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Universal AC response of a 1D Luttinger liquid ring

M.V. Moskalets

Fl.48, 93-A prospect Il'icha, 310020 Khar'kov, Ukraine

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Abstract

The response of a spinless ballistic Luttinger liquid ring to an oscillating in time magnetic flux is considered in the discrete spectrum limit. The dependence of the magnitude of both AC response and the DC current on the frequency and magnitude of a magnetic flux at nonzero temperatures is calculated.

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Keywords: Persistent current; Luttinger liquid; AC response

1. Introduction

The existence of thermodynamic equilibrium (persistent) currents [1–4] in normal-metal rings threaded by a magnetic flux Φ is one of the most intriguing phenomena in mesoscopic physics [5] where the underlying quantum effects appear on a macroscopic level. In the simplest one-dimensional case the persistent current I_{pc} is a manifestation of the Aharonov–Bohm effect [6] in solids with a discrete spectrum. The current I_{pc} is the derivative of the free energy F over the magnetic flux $I_{pc} = -\partial F/\partial\Phi$ [7,8] and it is periodic in Φ with a period of $\Phi_0 = h/e$.

The persistent currents were predicted as in the ballistic case [9,10] and in the diffusive one [11]. The experimental discovery [1–3] of such currents greatly increases the interest in this subject [12–44]. Note that in the ballistic case the magnitude of a measured current [3] agrees with the theoretical estimation [45] based on a model of noninteracting electrons. However, the analogous theory for disordered rings [46–52] predicts a current much less than the experimentally observed one [1,2]. It seems that the interplay of disorder and electron–electron interactions plays a crucial role for the theory of persistent currents in disordered rings [53–67]. But this question is open yet.

On the other hand, the study of the response of isolated (without leads) rings to a time-dependent magnetic flux [11,68–76] showed that the reactive (nondissipative) response is an intrinsic feature of phase-coherent samples ($L_\varphi(T) \gg L$, where L_φ is a phase breaking length; L is the size of a sample). Such a response corresponds to a free acceleration of electrons [71] and it is diamagnetic in nature. The magnitude of a correspondent nonequilibrium (dynamic) current is larger than that of a persistent current [72,77]. As emphasized in Ref. [72], the existence of dynamic currents can be quite relevant for a description of experiments on disordered rings.

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However, the experimental study of an AC response of diffusive rings [4] gives results which differ from the theoretical one [76]. In view of this, to better understand the role of nonequilibrium currents it seems useful to study an AC response of rings in the ballistic case where, as mentioned above, the theory and experiment are in agreement with each other.

The aim of this paper is a theoretical description of an AC response of a 1D ballistic system of spinless electrons. The accounting of an electron spin and high dimensions will be given elsewhere but, as it seems, this does not change results qualitatively. We consider a response to a magnetic flux.

$$\Phi(t) = \bar{\Phi} + \Phi_\omega \sin(\omega t). \tag{1}$$

On the one hand, such a dependence can be more easily realized in experiment compared to a widely used model of a magnetic flux growing linearly with time [11,68,77–80]. On the other hand, in this case the state of a system is periodic in time that greatly simplifies the analysis in the long time limit. Within the framework of the master equation for the density matrix in a relaxation time approximation [71] we consider both thermodynamic equilibrium (persistent) as well as nonequilibrium (dynamic) currents vs. both the frequency ω and the amplitude Φ_ω of a magnetic flux. At $\Phi_\omega \to 0$ we also use the linear response theory based on the Kubo formula.

The paper is organized as follows. In Section 2 we formulate the task under consideration and write down the main equations. We derive the expression for a time-dependent density matrix and discuss the frequency window where such an expression is valid. In Section 3 we analyse the current in a ring. In the high-frequency limit the AC response I_ω does not depend on interelectron interactions and the regime of coupling to a reservoir (either at a fixed number of electrons N_0 in a ring or at a fixed chemical potential μ). The time-average current $I_{\rm DC}$ does not depend on the frequency ω of a magnetic flux and in the limit of $\Phi_\omega \to 0$ it coincides with a thermodynamic equilibrium (persistent) current $I_{\rm pc}(\bar{\Phi})$. But the fluctuations of $I_{\rm DC}$ are very sensitive to the frequency because of dynamic excitation of energy levels. In conclusion we summarize the results.

2. The model and basic equations

Let us consider a one-dimensional impurity-free ring of length L with a fixed number of electrons N_0 threaded by a magnetic flux $\Phi(t)$ (Eq. (1)). We assume that the magnetic flux slowly varies in time and the usually applied adiabatic approximation is valid [11,68,77]. In this case the energy levels E_n of a system depend parametrically on $\Phi(t)$

$$E_n(t) = E_n(\Phi(t)), \tag{2}$$

where $E_n(\Phi)$ is an eigenenergy at a constant magnetic flux. In the ballistic case it is $E_n(\Phi) \sim (n + \Phi/\Phi_0)^2$ [9,10,45,81]. The adiabatic approximation implies that the frequency ω is less than the level spacing $\Delta_{\rm F} \sim E_{n+1} - E_n$ near the Fermi level [1]

$$\omega \ll \Delta_{\rm F}/\hbar. \tag{3}$$

In a one-electron picture this condition means that the magnetic flux (as well as a corresponding cyclic boundary condition for an electron wave function) change is small during a time of an electron flight around the ring.

We assume an energy exchange between a ring and a reservoir that fixes an electron temperature T in a ring in equilibrium. Note that the inelastic interaction between electrons in a ring and an environment (a reservoir) plays a specific role in the ballistic case. As is well known in the static case ($\omega = 0$), the ground-state energy (and the thermodynamic potential) is periodic in Φ with a period of Φ_0 [7,8] that is due to inelastic interactions with a reservoir (while the eigenenergies E_n are quadratic in Φ). Out of equilibrium the interplay between an energy absorption from the electric field $\mathscr{E} = L^{-1}\,{\rm d}\Phi/{\rm d}t$ and an energy

[1] Note that results of the present paper also may be applied to semi-ideal (with a weak disorder) systems if we limit the frequency ω from below. This is due to the following [77]. Because of disorder-induced gaps the static dependence $E_n(\Phi)$ of a nonideal system becomes periodic in Φ. However, if the magnetic flux depends on time the electric field $\mathscr{E} = L^{-1}\,{\rm d}\Phi/{\rm d}t$ can lead to Landau–Zener tunneling through such gap [78,80] that restores a square dependence $E_n(\Phi(t)) \sim (n + \Phi(t)/\Phi_0)^2$ typical for the ballistic case. The probability of Landau–Zener tunneling is $P = \exp(-\gamma)$ [80,82] with $\gamma \sim \delta^2/({\rm e}\mathscr{E}L\Delta_{\rm F})$ (δ is a characteristic forbidden gap). Assuming $\gamma \to 0$ ($P \to 1$) we have to limit the frequency $\omega \gg \omega_{\rm min}$ with $\hbar\omega_{\rm min} \sim \delta^2/\Delta_{\rm F}$. Such a restriction allows to consider a semi-ideal system as an ideal (ballistic) one with no gap at the crossing of energy levels.

dissipation into a reservoir defines a nonequilibrium state of electrons in a ring which we will describe by the density matrix $\hat{\rho}(t)$ [71]. Within the relaxation time approximation, which is valid if the coupling to a reservoir is weak (see e.g., Ref. [71] and references therein), we write down the master equation for the density matrix in the adiabatic regime (Eq. (3)) as follows:

$$\frac{\partial\hat{\rho}}{\partial t} = -\tau_\varepsilon^{-1}(\hat{\rho} - \hat{\rho}_{\rm eq}), \tag{4}$$

where $\tau_\varepsilon(T)$ is the inelastic relaxation time (T is the reservoir temperature); $\hat{\rho}_{\rm eq}$ is the equilibrium (diagonal) density matrix with elements

$$\rho_{\rm eq}^{(n)}(t) = \frac{\exp(-E_n[\Phi(t)]/T)}{\sum_n \exp(-E_n[\Phi(t)]/T)}. \tag{5}$$

Here $E_n(\Phi)$ is a many-electron energy level as a function of Φ. If the coupling between a ring and a reservoir is small we can use the eigenenergies of an isolated ring. We derive the dependence $E_n(\Phi)$ considering the system of interacting electrons in a ring as a Luttinger liquid [83]. The Lagrangian $L_{\rm LL}$ of a spinless Luttinger liquid in a bosonic form is [84].

$$L_{\rm LL} = \frac{\hbar v}{2g}\left\{\frac{1}{v^2}\left(\frac{\partial\theta}{\partial t}\right)^2 - \left(\frac{\partial\theta}{\partial x}\right)^2\right\}, \tag{6}$$

where v, g are Haldane's parameters [83] which depend on interelectron interactions in a ring. In the continuous (rotary invariant) limit they satisfy $vg = v_{\rm F}$, where $v_{\rm F} = \pi\hbar\rho_0/m^*$ is the Fermi velocity (ρ_0 is the mean electron density in the ground state; m^* is the electron effective mass). For noninteracting electrons $g = 1$. $\theta(x,t)$ is a boson field.

The Aharonov–Bohm interaction with a magnetic flux Φ is described by the Lagrangian $L_{\rm AB}$ [84]

$$L_{\rm AB} = \frac{2\hbar}{L}\pi^{1/2}\frac{\partial\theta}{\partial t}\left(\frac{k_j}{2} + \frac{\Phi}{\Phi_0}\right), \tag{7}$$

where k_j is a topological number dependent on the parity of the number N_0 of electrons in a ring: $k_j = 0$ (1) if N_0 is odd (even).

The Euclidean (in an imaginary time τ) action

$$S_{\rm E} = -\int_0^L {\rm d}x \int_0^\beta {\rm d}\tau (L_{\rm LL} + L_{\rm AB}) \tag{8}$$

(where $\beta = \hbar/T$) determines the partition function Z through the path integral over the field θ

$$Z = \int D\theta \exp(-S_{\rm E}/\hbar). \tag{9}$$

In the ballistic case the dependence $Z(\Phi)$ is determined by the extremal trajectories $\theta_k^{(0)}(\tau) = \sqrt{\pi}k\tau/\beta$ (zero modes) obeying the twisted boundary conditions $\theta(x, \tau+\beta) = \theta(x,\tau) + \sqrt{\pi}k$ (k is an integer). Substituting $\theta_k^{(0)}$ into Eq. (9) and performing summation over k we get [85–87]

$$Z(\Phi) = A\sum_{n=-\infty}^{\infty}\exp(-E_n/T), \tag{10}$$

$$E_n(\Phi) = \Delta_{\rm F}\left(n + \frac{\Phi}{\Phi_0} + \frac{N_0 - 1}{2} \bmod 1\right)^2, \tag{11}$$

where $\Delta_{\rm F} = 2\pi\hbar v_{\rm F}/L$. The factor A includes a flux-independent plasmon contribution.

Thus, using Eqs. (4), (5) and (11) we can calculate the nonequilibrium density matrix $\hat{\rho}(t)$. Note that in the used model (Eq. (4)) the density matrix stays diagonal with a normalization $\sum_n \rho^{(n)} = 1$. The formal integration of Eq. (4) gives

$$\rho^{(n)}(t) = \rho^{(n)}(0)\exp(-t/\tau_\varepsilon) + \frac{1}{\tau_\varepsilon}\int_0^t {\rm d}t'\ \rho_{\rm eq}^{(n)}(t')\exp\left(\frac{t'-t}{\tau_\varepsilon}\right), \tag{12}$$

where $\hat{\rho}(0)$ is the density matrix at $t = 0$ (e.g., if the oscillating magnetic flux $\Phi_\omega \sin(\omega t)$ starts at $t = 0$ then $\hat{\rho}(0) = \hat{\rho}_{\rm eq}(\tilde{\Phi})$). Below we focus on the behaviour of a system in the long-time limit.

$$t \gg \tau_\varepsilon, \Delta t \tag{13}$$

which is independent of $\hat{\rho}(0)$. Here $\Delta t = 2\pi/\omega$ is the period of an applied flux. In this case the density matrix is periodic in time with a period of Δt that is due to a periodicity of $\hat{\rho}_{\rm eq}(\Phi(t))$ and it is as follows:

$$\hat{\rho}(t) = \frac{\tau_\varepsilon^{-1}}{\exp(\Delta t/\tau_\varepsilon) - 1}\int_0^{\Delta t}{\rm d}t'\ \hat{\rho}_{\rm eq}(t+t')\exp(t'/\tau_\varepsilon). \tag{14}$$

From this equation it is obvious that in the quasistatic regime ($\omega \ll \tau_\varepsilon^{-1}$) the density matrix is (quasi)equilibrium and depends on time as $\hat{\rho}(t) = \hat{\rho}_{\rm eq}(\Phi(t))$. In the opposite extreme nonequilibrium

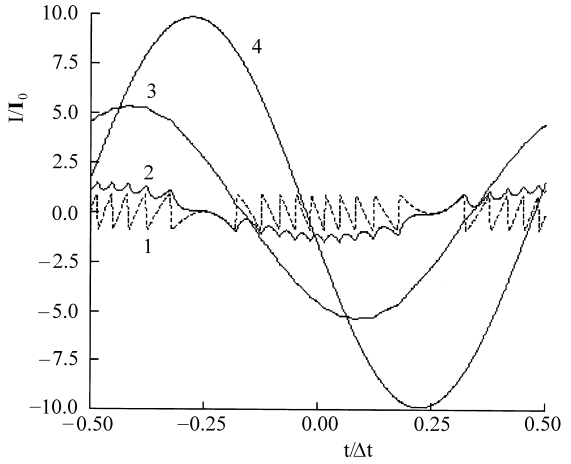

Fig. 1. Current in units of $I_0 = ev_F/L$ for $\tau_\varepsilon/\Delta t = 0$ (1); 0.02 (2); 0.1 (3) and 1 (4) is plotted vs. time. The parameters are: N_0 is odd; $T = 0.02\Delta_F$; $\bar{\Phi} = 0$; $\Phi_\omega = 5\Phi_0$.

(dynamic) regime ($\omega \gg \tau_\varepsilon^{-1}$) the density matrix is independent of time

$$\hat{\rho}_{dy} = \frac{1}{\Delta t}\int_0^{\Delta t} dt'\,\hat{\rho}_{eq}(t'). \tag{15}$$

Note that the dynamic regime is consistent with the adiabatic approximation (Eq. (3)) when broadening $\Gamma \sim \hbar/\tau_\varepsilon$ of energy levels is small compared to the level spacing Δ_F.

In the following we will discuss the dependence of the response of a ring on both the frequency and the amplitude of a magnetic flux.

3. Response of a ballistic ring

The current $I(t)$ in a ring is

$$I(t) = \sum_n \rho^{(n)} I_n, \tag{16}$$

where $I_n = -\partial E_n/\partial\Phi$ is a current carried by level E_n [7,8,11]. Substituting Eqs. (5), (11) and (14) into Eq. (16) we can numerically calculate the current. Some results of such calculations are presented below.

3.1. AC current

The dependence $I(t)$ is depicted in Fig. 1 at several values of a magnetic flux frequency $\omega = 2\pi/\Delta t$ (at fixed τ_ε). It is visible that at $\omega \ll \tau_\varepsilon^{-1}$ the current is periodic in Φ with a period of Φ_0 (i.e., the current frequency ω_I corresponds to the change of the magnetic flux Φ through the ring by Φ_0 : $\omega_I \sim 4\omega\Phi_\omega/\Phi_0$, if $\Phi_\omega > \Phi_0$). More precisely, the current has the following harmonics:

$$\begin{aligned} I(t) &\sim \sin\left(2\pi\frac{\bar{\Phi} + \Phi_\omega \sin(\omega t)}{\Phi_0}\right) \\ &= \sin\left(2\pi\frac{\bar{\Phi}}{\Phi_0}\right)\left\{J_0\left(2\pi\frac{\Phi_\omega}{\Phi_0}\right)\right. \\ &\quad \left. +2\sum_{n=1}^{\infty} J_{2n}\left(2\pi\frac{\Phi_\omega}{\Phi_0}\right)\cos(2n\omega t)\right\} \\ &\quad + 2\cos\left(2\pi\frac{\bar{\Phi}}{\Phi_0}\right)\sum_{n=1}^{\infty} J_{2n+1}\left(2\pi\frac{\Phi_\omega}{\Phi_0}\right) \\ &\quad \times\sin([2n-1]\omega t), \end{aligned} \tag{17}$$

where J_n is the Bessel function. Strictly speaking, this equation determines the current harmonics at $T > T^* = \Delta_F/\pi^2$. At lower temperatures ($T < T^*$) it is necessary to use $I = \sum_n a_n \sin(2\pi n\Phi(t)/\Phi_0)$ [45].

At the same time, at $\omega \gg \tau_\varepsilon^{-1}$ the current oscillates with the frequency ω and its amplitude further grows up. Such a dynamic diamagnetic response corresponds to a free acceleration of electrons in an electric field $\mathscr{E} = L^{-1}\,d\Phi/dt$ produced by a time-dependent magnetic flux $\Phi(t)$ [71] and it is a characteristic of normal-metal rings with a discrete spectrum either ballistic [77] or diffusive [71,72,76]. Thus, in the dynamic regime without both the relaxation ($\omega \gg \tau_\omega^{-1}$) and interlevel transfers ($\hbar\omega \ll \Delta_F$) the system of electrons displays a universal diamagnetic response. Now we show that in the ballistic case such a response does not depend on interelectron interactions and the amplitude of the current I_ω increases linearly in Φ_ω.

3.1.1. Linear response: $\Phi_\omega \to 0$

In this section we do not use the density matrix formalism and calculate I_ω using the Kubo formula for the linear response function α (a current–current correlation function). Note that in Ref. [71] using the density matrix formalism within a linear response theory the generalization of the Kubo–Greenwood formula for the conductance with respect to diamagnetic effects which are essential for a ring-lime geometry was done. In the present paper we will use the

density matrix formalism beyond the linear response approach. Of course, within the linear response and at appropriate frequencies (see below) the results of the present subsection (obtained within the Kubo linear response formalism) coincide with those obtained using the density matrix approach in consent with Ref. [71].

In accordance with the linear response theory [88] at $\Phi_\omega \to 0$ we write $I_\omega = \alpha(\omega)\Phi_\omega$, where $\alpha(\omega) = \mathrm{i}\hbar^{-1}\int_0^\infty \mathrm{d}t \exp(\mathrm{i}\omega t)\langle \hat{I}(t)\hat{I}(0) - \hat{I}(0)\hat{I}(t)\rangle$ (here $\hat{I}$ is a current operator and $\langle\cdots\rangle$ means averaging over the equilibrium state). At first we calculate the Matsubara response function [89] $\alpha_{\mathrm{M}}(\omega_n) = \hbar^{-1}\int_0^\beta \mathrm{d}\tau \exp(\mathrm{i}\omega_n\tau)\langle \hat{I}(\tau)\hat{I}(0)\rangle$, where $\omega_n = 2\pi n/\beta$ is the Matsubara frequency (n is an integer). Then $\alpha(\omega)$ is an analytical continuation of $\alpha_{\mathrm{M}}(\omega_n)$ with respect to $\alpha(\mathrm{i}\omega_n) = \alpha_{\mathrm{M}}(\omega_n)$. We calculate α_{M} within a model, Eqs. (6) and (7). In a bosonic representation the current is $I(\tau) = \mathrm{ie}\pi^{-1/2}\partial\theta/\partial\tau$. Using the expansion $\theta(\tau) = \theta^{(0)}(\tau) + \beta^{-1}\sum_n \theta_n \exp(-\mathrm{i}\omega_n\tau)$ we can write $\alpha(\omega_n) = -\mathrm{e}^2\omega_n^2/\pi\hbar\beta\langle|\theta_n|^2\rangle$ (here $\langle x\rangle = \int D\theta x \exp(-S_{\mathrm{E}}/\hbar)$). Because in a rotary invariant case under consideration the current does not depend on x we may calculate I at any point, say at $x = x_0$. This allows to integrate out fluctuations in θ for all x except $x = x_0$ [90]. As a result, we obtain an effective action as follows:

$$S_{\mathrm{eff}} = \hbar\sum_n |\theta_n|^2 \frac{|\omega_n|}{g\beta}\tanh\left(\frac{L|\omega_n|}{2v}\right) + S_0. \tag{18}$$

The term S_0 due to zero modes and the Aharonov–Bohm interaction (Eq. (7)) is irrelevant for α_{M}. Using S_{eff} we can easily perform averaging: $\alpha_{\mathrm{M}} = -(\mathrm{e}^2 g|\omega_n|/2\pi\hbar)\coth(|\omega_n|L/2v)$. Substituting $|\omega_n| = \mathrm{i}\omega$ we finally obtain

$$\alpha(\omega) = -\frac{\mathrm{e}^2 g\omega}{2\pi\hbar}\cot\left(\frac{\omega L}{2v}\right). \tag{19}$$

At $\omega \ll \Delta_{\mathrm{F}}/(\pi\hbar g)$ we have $\alpha(\omega) = -\mathrm{e}^2 v_{\mathrm{F}}/(\pi\hbar L)$. Thus for such frequencies the current amplitude does not depend on ω

$$I_\omega = -2I_0\Phi_\omega/\Phi_0, \tag{20}$$

where $I_0 = \mathrm{e}v_{\mathrm{F}}/L$. The same expression for the current (with a different expression for I_0) was obtained in Ref. [72] for a diffusive ring containing noninteracting electrons in the limit of $\omega \to 0$. As follows from the above expression, in the ballistic (rotary invariant) case the diamagnetic current does not depend on interelectron interactions (the parameter g) even as a thermodynamic equilibrium (persistent) current (which depends on the product $vg = v_{\mathrm{F}}$ only) [84,81].

As emphasized in Refs. [72,91], the dynamic current qualitatively differs from the thermodynamic equilibrium (persistent) current. In particular, this is due to relaxation processes. The persistent current I_{pc} is a characteristic of a quasistatic regime ($\omega \ll \tau_\varepsilon^{-1}$). In this regime the amplitude of an AC current is [71]

$$I_\omega = \frac{\partial I_{\mathrm{pc}}(\bar{\Phi})}{\partial\bar{\Phi}}\Phi_\omega, \quad \omega \ll \tau_\varepsilon^{-1}. \tag{21}$$

Note that this regime is not described by a response function (Eq. (19)) that is due to the following. The model Lagrangian equations (6) and (7) do not contain the terms describing an energy exchange with a reservoir. In fact, omitting such terms (which are proportional to τ_ε^{-1}) supposes $\omega_n \gg \tau_\varepsilon^{-1}$. Thus the dynamic response (Eq. (20)) is valid at

$$\tau_\varepsilon^{-1} \ll \omega \ll \Delta_{\mathrm{F}}/(\pi\hbar g). \tag{22}$$

Within the framework of a density matrix formalism we now calculate an equilibrium response as well as a dynamic one and verify that the inelastic rate τ_ε^{-1} separates the (quasi) equilibrium regime from the dynamic one. Moreover, we show that Eq. (20) holds at any values of Φ_ω.

3.1.2. Dynamic response at $\Phi_\omega \sim \Phi_0$

One of the important differences between the ballistic case and the diffusive case is the character of the dependence of an electron spectrum on the magnetic flux. In the former case the spectrum is quadratic in Φ (Eq. (11)) and the properties (of a ring) become periodic in Φ with a period of Φ_0 because of the relaxation to the ground state (the state with a minimum energy) that is due to an energy exchange with a reservoir (the characteristic rate is τ_ε^{-1}). While, in the diffusive regime the electron spectrum itself is periodic in Φ with a period of Φ_0 [11] because impurity scattering opens gaps at energy levels crossing and forms microbands $E_n(\Phi) = E_n(\Phi + \Phi_0)$ [7,8,11]. Therefore, if the magnetic flux amplitude is large ($\Phi_\omega \sim \Phi_0$) the response crucially depends on the case under consideration. In the diffusive case the current oscillates with a Josephson frequency $\omega_{\mathrm{J}} = \mathrm{e}V/\hbar$, where $V = |\mathrm{d}\Phi/\mathrm{d}t|$ [11]. At the same time, in the ballistic case such oscillations are rather characteristic for a quasistatic regime

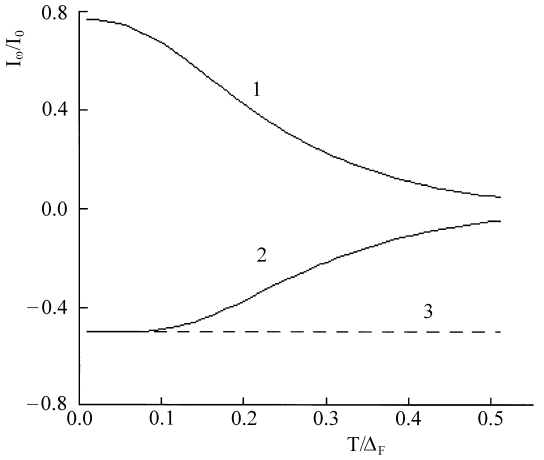

Fig. 2. The temperature dependence of the first harmonic I_ω of a current in units of I_0 for the quasistatic regime ($\omega \ll \tau_\varepsilon^{-1}$) at $\bar{\Phi} = \Phi_0/2$ (1); 0 (2) and for the dynamic regime ($\omega \gg \tau_\varepsilon^{-1}$) (3). The parameters are: N_0 is odd; $\Phi_\omega = \Phi_0/4$.

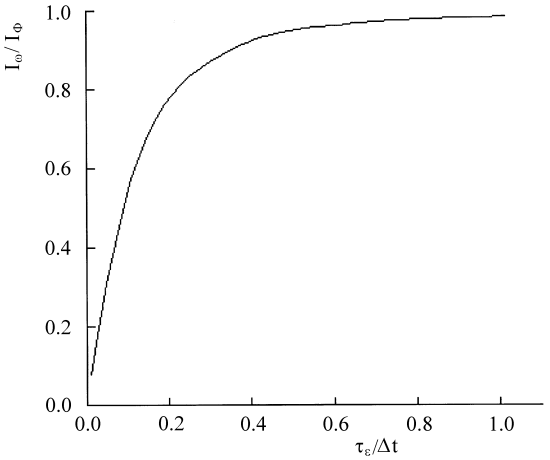

Fig. 3. The dependence of the first harmonic I_ω of a current in units of $I_\Phi = 2I_0\Phi_\omega/\Phi_0$ on the degree of nonequilibration (the parameter $\tau_\varepsilon/\Delta t$, $\Delta t = 2\pi/\omega$). The parameters are: N_0 is odd; $T = 0.5\Delta_F$.

($\omega \ll \tau_\varepsilon^{-1}$) and they disappear completely with increasing frequency (see Fig. 1).

In the dynamic regime ($\omega \gg \tau_\varepsilon^{-1}$) the density matrix (Eq. (15)) is independent of time. Using Eqs. (1), (5), (11), (15) and (16) we find that the current oscillates with a frequency ω and its amplitude I_ω is given by Eq. (20) which, therefore, is valid at any value of Φ_ω. Note that the quadratic dependence of energy levels on Φ is important.

3.1.3. Dependence of an AC response on frequency ω

Using Eqs. (1), (5), (11), (14) and (16) we can calculate the response of a ring at arbitrary ω. At first, we consider the limit $\Phi_\omega \to 0$. In the quasistatic regime ($\omega \ll \tau_\varepsilon^{-1}$) it is $\hat{\rho} = \hat{\rho}_{\rm eq}$ and from Eq. (16) it follows that (with the accuracy of $O(\Phi_\omega^2)$) the current amplitude I_ω is defined by Eq. (21), where $I_{\rm pc}(\bar{\Phi}) = \sum_n \rho_{\rm eq}^{(n)}(\bar{\Phi}) I_n(\bar{\Phi})$. In the dynamic regime Eq. (20) has to be used. At low temperatures ($T \ll T^*$) Eqs. (20) and (21) give same results at $\bar{\Phi}$ corresponding to a "diamagnetic" piece of dependence $I_{\rm pc}(\bar{\Phi})$ (e.g., $\bar{\Phi} \sim 0$ if N_0 is odd), Fig. 2 (curves 2 and 3). With increasing temperature ($T \gg T^*$) the quasistatic response vanishes (because of the vanishing of a persistent current [45]). However, the dynamic response survives, Fig. 2 (curve 3) (it is temperature independent while $L_\varphi(T) \gg L$). So, at high temperatures ($T \gg T^*$) the AC response of a ballistic ring must significantly grow with increasing frequency above the inelastic relaxation rate τ_ε^{-1}. The dependence $I_\omega(\omega)$ is depicted in Fig. 3.

Further we consider finite values of Φ_ω. The numerical calculations show that at $T \gg T^*$ the amplitude I_ω is linear in Φ_ω at arbitrary ω. However, at low temperatures ($T \ll T^*$) the dependence $I_\omega(\Phi_\omega)$ is significantly affected by the frequency (see Fig. 4). In the quasistatic regime such a dependence comes from the Bessel functions (Eq. (17)). At the same time, in the dynamic regime it is $I_\omega \sim -\Phi_\omega$ (Eq. (20)). Note that with increasing ω the higher harmonics $I_{n\omega}$ tend to zero (see Fig. 5 for the third harmonic).

3.1.4. Dynamic response in the regime $\mu = const$

Throughout the paper we consider $N_0 = \text{const}$. In this section we allow a particle exchange between a ring and a reservoir that fixes the chemical potential μ in the ring. We will show that this does not affect the dynamic response of a ballistic ring. In the limit $\Phi_\omega \to 0$ this statement follows from the fact that including the Lagrangian $L_{\rm ex}$ [85,86]

$$L_{\rm ex} = \frac{\mu}{\pi^{1/2}} \frac{\partial\theta}{\partial x} - \frac{E_{\rm c}}{L}(N_{\rm e} - CV_{\rm g}/{\rm e})^2 \qquad (23)$$

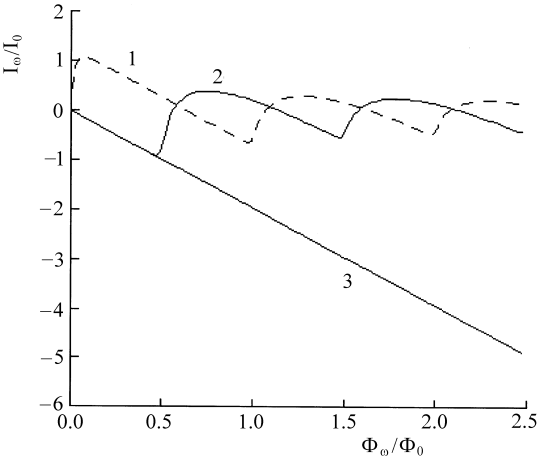

Fig. 4. The dependence of the first harmonic I_ω of a current in units of I_0 on Φ_ω in units of Φ_0 for the quasistatic regime at $\bar{\Phi} = \Phi_0/2$ (1); 0 (2) and for the dynamic regime (3). The parameters are: N_0 is odd; $T = 0.02\Delta_F$.

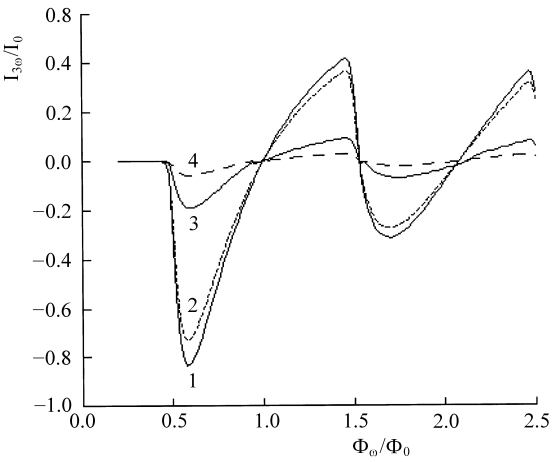

Fig. 5. The dependence of the third harmonic $I_{3\omega}$ of a current in units of I_0 on Φ_ω in units of Φ_0 for $\tau_\varepsilon/\Delta t = 0$ (1); 0.02 (2); 0.1 (3) and 0.2 (4). The parameters are: N_0 is odd; $T = 0.02\Delta_F$; $\bar{\Phi} = 0$.

(where $N_e = \pi^{-1/2}\int_0^L dx\nabla\theta + N_0$ is the number of electrons in a ring; $E_c = e^2/(2C)$; C and V_g are the capacitance and the potential difference between a ring and a reservoir, respectively) describing a particle exchange between a ring and a reservoir does not change a current–current correlation function (Eq. (19)). Thus, expression (20) is also valid in the regime $\mu = \text{const}$.

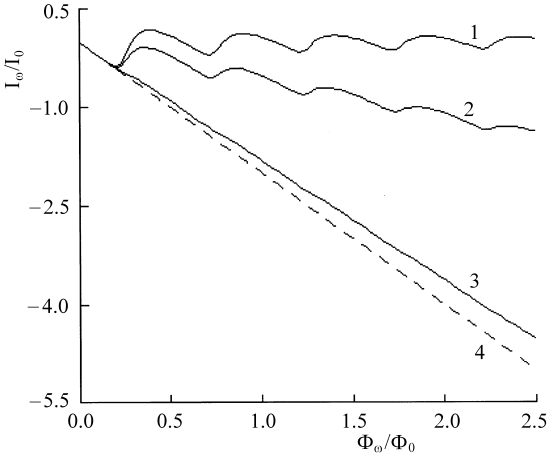

Fig. 6. The dependence of the first harmonic I_ω of a current in units of I_0 on Φ_ω in units of Φ_0 for the regime $\mu = \text{const}$ at $\tau_\varepsilon/\Delta t = 0$ (1); 0.1 (2); 0.5 (3) and ∞ (4). The parameters are: $T = 0.02\Delta_F$; $\bar{\Phi} = 0$; $\Delta_c = \Delta_F$; $\delta_c = 1/4$.

At finite values of Φ_ω we will use the master equation (4). To obtain $\hat{\rho}_{eq}$ we calculate the spectrum in the regime $\mu = \text{const}$. The calculation of the partition function Z_μ in the model equations (6), (7) and (23) is quite analogous to that for the regime $N_0 = \text{const}$ and it is done in Refs. [85–87]. Here we write down the result

$$Z_\mu = A\sum_{N_e}\exp(\mu N_e/T)\sum_{n=-\infty}^{\infty} e^{-E_{ln}/T}, \tag{24}$$

$$E_{ln}(\Phi) = l\mu + \Delta_c(l/2+\delta_c)^2 + \Delta_F\left(n + \frac{\Phi}{\Phi_0} + \frac{N_e-1}{2} \bmod 1\right)^2. \tag{25}$$

Here $\Delta_c = \Delta_F/g^2 + 4E_c$; $l = N_e - N_0$; $\delta_c = (4(eV_g - \mu)/\Delta_c) \bmod 1$. The equilibrium density matrix $\hat{\rho}_{eq}$ is the same as in Eq. (5) with respect to an obvious replacement $n \to \{ln\}$. With regard to other details the calculations are quite similar to those for $N_0 = \text{const}$. In particular, the reasons of Section 3.1.2 are still correct. The dependence $I_\omega(\Phi_\omega)$ for the regime $\mu = \text{const}$ at some values of ω is depicted in Fig. 6. So, the dynamic response does not depend on the regime of coupling to a reservoir (either $N_0 = \text{const}$ (Fig. 4, curve 3) or $\mu = \text{const}$ (Fig. 6, curve 4)).

However, the regime of coupling to a reservoir is important because of the following. As emphasized in

Ref. [77], in the ballistic case the relaxation towards the equilibrium associates with a large momentum transfer. Therefore, the particle exchange with a reservoir may be the main source of such a relaxation. Thus, in the regime $\mu = \text{const}$ the inelastic rate τ_ε^{-1} is expected to be larger compared with the regime $N_0 = \text{const}$. The estimation of an inelastic relaxation time in the regime $N_0 = \text{const}$ is $\tau_\varepsilon \sim 10^{-8}$–$10^{-7}$ s [77].

3.2. DC current

Because the current (Eq. (16)) is periodic in time with a period of $\Delta t = 2\pi/\omega$ the time-averaged current $I_{\rm DC}$ is

$$I_{\rm DC} = \frac{1}{\Delta t}\int_0^{\Delta t} {\rm d}t\, I(t). \tag{26}$$

This current is periodic in both $\bar{\Phi}$ and Φ_ω with a period of Φ_0 and its magnitude is of the order of $I_{\rm DC} \sim I_0 = ev_{\rm F}/L$. Note that $I_{\rm DC}$ is not a zero-frequency limit of an AC current I_ω (which is rather a derivative of the persistent current over the magnetic flux (Eq. (21))) but it is a persistent current itself (which is strongly modified by Φ_ω).

It can be easily shown that in the ballistic case (because of $E_n \sim (n + \Phi/\Phi_0)^2$) the DC current does not depend on the frequency ω and, in fact, it is determined by a dynamic (independent of time) density matrix (Eq. (15)). Substituting Eqs. (14) and (16) into Eq. (26) we have $I_{\rm DC} = \sum_n \rho_{\rm dy}^{(n)} I_n$. At $\Phi_\omega \to 0$ from Eq. (15) we obtain $\hat{\rho}_{\rm dy} = \hat{\rho}_{\rm eq} + {\rm O}(\Phi_\omega^2)$ and, as a consequence, $I_{\rm DC} = I_{\rm pc}(\bar{\Phi})$. However, at $\bar{\Phi}_\omega > 0$ the dynamic density matrix differs from an equilibrium one (see Fig. 7). As a result, the dependence $I_{\rm DC}(\bar{\Phi})$ is considerably changed (see Fig. 8a) and at some values of Φ_ω it may display a $\Phi_0/2$ periodicity in $\bar{\Phi}$ (see Fig. 8b).

3.2.1. Fluctuations of a DC current in the dynamic regime

The dynamic density matrix determines the DC current at arbitrary ω by a formal way. In fact, $\hat{\rho}_{\rm dy}$ describes the state of a system in an extreme nonequilibrium (dynamic) regime only. The distinctive feature of this regime ($\omega \gg \tau_\varepsilon^{-1}$) is the dynamic excitation of quantum levels in the ring (see Fig. 7).

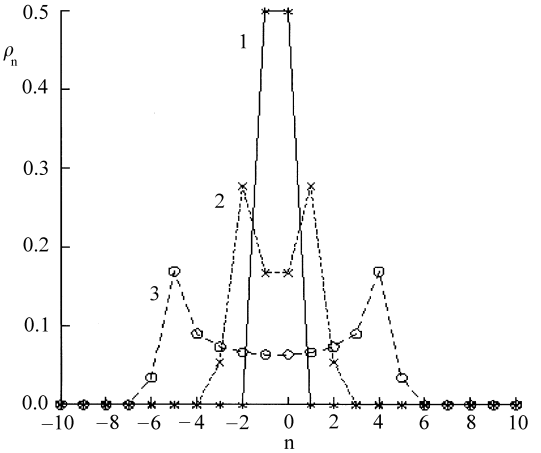

Fig. 7. Diagonal elements ρ_n of a density matrix for the quasistatic regime (1) and for the dynamic regime at $\Phi_\omega = 2\Phi_0$ (2) and $\Phi_\omega = 5\Phi_0$ (3). The parameters are: N_0 is odd; $T = 0.2\Delta_{\rm F}$; $\bar{\Phi} = \Phi_0/2$. Lines are guides to the eye.

The energy levels of a system change (Eq. (11)) in accordance with a cyclic evolution of a magnetic flux. In the quasistatic regime with increasing magnetic flux $\Phi \sim \Phi_\omega \sin(\omega t)$ the system relaxes to the state with a minimum (at given Φ) energy $E_{\rm min} = E_n$ with $n \sim -\Phi/\Phi_0$ and, as a consequence, the system passes in series through all the states with $|n| < n_{\rm max}$, where $n_{\rm max} \sim \Phi_\omega/\Phi_0$ (note, $n = 0$ corresponds to the ground state at $\Phi = 0$). However, in the dynamic regime during the period $\Delta t = 2\pi/\omega$ the system does not have time to relax to the ground state (the relaxation time is $\tau_\varepsilon \gg \Delta t$). Therefore, in the dynamic regime all the states with $|n| < n_{\rm max}$ are excited (see Fig. 7). Because of a spectrum discreteness the number of excited levels changes discontinuously when the amplitude Φ_ω changes by Φ_0. This effect is irrelevant for $I_{\rm DC}$ but it is important for higher degrees of a current. In particular, it affects considerably the fluctuations of $I_{\rm DC}$. In the quasistatic regime the current fluctuations are considered in Ref. [87]. The nature of such (thermodynamic equilibrium) fluctuations is as follows. When the system stays (during a time $\sim \tau_\varepsilon$) in one of the (many-particle) eigenstates (say, n) the current in a ring is $j = I_n$. Because of interaction with a reservoir, during a time $\gg \tau_\varepsilon$ the system will "visit" different eigenstates m with a probability $\rho_{\rm eq}^{(m)}$ that causes the current to fluctuate near $\langle j \rangle_t = I_{\rm pc}$, where $\langle \cdots \rangle_t$ means averaging over

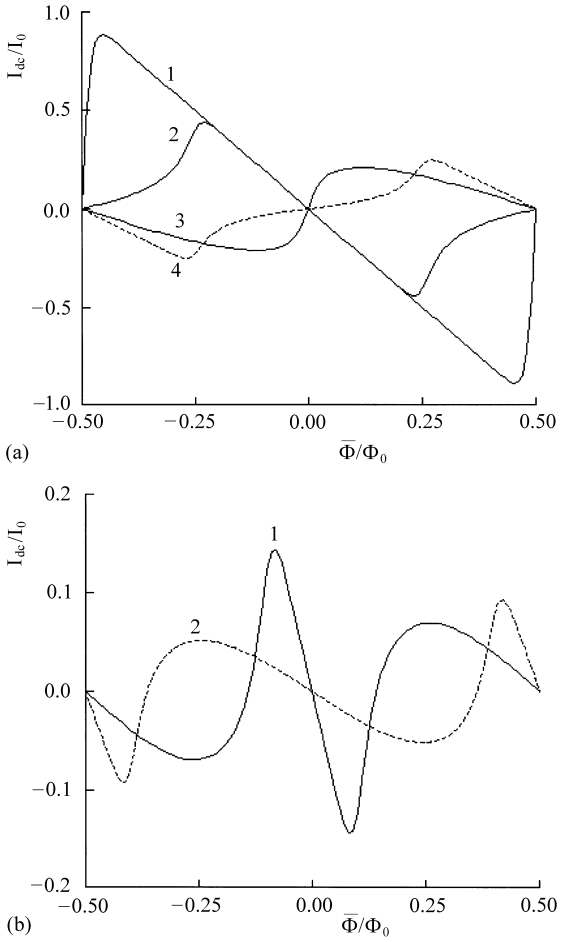

Fig. 8. DC current I_{DC} in units of I_0 is plotted vs. DC magnetic flux $\bar{\Phi}$ for (a) $\Phi_\omega = 0$ (1); $\Phi_0/4$ (2); $\Phi_0/2$ (3); $3\Phi_0/4$ (2) and (b) $\varphi_\omega = 0.4\Phi_0$ (1); $0.9\Phi_0$ (2). The parameters are: N_0 is odd; $T = 0.02\Delta_F$.

time. The analogous reasoning may be applied to the dynamic regime (because the dynamic density matrix (Eq. (15)) is time-independent, averaging over time is equivalent to averaging over the quantum states n with a probability $\rho_{dy}^{(n)}$). In this case during a time $\sim \tau_\varepsilon$ the current in a ring $j(t) = I_n = -2I_0(n + \bar{\Phi}/\Phi_0 + \Phi_\omega \sin(\omega t)/\Phi_0)$ oscillates with a frequency $\omega \gg \tau_\varepsilon^{-1}$ near $j_{DC} = -2I_0(n + \bar{\Phi}/\Phi_0)$. Note that the amplitude of an AC current does not fluctuate because $j_\omega = -2I_0\Phi_\omega/\Phi_0$ does not depend on the number n of a quantum state. In contrast, the DC current j_{DC} fluctuates on a time scale $\gg \tau_\varepsilon$. The mean square

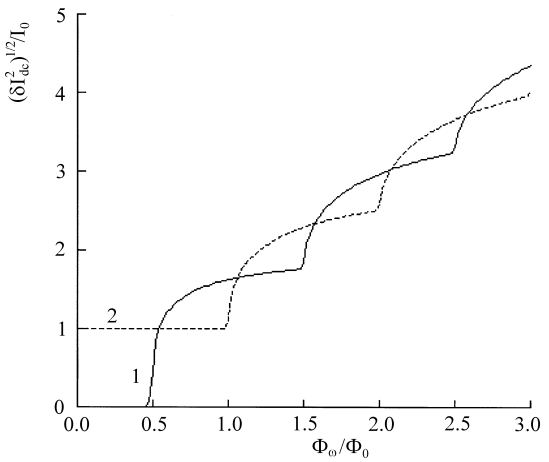

Fig. 9. Mean square fluctuations of a DC current for the dynamic regime. The parameters are: N_0 is odd; $T = 0.01\Delta_F$.

fluctuations are expressed as

$$\langle \delta I_{DC}^2 \rangle = \sum_n \{4I_0^2(n + \bar{\Phi}/\Phi_0)^2 \rho_{dy}^{(n)} - I_{DC}^2\}. \quad (27)$$

The quantity $\langle \delta I_{DC}^2 \rangle$ is periodic in $\bar{\Phi}$ with a period of Φ_0 and in the limit of $\Phi_\omega \to 0$ it coincides with the mean square fluctuations of a persistent current. The dependence $\langle \delta I_{DC}^2 \rangle$ on Φ_ω is depicted in Fig. 9. Note that with increasing temperature ($T > \Delta_F$) the peculiarities corresponding to dynamic excitation of energy levels are washed out.

4. Conclusion

We have considered the response of a ring-like ballistic spinless Luttinger liquid to a time-dependent magnetic flux $\Phi(t)$ (Eq. (1)) in the discrete spectrum limit at nonzero temperatures. We assume that the magnetic flux oscillates slowly in time (Eq. (3)). In this case the states of a system adiabatically follow the magnetic flux: $E_n(t) = E_n(\Phi(t))$. The dependence of the current magnitude on both the frequency $\omega \ll \Delta_F/\hbar$ and the amplitude Φ_ω of the magnetic flux has been studied. We use both the master equation for the density matrix and the Kubo formalism (at small Φ_ω) and consider both equilibrium as well as nonequilibrium (dynamic) currents in a ring.

The amplitude of the first harmonic I_ω of a current depends considerably on both the frequency ω (Fig. 3) and the temperature T. In the low-frequency limit ($\omega \ll \tau_\varepsilon^{-1}$) the response ($I_\omega$) (Eq. (21)) decreases with increasing temperature (Fig. 2) (because of vanishing of persistent currents at $T \gg T^*$). At the same time the high-frequency ($\omega \gg \tau_\varepsilon^{-1}$) response (the dynamic response) is temperature independent. In the dynamic regime the response is diamagnetic and the current amplitude I_ω is linear in Φ_ω (Eq. (20)). In addition, the dynamic response does not depend on both the strength of interelectron interactions and the regime of coupling to a reservoir (either $N_0 = \text{const}$ or $\mu = \text{const}$).

The DC current I_{DC} does not depend on the frequency ω but it is very sensitive to the amplitude Φ_ω (Fig. 8). In contract, the DC current fluctuations depend on ω and in the dynamic regime they greatly differ from those in the quasistatic regime. This is a consequence of dynamic excitation of system levels by a periodic magnetic flux (Fig. 7). In the discrete spectrum limit the number of excited levels changes by ~ 1 when Φ_ω changes by Φ_0. As a result, the mean square fluctuations of a DC current displays a quantized-like behaviour as a function of Φ_ω (Fig. 9).

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