhys. Lett.* 36 (1996) 533.
- [73] H. van Houten, C.W.J. Beenakker, *Phys. Rev. Lett.* 63 (1989) 1893.