0The 200th anniversary of Kharkov National University

Quantum Mechanics, Common Sense and Philosophical Problems

Viktor P. Demutsky, Revol'd V. Polovin, Oleg M. Pignasty

Kharkov National University
Kharkov, 2003

УДК 530.145
ББК 22.314
Д 30

Reviewers:
professor of Physics and Mathematics Yu.A.Berezhnoy
professor of Philosophy V.V.Shkoda

V.P. Demutsky, R.V. Polovin, O.M. Pignasty
D 30 **Quantum mechanics, common sense and philosophical problems.**-Kharkov, 2002.-192 p.

The book is devoted to a discussion of the interpretation of quantum mechanics. The main ideas of the book are: there is nothing mystical in Quantum Mechanics; classical mechanics also contains statements which contradict to some philosophical principles and to the common sense; most of separate quantum effects exist also in Classical Mechanics, "wildness" of Quantum Mechanics lies in joining together of incompatible effects; return to the Classical Mechanics by constructing of a more profound theory is impossible.

The problems discussed: the principle of observability and operationalism; the probabilistic approach to quantum mechanics as a way of reconciling the concepts of particle and wave; the reduction of the wave packet as a logical act; the paradoxes of quantum mechanics – EPR and Aharonov-Bohm; the causality principle and its various manifestations. The impossibility to introduce hidden variables into quantum mechanics is shown. Mathematics is reduced to the necessary minimum, many examples are provided to illustrate the discussion. Outstanding contributors to physics are extensively quoted.

Рецензенты:
доктор физ.-мат. наук, профессор Бережной Ю.А.;
доктор философских наук, профессор Шкода В.В.

Демуцкий В.П., Половин Р.В., Пигнастый О.М.
Д 30 **Квантовая механика, здравый смысл и философские проблемы.**-Харьков, 2002.-192 с.

Книга посвящена обсуждению интерпретации квантовой механики. Основные положения книги: в квантовой механике нет ничего мистического; классическая механика также содержит утверждения, которые противоречат некоторым философским принципам и здравому смыслу; большинство отдельных квантовых эффектов присущи также и классической механике, «дикость» же квантовой механики заключается в объединении несовместимых эффектов; возврат к классической механике с помощью более глубокой теории невозможен.

Обсуждаются вопросы: принцип наблюдаемости и операционализм; вероятностный подход в квантовой механике как способ объединения противоречивых понятий волны и частицы; о редукции волнового пакета как логическом акте; наиболее известные парадоксы квантовой механики такие как ЭПР и Аронова-Бома; причинность в квантовой механике и ее различные проявления. Утверждается невозможность введения «скрытых» параметров в квантовую механику. Математика сведена до необходимого минимума. Приведено много примеров, цитируются высказывания выдающихся физиков.

ISBN 966-623-176-X

ББК 22.314

Предисловие

Фундаментальные положения квантовой теории коренным образом изменили представление о мире, унаследованное от 19 века. Они вызывают переворот в мышлении и потому касаются широкого круга людей. Однако в литературе, посвященной осмыслению квантовой механики, имеется существенный пробел. А именно, в ней обстоятельно излагается парадоксальность квантовой механики, неожиданность ее выводов, и противоречия между квантовой механикой и нашей интуицией. В то же время очень мало книг посвящено развитию интуиции читателей, чтобы новые факты стали для него понятными и очевидными.

Мы поставили своей целью в этой книге проследить связь квантовой механики с интуицией, здравым смыслом и философией. Мы широко пользуемся цитатами. Цитаты были дискредитированы теми авторами, которые прикрывали ими отсутствие собственных мыслей. Однако, статьи по концептуальным вопросам естествознания без цитат столь же неуклюжи, как статьи по математике без формул. Классики обычно формулируют свои мысли белыми стихами. Поэтому при пересказе цитат теряется «чувство непосредственного контакта с прекрасным» [2]. Кроме того, при пересказе утрачивается документальность, нюансы и колорит. Цитаты у нас служат не только для иллюстрации, но в ряде случаев составляет часть текста.

В настоящее время серьезные работы по концептуальным проблемам физики должны содержать некоторую дозу математики. Однако мы свели число математических формул до минимума. Несмотря на это, отдельные места в книге могут показаться трудными для читателей, имеющих гуманитарное образование. Такие места могут быть пропущены при первом чтении. Мы не жалеем места для примеров, оживляющих изложение. «Примеры лучше

объясняют …, чем общие, абстрактные рассуждения» (Ernst Mach, [3], p.247).

В книге приводятся много малоизвестных интересных исторических фактов. Тем не менее, мы ограничиваемся только самыми существенными интерпретациями квантовой механики.

Книга предназначена для лиц с высшим образованием как естественно-научного, так и гуманитарного профиля, интересующихся концептуальными вопросами современного естествознания. Было приложено много сил для изложения важнейших вопросов так, чтобы они могли быть поняты и читателями-неспециалистами.

В связи с ориентацией книги на широкий круг читателей в нее включены фрагменты с элементарным изложением ряда основных вопросов. Некоторые из этих фрагментов могут показаться излишними для специалистов, но они необходимы для широкого круга читателей. Ссылки в тексте указывают литературу, где обсуждаемые вопросы освещены более подробно. Аналогичный характер имеют работы [4-5].

Настоящая книга отличается тем, что в ней освещена роль Маха как идеолога научных революций, дан анализ процесса измерения, доказана невозможность введения в квантовую механику скрытых параметров. (Аннотированную библиографию по интерпретации квантовой механики можно найти в статье [7]).

В последнее время появилось много книг, посвященных столетнему юбилею квантовой механики. В них широко обсуждаются концептуальные вопросы квантовой механики, последние достижения квантовой теории. Одной из таких публикаций является работа Ю.Степановского [162], в которой кратко обсуждаются недавние принципиальные эксперименты, такие, как остановка света, квантовая телепортация и другие.

В недавно опубликованном обзоре М.Б.Менского [163] обсуждаются новые эксперименты, в частности, по проверке неравенств Белла, а также проблемы квантовой информатики и теории декогеренции, включение мышления в процесс измерения, механизм селекции в измерении в квантовой механике, множественность параллельных миров. Комментарии и обсуждения этих проблем опубликованы в письмах Липкина А.И., Панова А.Д., Лесовского Г.Б., Цехмистро И.З., Нахимсона Р.С. [164].

Обстоятельное и интересное обсуждение актуальных философских проблем современной квантовой механики можно найти в последней книге И.З.Цехмистро [167].

Авторы благодарят профессоров Бережного Ю.А., Шкоду В.В. и Степановского Ю.Т. за ценные дискуссии, А.В. Демуцкую за английский перевод издания и В.С. Пигнастую за компьютерный набор книги.

"The cruel necessity rather than speculation or the desire of the novelty forces us to change old classical views."

A. Einstein, L. Infeld, [1]

Answering the question : "How do you create sculp-tures?" Auguste Rodin said: "I take a stone and cut off all superfluous".

The fundamental ideas of quantum theory have altered the picture of the word that we have inherited from the nineteenth century. They have caused a conceptual revolution and thus touch the mental lives of many people. However, the literature devoted to the interpretation of quantum mechanics suffers from a significant gap. Whilst there are extensive discussions of the paradoxes of quantum mechanics, the unexpected nature of its conclusions, and the contradictions between quantum mechanics and our intuition there are relatively few books which try to develop the reader's intuition so that the new facts become more readily understood and accepted.

Our aim in this book is to examine the connection between quantum mechanics, on the one hand, and common sense and philosophy on the other. We make extensive use of quotations while being fully aware that some authors employ quotations as a means of dissimulation or of covering up their poverty of thought. However, a book on conceptual problems in natural science that is devoid of quotations is just as clumsy as mathematical paper without formulas. We know that eminent scientists often express their thoughts epigrammatically, which is why the re-telling of such quotations often causes a loss of the "feeling of direct contact with beauty" (Ref 2). Such retelling tends to lose nuances, color and documentary character. In our own presentation, citations serve not only as illustrations, but are treated as an integral part of the text.

Any serious publication on philosophical problems in physics must by now contain a certain amount of mathematics. However we have tried to reduce the number of mathematical formulas to a

minimum. Even so, some of the sections presented below may be difficult for readers whose education is mostly in the humanitics. These sections may be omitted by them.

We have taken every opportunity to enliven our discussion with examples. "Examples provide better explanations... than abstract general discussions" ([3], p. 247). There are many little known interesting historical facts. Indeed, we confine our attention to the more substantial interpretations of quantum mechanics.

Our book is intended for broad sections of readers with higher education in the natural sciences and the humanities, who are interested in conceptual problems in modern science. We have given considerable thought to the digest the more important questions for nonspecialist readers.

Since our book is intended for a wide range of readers, it contains fragments that are rather elementary. Some of them may seem unnecessary to specialists, but they are useful to others. More detailed discussions of particular topics may be found in the references scattered throughout the text. Similar accounts are presented in [4], [5] and [6].

Our book differs from other presentations mainly by its examination of the contributions of Ernst Mach as the ideologue of scientific revolutions, by its analysis of the process of measurement, and by the proof that it gives of the impossibilitty of hidden parameters in quantum mechanics. (An annotated bibliography of interpretations of quantum mechanics may be found in [7]).

Recently a number of works have been published devoted to the centenary of the quantum mechanics. Conceptual problems of quantum mechanics are widely discussed in these publications. One of these publications is the work of Stepanovsky Yu.P. [162] in which recent principal experiments are considered briefly, such as the stopping of light, the creation of quatum mechanical superposition of macroscopic states and quantum teleportation.

Quite reliable Mensky's review has been published in [163]. In this review new experiments are discussed, in particular, the

experiments testing Bell's inequalities. Also such problems are discussed: quantum informatics and the theory of decocherence, consciousness inclusion in the process of measurement, the mechanism of celection in measurement in quantum mechanics, multiplicity of parallel worlds.

Comments and discussions as to considered problems are published in [164] in the letters of Lipkin A.I., Panov A.D., Lesovski G.B., Tsekhmistro I.Z., Nakhimson R.S.

Inetresting and detailed account of philosophical problems of quantum mechanics can be found in the last Tsekhmistro's book [167].

It is a pleasure to thank prof. Yu.A. Berezhnoy, V.V. Shkoda and Yu.P. Stepanovskii for valuable discussions, A.V. Demutskaya for English tranlation and V. Pignastaya for computer version.

Contents

"Within there was a small corridor, which ended in a very massive iron gate. This also was opened, and led down a flight of winding stone steps, which terminated at another formidable gate...

"We are at present in the cellar of the city branch of one of the principal London banks...

"You are not very vulnerable from above", Holmes remarked...

"Nor from below", said Mr. Merryweather, striking his stick upon the flags which lined the floor.

"Why, dear me! it sounds quite hollow", he remarked, looking up in surprise...

...and then, without any warning or sound, a gash seemed to open... With a rending, tearing sound, one of the broad, white stones turned over upon its side, and left a square, gaping hole, through which streamed the light..."

A. Conan Doyle. The red-headed league.

"To ancient man the universe was chaos, governed by caprice. In order to explain its phenomena, he found it necessary to people heavens with a host of minor gods and goddesses, and the mountains and streams with a varied throng of giants, nymphs and spirits...

Gradually science revealed the order of the cosmos. It taught that the universe was orderly, functioning in response to well-established laws"([8]).

Newton's laws of motion and of universal gravitation led to an explanation of the motion of celestial bodies, and to predictions of solar and lunar eclipses that had been previously "explained' by divine intervention. Newton's laws explained all the known celestial phenomena.

The application of the laws of physics to terrestrial phenomena has resulted in a veritable avalanche of inventions that would have been regarded as miraculous in the past.

The steam engine and gramophone, electricity and aeroplane, cinematography and radio appeared. It seemed that the victory of reason was total and irreversible.

However there were accumulated facts which could not be explained in the frame of the classical theory.

First of all atoms in a solid body are held in their positions by electrostatic forces. However, according to the Earnshaw's theorem ([9], p.116) any stationary distribution of electric charges is unstable (Nuclear forces cann't act because their range is 10^{-13} cm. whereas the distance between atoms in solid bodies has order of magnitude 10^{-8} cm.). Therefore classical physics contradicts to the existence of solid bodies.

Then the Rutherford's model of atom is also unstable with very short time of life.

Besides this, experiments show that energy of an excited atom cannot be arbitrary. It can have only strictly prescribed isolated values, between which the forbidden intervals are situated. This discreteness of energy is incompatible with the Rutherford's model of atom.

Further, according to the classical theory of the Rayleight-Jeans the spectral density of the radiation of an absolutely black body was defined by the formula ([10])

$$\rho(\omega)=\frac{\omega^2}{\pi^2 c^3}kT \qquad (0.1)$$

Here ω is the radiation frecuency, c – velocity of light, k – Boltzmann constant and T – absolute temperature. This formula was in accord with experimental data only for small values of the frequency ω. As for high frequencies the experimental results sharply differed from the calculations according to the formula (0.1). Moreover, the total intensity of the radiation I,

$$I = \int_0^\infty \rho(\omega) d\omega \qquad (0.2)$$

was infinite. It was an evident absurd.

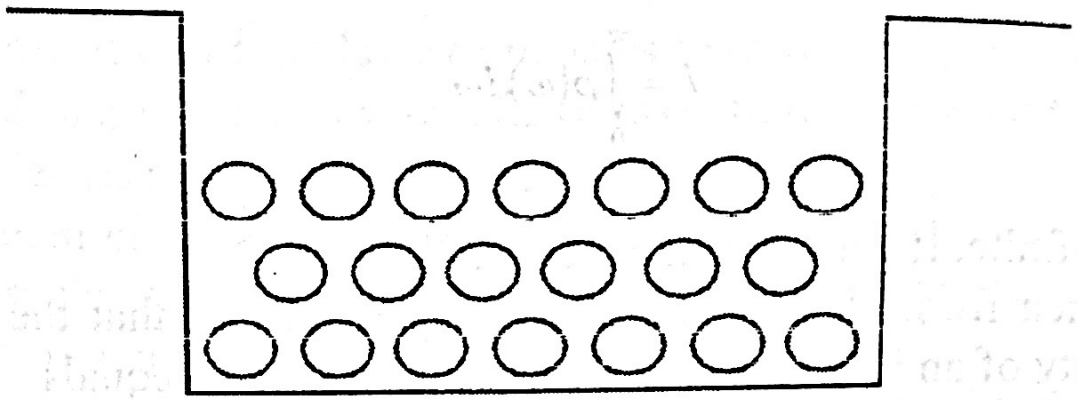
Then from classical kinetic theory it follows that the thermal capacity of an ideal gas under constant volume c_v equals

$$c_v = \frac{l}{2} kT \qquad (0.3)$$

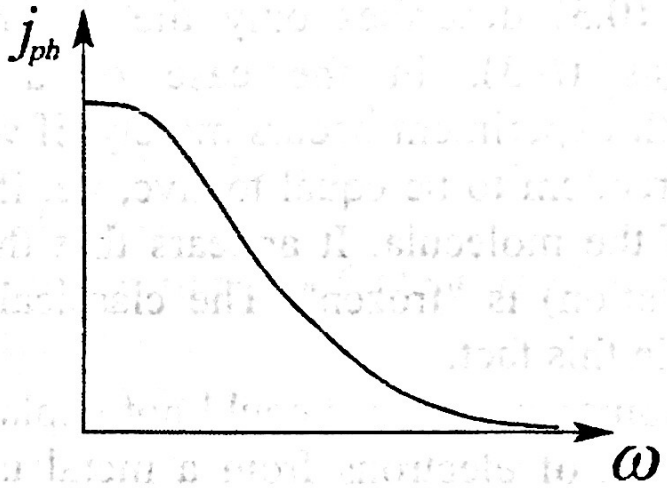
Here l- the number of degrees of freedom. This formula fits to the experiment if the temperature T is high. As to low temperature the expression (0.3) describes only the thermal capacity of a monoatomic gas (l=3). In the case of a two-atomic gas, coincidence with experiment occurs merely, if we put the number of degrees of freedom to be equal to five, i.e. if we disregard the deformation of the molecula. It appears that the sixth degree of freedom (oscillation) is "frozen". The classical physics was not enable to explain this fact.

Now, the Newtonian mechanics could not explain the photoeffect (the knocking out of electrons from a metal under influence of light). Indeed, the metal is a potential well for electrons. Electrons in metal may be treated graphically as balls in a wash- tub under action of the force of gravity (Fig.1). Action of light, i.e. of electromagnetic field, on electrons is equivalent to swinging of the tub. During this swinging part of electrons fall out. This is the photoeffect.

We see that the photoeffect may be explained qualitatively in the frame of Newtonian mechanics. But the quantitative relations which are obtained in experiments do not correspond to the classical mechanics. Indeed, according to Newton, the dependence of photocurrent on frequency of light must have the following character: the maximal current would be in the case of infinitesimal frequency. The current must



Electrons in a metal
Fig 1.



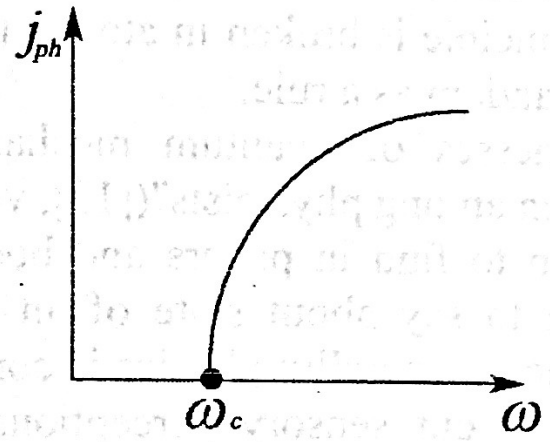
The dependence of photocurrent j_{ph} from frequency ω according to the classical theory
Fig 2.

diminish if the frequency increases (Fig.2). In reality the dependence of photocurrent j_{ph} on frequency ω has a character, which is depicted on the Fig.3. Namely, the photocurrent is possible only in the case, when the frequency of light ω exceeds some critical value ω_c (see Fig.3).

Finally, in some cases particles penetrate through a potential barrier. During this their total energy is less than the potential energy. It means that the kinetic energy is negative which is impossible in the classical physics.

All these phenomena were brilliantly explained in the frame of quantum theory. Now quantum mechanics is a basis of all natural sciences. Principles of quantum mechanics lie in the foundation of the theories of atoms, elementary particles, transistors, superconductivity and so on.

However, quantum mechanics contains merely mathematical results refuting the common sense.



Experimental dependence of photocurrent from frequency

Fig 3.

"What are ...these features of quantum mechanics that prevent us from treating it in a classical spirit and see the wave function as a field distributed in space and time, in many ways similar to the classical field... For a complex system consisting of a large number of particles, the wave function depends on all the degrees of freedom of the system and not just on three coordinates. It is function in multidimensional configuration space and not in the real physical space... The wave function does not always exist and is not always described by the Schrödinger equation, under certain well- known conditions, it is simply deleted and replaced by another (this is the so-called reduction of a wave packet). It is clear that this type of instantaneous change is not consistent with the concept of a field." (Ref 11, p.461)

Besides this, quantum mechanics questioned of objectivity of our knowledge:

1) If the wave function is not "a field distributed in a space", if it is defined not in the real physical space but in multidimensional abstract space. if its evolution is not always described by the Schrödinger equation, but it simply deleted, then does quantum mechanics describe merely our sensory perceptions or our knowledge, rather than nature?

2) Causality principle is broken in atomic processes. Result of a measurement is random as a rule.

These strangenesses of quantum mechanics "produced an increasing confusion among physicists"([12], vol.3, p. 199).

It was common to find in papers and books statements that there is no reason to say about state of an electron if nobody observes it. And more generally: physics is concerned exclusively with the ordering of our sensory perceptions and not with the discovery of objective laws independent of the observer. Jordan wrote:

"When it is characterized as the framework for mathematical formulas, the atom is an auxiliary device for ordering experimental facts, like the geographical grid of the Earth."

Some scientists used to say about crises of physics, about crises of science, about the end of "the dictates of reason". There a mysticism arose that was contrary to the spirit of science" ([13], p.204).

Our aim here, on the other hand, is to understand quantum mechanics and to reconcile it with common sense. Fundamentals of quantum mechanics are no less admissible than that of classical physics.

The "strangeness" of quantum mechanics does not lie exclusively in the existence of specific quantum effects that cannot be explained by classical physics. Many separate effects, e. g., discreteness, randomness and uncertainty relations, that are usually referred to as quantum, are actually found in classical physics too (see, for example, about this in [13]). However, in

quantum mechanics, these classical effects combine in totally "senseless" way. To achieve a better understanding of such quantum effects, we begin by discussing them in terms of the ordinary classical language, emphasizing those aspects of the phenomena that are usually left on the sidelines, but assume particular prominence in quantum theory.

Chapter 1

Observability principle

"There's no motion" said the bearded sage
The other interrupted clever talk,
Instead of answer he began to walk,
It was the best available refutation,
However, I remain in hesitation:
The Sun is walking, and the Earth is fixed.
Yet Galileo us of the reverse convinced.

After Alexander Pushkin

1.1 Believe only your own eyes

Studying history of physics, Mach drew attention to the fact that this history is an alternation of evolution (i. e., gradual gathering of knowledge) and revolution (i.e., a radical change in our picture of the physical world) [14]. At the time the last revolution had been engineered by Newton.

In an earlier epoch, Aristotle considered that uniform motion of a body required a constant force for its continuance. Newton, on the other hand, considered that a body continued in uniform motion if there were no external forces acting upon it.

Mach was one of the very few who foresaw the radical changes in physics that were to come. He therefore understood that Newtonian mechanics could not be regarded as absolute truth. He wrote [15]: "If we now assume that the facts established in mechanics are so much better understood than other facts that they can be used as the foundation for all other physical facts, then this must be an illusion. The explanation is that the history of mechanics is so much older and richer than the history of physics that we tend to take the facts of mechanics as primary" [16]. Mach's remarks stimulated the revision of classical mechanics.

Einstein had a high regard for Mach's critique of mechanics: "I see Mach's true greatness in his incorruptible skepticism and independence." ([17], vol. 4, p.266) Einstein continues: "In his historiocritical publications, in which he followed with great care the evolution of science and explored the internal laboratory of individual researchers who have laid new pathways in their own branches of science, Mach had an enormous influence on the scientists of our generation"([18], p. 113).

Indeed, Mach developed a program for a new revolution in physics. In particular, if we were to abandon all our acquired knowledge, we would regress to the level of the ape. We must therefore retain something of the prevailing theory, but the question is what? The answer to this question is supplied by Mach's principle of observability: the only true phenomena are those that can be observed directly. ([4], p.70)

Feynman discusses this in greater detail: "We just have to take what we see, and than formulate all the rest of our ideas in terms of our experience". ([12], vol.1, p.47)

The observability principle is a revival of the middle age Occuma's razor" - all notions which cannot be checked by an experiment, must be cut off from science.

We note that when Newton developed classical mechanics, he also relied on the principle of observability: "...hitherto I have not been able to discover the cause of those properties of gravity from phenomena, and I frame no hypotheses; for whatever is not deduced from the phenomena is to be called an hypothesis; and hypotheses, whatever metaphysical or physical, whether of occult qualities or mechanical, have no place in experimental philosophy. In this philosophy particular propositions are inferred from the phenomena, and afterwards rendered general by induction". [19]

Mach's observability principle may turn out to be a return to Berkley's ultrasubjectivism: "to exist is to be perceived"[20]. In actual fact, it is only the unrestricted application of this principle that leads to subjectivism. On the other hand, a reasonable use of the observability principle is a powerful tool for constructing new

theories. This principle enables us to select from the ruins of the old theory the components that will remain in the new.

It is precisely with the help of the observability principle that Einstein was able to create the theory of relativity, and Bohr and Heisenberg were led to the conclusion that the electron in an atom does not have a definite position or a definite momentum.

At the same time, the energy of an electron in the atom is observable, i.e. is defined precisely: "... as far as the periodic orbit of the electron is concerned, it may be that it does not exist at all. The only directly observable entities are the energies of the discrete stationary state, the spectral-line intensities, and, possibly, the corresponding amplitudes and phases, but not the electron obits"([21], p.82).

To be fair, we note that, the two other founding fathers of quantum mechanics, namely, de Broglie and Schrödinger, took the route of classical physics and treated quantum effects in terms of the flow of some subquantal fluid.

However, having opposed metaphysical materialism, Mach proceeded to argue against materialism generally: "The majority of scientists, acting as philosophers, adhere to a materialism that is now 150 years old and has long been regarded as inadequate not only by philosophers but also by people who are more or less familiar with philosophical thinking. My aim has been not so much to introduce a new philosophy into natural science as to remove from it the old philosophy that have outlived its purpose" ([3], p. 12; [4]).

1.2 Limitation of the observability principle

Theory cannot, however, be confined to the description of observations. It must necessarily include generalization. Einstein wrote:"It would be quite wrong to try to construct a theory entirely on the basis of observable quantities. Indeed, the reverse is the case. It is only theory that determines what can be observed" ([22]).

If we were to follow the principle of observability to the letter, science would be just as unpredictable as the result of a horse race. "Theory is not a listing of individual observations, but an account of general regularity"([23], p.294). "A scientific law is not only the expression of a particular number of experimental facts; it reflects the thinking of scientist: the selection of facts, comparison, *fantasy*, and the spark of genius"([24], p.349).

Before Newton, people could see that apples fell on the ground and that the Moon orbited the Earth. However, only Newton saw the common law underlying both the fall of the apple and the motion of the Moon, and used it to predict a multitude of effects that had not been previously observed.

No theory can be verified precisely in a finite number of experiments ([23], p.288). Only experimental facts are not sufficient for constructing any theory. These facts must be supplemented by some supposition. For example, when Heisenberg constructed quantum theory, he did not confine himself to the principle of observability, but also assumed that Newton's equations of motion were also valid in quantum theory if the position coordinate and the momentum were assumed to be matrices rather than numbers ([25]).

Mach's observability principle is essential at the first stage of an investigation, but it must be abandoned once a formulation of a physical law has been found. Mach himself wrote: "... naturally, it is only an infinite number of observations, performed by excluding all interfering factors, that can yield a law" ([3], p. 241).

Strictly speaking, the principle of observability is not satisfied even in classical mechanics: "Although one can see throughout that Newton was trying to present his system as the necessary outcome of experiment, and to introduce the smallest number of concepts that were not related directly to experiment he nevertheless introduced the concepts of absolute space and absolute time" ([17], vol. 4, p.85).

In relativistic quantum mechanics, the background of electrons with negative energy is unobservable ([26], p. 62).

Einstein related Mach's observability principle not to the positivism or contemporary philosophy, but to passive realism: "From the philosophical point of view, this picture of world is closely related to naive realism because the supporters of the latter consider that objects in the outside world are presented to us directly by sensory perception. However, the introduction of immutable material points signified a step toward more refined realism, since it is clear from the very outset that the introduction of such atomistic elements is not based on direct observation" ([17], vol. 4, p. 317). "Mach assigned absolute significance to the principle of observability and refused to acknowledge the existence of atoms" ([15], p. 55).

The wave function is unobservable in quantum mechanics. "Heisenberg taught that theory must operate exclusively with experimental facts and insisted that this principle be applied in elementary-particle physics by removing from it any mention of the time dependence of the state vector between preparation and measurement. This more radical form of quantum mechanics was called by him the S-matrix theory, and he presented it as a competitor to quantum field theory... It did not turn out to be a successful theory of elementary particles" ([23], p. 224).

"After nearly 15 years of wandering, quantum theory returned to its basic principles: to the spatial-temporal description of phenomena" ([27], p. 631).

1.3 Objective processes versus subjective sensations

Mach was a great physicist. But Mach made all physical investigations for the sake of solving some philosophical problems. Namely, he elucidated the relations between objective processes and subjective sensations. As an example we describe the following experiment ([28]), which we have modified slightly. A star is drawn on a disc of light colour. The star does not reach the border of the disc. The interior of the star is painted in dark colour (see Fig.4). The following picture must be observed while

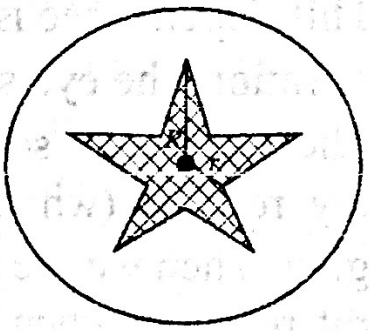
the disc is rotated rapidly: there is a dark circle of radius r in the centre of the disc; then there is a grey ring with radii r and R, the colour of which is changed continuously from dark to light; and finally there is a light ring with inner radius R. Noting that dark surface reflects light badly, while light surface has good reflectivity, we obtain the dependence of the intensity of reflected light I from radius ρ . This dependence is depicted on the Fig.5.

As for subjective sensation, the eye sees the dependence $I(\rho)$, which is depicted on the Fig.6: we see thin black ring on the border of dark and grey regions (when $\rho=r$). This ring seems darker than the dark region. Then we see a bright white ring on the border of grey and light regions (when $\rho=R$). This ring seems much lighter than the light region. "An eye in some sense "is afraid" of transition from penumbra to full illumination and overestimates the difference. The same occurs in transition to full darkness. Eye "judges" according to contrasts rather than according to objective values of intensities. It "judges" more according to the differential relations of the intensity curve rather than according to absolute values of its ordinates. For all this white and black lines (which are, of course, concentric circles on the rotating disc) are expressed so clearly that a naive spectator should swear that they are real"[29].

In order to prove that the Fig.5 rather than the Fig.6 corresponds to reality, Mach took a photograph of the rotating disc. To his great surprise, Mach saw in the photograph the same thin rings at $\rho=r$ and $\rho=R$. Another words, he saw distorted picture, which is shown on the Fig.6, rather than veritable one which is shown in the Fig.5. It seemed that camera marks out a boundary between two regions analogously as man's eye. In reality the picture on the photograph (the number of darkened grains) corresponds to the true picture, which is drawn on the Fig.5. But while looking at the photograph, we "work up" it subconsciously in the same way as an image of the rotating disc.

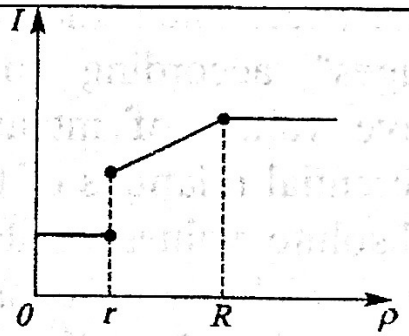
The philosophical interpretation of quantum mechanics was greatly influenced by Mach's assertion about their inseparability of objective processes and subjective sensations:

"Everything physical that I find I can resolve into elements, but entities that remain unresolved at present are colours, pure tones, pressure, heat, odor, space, time,



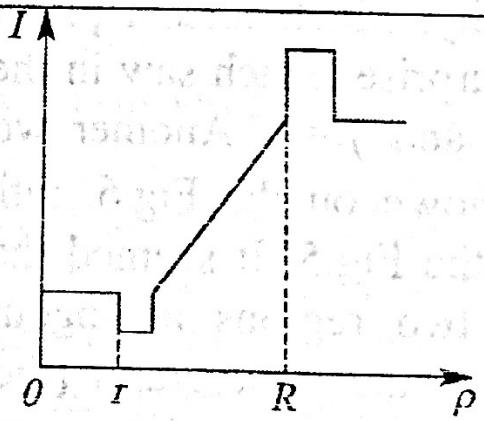
A star on a disc

Fig 4.



Real dependence of intensity of reflected light I from radius ρ

Fig 5.



Apparent dependence of intensity of reflected light I from radius ρ

Fig 6.

and so on. Depending on circumstances, these elements lie[1] outside or inside *U*. Because, and only because, these elements depend on conditions prevailing inside and outside *U*, we also call them sensations" ([3], p. 17). This interpretation makes physical events the consequences of their observation instead of considering that events are observed because they have actually occurred ([23], p.292).

Mach's assertion that sensations cannot be separated into subjective and objective parts is not consistent with reality. For example, we perceive five minutes spent in a dentist's chair as being longer than half an hour spent in the company of a beautiful lady. Here the "unresolvable element" is actually readily resolved into "conditions inside I and those not inside *U*". However, Mach himself, when he passes on to specific examples, in fact resolves the unresolvable elements and rejects all conditions that lie both inside our body and inside our mind: "...a hot body A (an incandescent iron ball) will heat a cooler body B (thermometer) by radiation even if the two are not in contact"([3], p. 196). Where is U and where is I in this example? Here we have only non-I, i.e. objective reality, which is independent of the observer's sensations.

Mach's view of the world is clearly illustrated by an episode in his life. He was interested in ballistics and was often present at shooting practice. Once he turned to a colleague and said: "I am constantly bothered by the question whether the bullet exists in the interval between the firing of the gun and the striking of the target. We can't see what is happening and cannot perceive it". "You are mad" replied the colleague, "How can you doubt the existence of the bullet? And this quite apart from the fact that you yourself have done calculations on bullet trajectories, and your calculations agree with experiment. Doesn't this prove the existence of the bullet?". "This proves nothing "replied Mach. "It may be that the

[1] In this context, it seems to us that the difference between *U* (our body) and the philosophical concept I adopted in the literature (our perception) seems unimportant.

trajectory is merely an auxiliary mathematical concept that serves only for the prediction of further observations. It may be that the bullet does not travel on the trajectory at all. It is possible that the bullet vanishes at the point of firing and reappears just before it hits the target". The colleague just shrugged his shoulders, but Mach remained dubious. He constructed an instrument that could be used to photograph the bullet in flight and saw on the photograph some lines emerging from the bullet. They are now known as Mach lines. [15].

It was thus his doubts about the existence of a flying bullet that led Mach to the foundations of supersonic gas dynamics. The ratio of the speed of a flying object to the speed of sound is now called the Mach number in his honor.

1.4 Operationalism

The principle of observability has led to the development of operationalism. The founding father of operationalism was Bridgman who defined it in the following words: "We understand by any concept no more than a sequence of operations. The concept is synonymous with a known sequence of operations" ([30], p.5).

Operationalism is discussed in greater detail in [23], [31] and [32].

We shall elucidate the concept of operationalism in terms of an example. Suppose we ask: what is the time? "Webster defines "a time" as "a period", and the later as "a time" ([12], vol.1, p.86).

This is more of a vicious circle than a definition. To give "time" a meaning we must specify how it must to be measured. In other words time is defined by the operation of its measurement. The extension of this to all other physical entities is operationalism. Without the operational approach there would be no theory of relativity and no quantum mechanics ([33], p.2).

Returning to the concept of time we note that it is measured with a clock, and the terrestrial globe is a natural clock. Indeed the unit of time, the second, is the time taken by the globe to complete 1/86400 th part of its revolution around its axis. In particular, it follows from this definition that there is no point in asking whether the Earth rotates uniformly because time is defined in terms of its rotation. "Recently", writes Feynman, "we have been gaining experience with some natural oscillators which we now believe would provide a more constant time reference than the Earth, and which are also based on a natural phenomenon available to everyone. These are called *atomic clocks*. Their basic internal point period is that of an atomic vibration which is very insensitive to temperature or any other external effects. These clocks keep time to an accuracy of one part in 10^9 or better" ([12], vol.1, p.93).

Atomic clocks have been used to measure the extent to which the Earth's rotation is nonuniform. "...the Earth's rotation on its axis is slightly slowing down. It is due to tidal friction"([34], p.98). We see that the question whether the Earth rotates uniformly is not as meaningless as suggested by operationalists. Time does not reduced to some specific measuring device, not even such an accurate instrument as the rotating Earth. Time is a much deeper concept; it is meaningless unless we specify the method used to measure it, but it does not reduce entirely to the method of measurement.

"The operational point of view, taken as the only criterion, always presupposes an abstractions at a lower level. However, the most striking theoretical achievements involve abstractions at a very high level" ([35], p.184).

Bridgman himself subsequently acknowledged that purely operational definitions of different concepts were incomplete: "If I were to write all this again I would try to emphasize the importance of both mental and pencil-and-paper operations. One of the most important mental operations is the verbal operation. It

plays a much greater part that I suggested previously..."([36], p.184).

We note in this connection that all physical quantities such as momentum, energy, and so on have a precise meaning only whithin the framework of a particular theory (Newtonian mechanics, theory of relativity, quantum mechanics), but some quantities are more fundamental than theories. This is why the concepts of momentum and energy survived (with modifications) when Newtonian mechanics was replaced by relativity and quantum mechanics.

Chapter 2

Particles versus waves

Three blind men encounter an elephant for the first time.
He is like a wall - says one
"No, he is like a column" - says another.
"You are both wrong" - says the third -
"he is like a rope".

2.1 Distinction between particles and waves

There are two forms of physical reality: substance and field. Substance consists of individual particles of enormously small size, namely, electrons, protons, neutrons, etc. Field, on the other hand, is distributed in all space.

Excited states of the field propagate in space in the form of waves. Waves on a corn field, driven by wind, are a clear example of this. Although the waves constantly travel in the same direction, the corn ears themselves do not take part in net translational motion because they are attached to the ground.

Waves play a major part in physics. For example, sound waves propagate in air. Electromagnetic waves with wavelengths between a meter and a kilometer are known as radio waves, whereas visible light has wavelengths of the order of 10^{-4}cm.

We note that fields exist even when waves are absent. A particular physical field can be determined by specifying it at all points in space. For example, sound is the excited state of a pressure field $p(x,y,z)$.

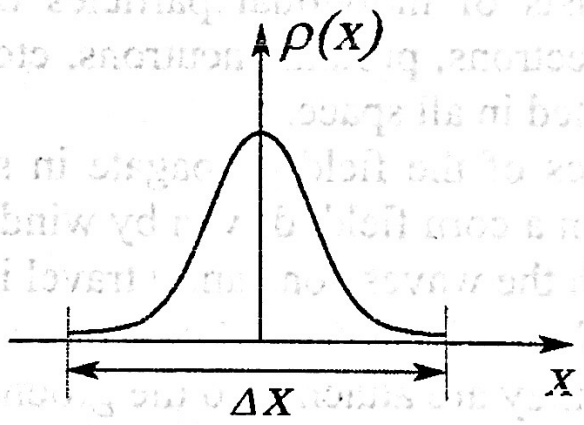
The fundamental conflict between the concepts of *particle and wave* disappears in quantum mechanics. To comprehend this, we

must first examine these concepts whithin the framework of classical physics in which they cannot be combined.

Consider particles first. Particles typically occupy a negligible volume, i.e. they are practically point objects (Fig.7).

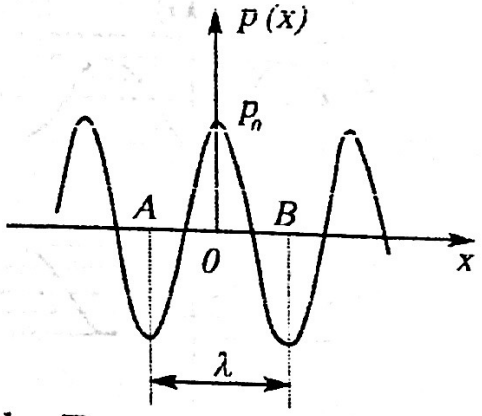
However, their more important property is that they are indivisible. A liter of water can be readily divided into two parts with identical properties. However, a molecule of water cannot be divided into two parts simply by tilting a glass: a much more powerful means of division is necessary to achieve this end, e. g. electrolysis. The main point is that the division of a molecule of water does not result in two half molecules of water, but in the atoms of two new materials, namely, oxygen and hydrogen.

A further important property of classical particles is their individual identity. We can always label each particle and follow its individual fate.



A distribution of the density of matter in classical particle [$\rho(x)$ - density of matter, Δx - particle size]. For the sake of simplicity, the particle is to be one-dimensional

Fig 7.

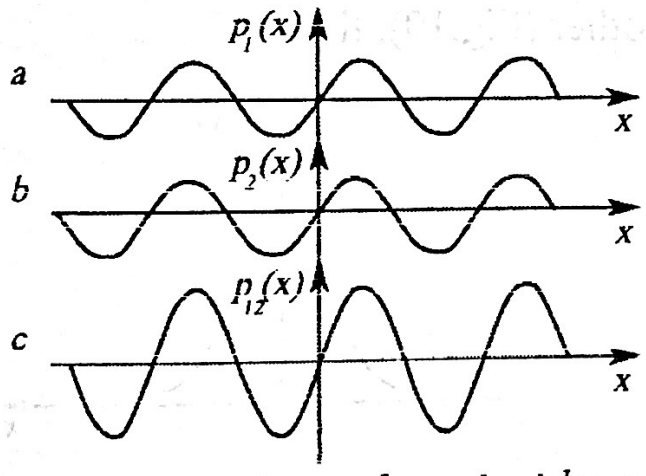


Acoustic wave profile. Excess pressure above normal, plotted against position x (p_0 is the wave amplitude and λ is the wave-length which determine the frequency)

Fig 8.

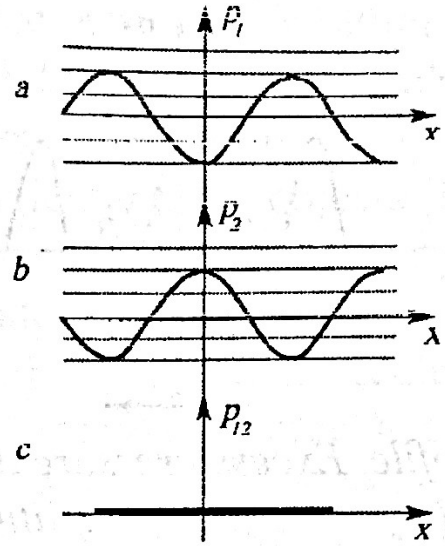
Waves constitute the exact opposite of all this. The ideal wave has the sinusoidal shape shown in Fig.8 and is called a harmonic wave.

We note that measurement on a low-intensity wave always involves some distortion of it. For example, when we tune to a particular radio station, we use resonance to amplify specific frequency and suppress all other frequencies. The result is a highly distorted wave, but its intensity is high enough for the purpose.



Interference when crests and troughs coincide: a –first wave, b – second wave, c – resultant wave

Fig 9.

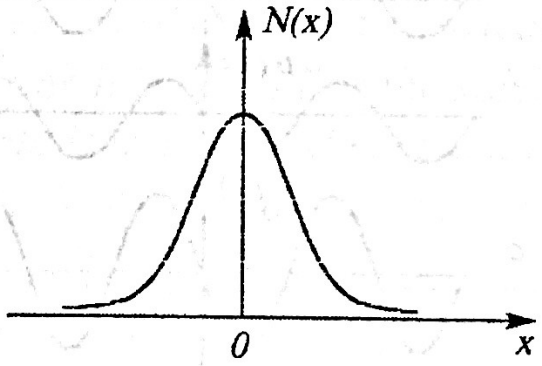


Interference when crests of one wave fall on the troughs of the other:
a – first wave, b – second wave, c – resultant wave
Fig 10.

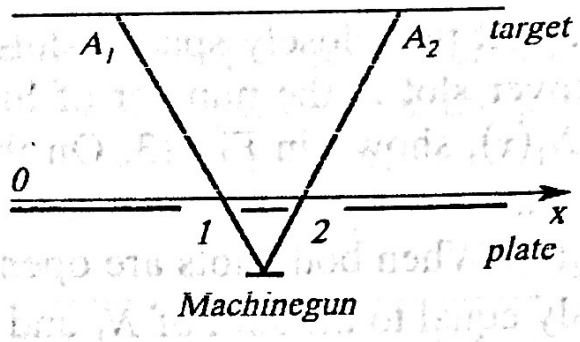
2.2 Interference of waves

The characteristic feature of waves is that they can not only amplify, but also "extinguish" one another. This mutual amplification or extinguishing of waves is called interference.

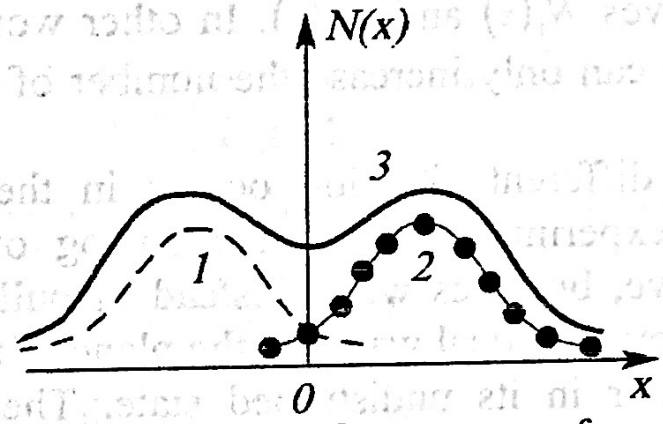
When the crests and troughs of two waves coincide (Fig.9), they add constructively, but when the crests of one fall on the troughs of the other (Fig.10), they



Number of bullets N striking the target as a function of distance from center of target
Fig 11.



Firing through two slits
Fig 12.



Number of bullets reaching the screen after two slits: 1 – number of hits N_1 when slit 1 is open, 2 – number of hits N_2 when slit 2 is open, 3 – number of hits N_{12} when both slits are open
Fig 13.

interfere destructively. We note that the phenomenon of interference-especially destructive interference - is inconceivable in the case of particles. We can illustrate this by considering a machine gun and a target.

An armored plate, with a vertical slot cut into it, is placed between the machine gun and the target and bullets can pass freely through the slot. Most of the bullets hit the target center A which lie directly opposite the center of the slot. If we take the x axis to be horizontal and parallel to the plate, we can define a function $N(x)$ to represent the number N of hits at x. This function has the bell shape shown in Fig.11.

Now suppose that two closely spaced slots are cut in the plate (Fig.12). If we cover slot 2, the number of hits will be described by the function $N_1(x)$, shown in Fig.13. On the other hand, if we cover slot 1, the number of hits will be described by a similar, but shifted, curve $N_2(x)$. When both slots are open, the number of hits $N_{12}(x)$ is obviously equal to the sum of N_1 and N_2:

$$N_{12} = N_1 + N_2 \tag{2.1}$$

We note that the resultant curve $N_{12}(x)$ lies above each of the individual curves $N_1(x)$ and $N_2(x)$. In other words, by opening a further slit we can only increase the number of hits at each point on the target.

A totally different situation occurs in the case of waves. Consider an experiment that is the analog of the experiment described above, but uses water instead of bullets. The target is now replaced by a vertical wall and the plane of the drawing is the surface of water in its undisturbed state. The machine gun is replaced by a source of waves (an oscillating object). The vertical displacement of a point on the surface of water, measured from the undisturbed state, will be denoted by $p(x)$.

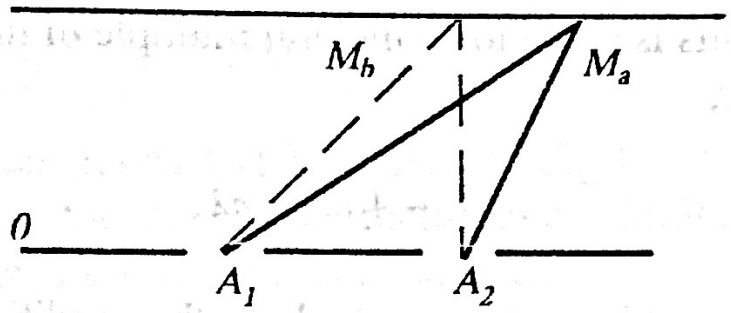
Consider the case where only slot 1 is open. In contrast to the number of bullets, $N_1(x)$, the displacement $p_1(x)$ can be either positive (crest) or negative (trough). When only the second slot is open, the displacement is $p_2(x)$. When both slots are open, the resultant displacement $p_{12}(x)$ is the algebraic sum of the two displacements:

$$p_{12}(x) = p_1(x) + p_2(x) \tag{2.2}$$

Since the quantities $p_1(x)$ and $p_2(x)$ can be either positive or negative, we have the possibility of mutual cancellation of displacements, i.e., destructive interference (see Fig.10).

The energy W of the wave per unit volume is proportional to the square of displacement. Omitting, for the sake of simplicity, the proportionality coefficient, we can write

$$W_1(x) = p_1^2(x) \tag{2.3}$$



Condition for destructive and Constructive interference
Fig 14.

$$W_2(x) = p_2^2(x) \tag{2.4}$$

And the energy of the resultant wave is

$$W_{12}(x) = (p_1(x) + p_2(x))^2 \tag{2.5}$$

We see, that

$$W_{12}(x) = W_1(x) + W_2(x) + 2p_1p_2 \tag{2.6}$$

i.e.

$$W_{12}(x) \neq W_1(x) + W_2(x) \tag{2.7}$$

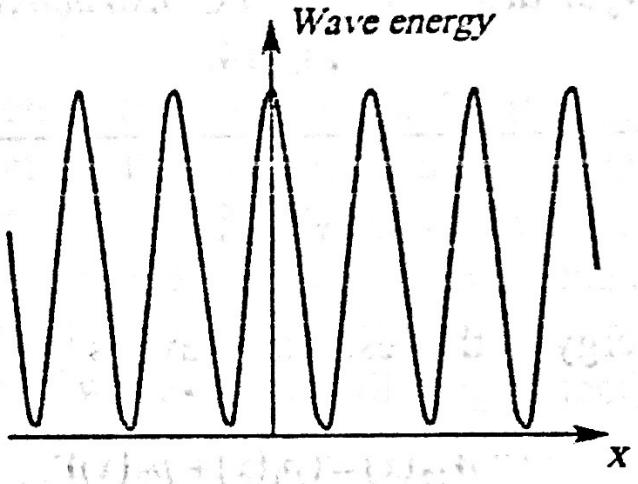
Thus, in the case of waves, we add not the energies, but the amplitudes. This gives rise to an apparent violation of the law of conservation of energy (2.7). However, in reality, this is not so. Indeed, what we are dealing with is the outflow of wave energy from the volume under consideration (which can be either positive

or negative), so that the energy remaining in the chosen volume is not constant. The law of conservation of energy actually demands that the rate of loss of energy from a given volume must be equal to the rate at which energy flows out of the volume ([37], p.358).

At a point M_a, for which the difference between its distances to the two slits is equal to an integral multiple of the wavelength λ (Fig.14), i.e.,

$$A_1M_a - A_2M_a = n\lambda, \tag{2.8}$$

where n is an arbitrary integer, we have the condition



Interference of two waves
Fig 15.

$$p_1(x) = p_2(x). \tag{2.9}$$

The displacement of a floating detector when both slots are open is twice as large as it was when only one slot was open (Fig.9) In this case the energy of the resultant wave is greater by a factor of four.

On the other hand, at a point M_b at which

$$A_1M_b - A_2M_b = n\lambda + \lambda/2, \tag{2.10}$$

the crests of the wave from one slot fall on the troughs of the wave from the other, and we have

$$p_1(x) = -p_2(x). \tag{2.11}$$

This produces destructive interference (Fig.10). Such effect is inconceivable in the case of particles: the addition of salt to seawater can never convert it into fresh water.

We see that the opening of the second slot produces an increase in the wave amplitude at some points and a reduction at other points. When the number n in (2.8) and (2.10) runs through all integral values between $-\infty$ and $+\infty$, the corresponding points "move" along the wall. Regions of high energy of oscillations alternate with regions of low oscillation energy (Fig.15, which plots the wave energy, proportional to the square of its amplitude).

2.3 Coherence

Let us consider in greater detail the interference of the two waves

$$p_1(x) = P_1 \sin(kx + \varphi_1) \tag{2.12}$$

$$p_2(x) = P_2 \sin(kx + \varphi_2). \tag{2.13}$$

where P_1 and P_2 are constants (the respective amplitudes of the two waves), $k = 2\pi/\lambda$ is the wave number, and φ_1 and φ_2 are the phases of waves. The energy of the first wave is

$$W_1 = (p_1(x))^2 = P_1^2 \sin^2(kx + \varphi_1), \tag{2.14}$$

$$< W_{12} >= \frac{1}{2} P_1^2 + \frac{1}{2} P_2^2 + 2 P_1 P_2 < sin(kx + \varphi_1) sin(kx + \varphi_2) > . \quad (2.19)$$

Since

$$sin(kx + \varphi_1) sin(kx + \varphi_2) = \frac{1}{2} cos(\varphi_1 - \varphi_2)$$

$$- \frac{1}{2} cos(2kx + \varphi_1 + \varphi_2), \quad (2.20)$$

we have

$$< sin(kx + \varphi_1) sin(kx + \varphi_2) >= \frac{1}{2} cos(\varphi_1 - \varphi_2) \quad (2.21)$$

Consequently,

$$< W_{12} >= \frac{1}{2} P_1^2 + \frac{1}{2} P_2^2 + P_1 P_2 \, cos(\varphi_1 - \varphi_2) \quad (2.22)$$

The term containing $cos(\varphi_1 - \varphi_2)$ describes interference. In particular, when $P_1 = P_2$ and $cos(\varphi_1 - \varphi_2) = -1$, the two waves cancel one another out.

Since light is an electromagnetic wave, illumination by two electric lamps theoretically can produce either an increase or a reduction in intensity at certain points in space. In practice, this is not observed because each atom in the filaments of the lamps emits a photon within a very short interval of time, so that the phase difference $\varphi_1 - \varphi_2$ is a rapidly-varying random function of time. We thus observe the average value of $cos(\varphi_1 - \varphi_2)$ which is zero, i.e., the interference term disappears from (2.19) and there is no interference.

Usually the wave length λ is small in comparison with the typical linear dimensions of the apparatus, so that $sin^2(kx+\varphi_1)$ is a rapidly oscillating function. In a measurement, we always average x over a certain interval Δx which is small in comparison with macroscopic dimensions, but is large in comparison with the wavelength λ.

The average of the energy over Δx has a physical meaning:

$$< W_1 >= P_1^2 < sin^2(kx+\varphi_1) >, \tag{2.15}$$

We know from trigonometry that

$$sin^2(kx+\varphi_1) = \frac{1}{2} - \frac{1}{2} cos[2(kx+\varphi_1)] \tag{2.16}$$

and if we note that

$$< cos[2(kx+\varphi_1)] >= 0,$$

we obtain

$$< W_1 >= \frac{1}{2} P_1^2. \tag{2.17}$$

Similarly

$$< W_2 >= \frac{1}{2} P_2^2 \tag{2.18}$$

Combining these results, we obtain

Waves for which there is a strict relation between φ_1 and φ_2 are called coherent. We thus see that interference is observed only for coherent waves. In this sense, incoherent waves behave like particles.

Chapter 3

Discreteness

A man being out of breath asks a station man on duty: "Has the train for Moscow left yet? "Yes, just a minute ago. You can still see its tail". "I have done right to run so quickly. If I had not donc it, I should be late still more".

3.1 Light quanta

Quantum mechanics was born when Planck discovered minute particles of light, i.e., when he found that the energy of light wave cannot be indefinitely small, but consisted of indivisible portions (quanta) given by

$$\varepsilon = \hbar\omega, \tag{3.1}$$

where $\hbar$ is Planck's constant $\left(\hbar = 1.055\times10^{-27}\ \ erg\cdot s\right)$ and ω is the wave frequency. (We note that, in early works, it was common to use the quantities $h = 2\pi\hbar$ and $\nu = \omega/2\pi$. The formula given by (3.1) then took the form

$$\varepsilon = h\nu. \tag{3.2}$$

"...The energy of a beam of light emerging from a particular point is not distributed continuously in the entire expanding volume, but consists of finite number... of indivisible quanta of energy that are absorbed or emitted only as complete quanta" [38].

The law expressed by (3.1) is true for any wave process. "Wherever it occurs in nature, the energy of a sinusoidal

oscillatory process of frequency ν always assumes values that are integral multiples of $h\nu$. Intermediate values of the energy of sinusoidal oscillatory processes are not found in nature" ([17], vol. 4, p. 58).

Einstein and de Broglie used the equality (3.1) to derive the relation between the momentum p and the wave number k:

$$p = \hbar k . \tag{3.3}$$

The Planck's law (3.1) and the Einstein-ge Broglie relation given by (3.3) show that each particle of energy ε and momentum p is also a wave of frequency

$$\omega = \varepsilon / \hbar \tag{3.4}$$

and wavelength

$$\lambda = 2\pi\hbar / p . \tag{3.5}$$

Thus the distinction between particles and waves disappears in quantum mechanics.

3.2 Energy levels of atom

L. de Broglie explained energy levels of an atom of hydrogen assuming that integral number of electron wave lengths λ packs up on the circle of radius r along which the electron moves:[2]

$$2\pi r = n\lambda . \tag{3.6}$$

Equating centrifugal force to the attraction force between the nonrelativistic electron and nucleus

[2] This was one of earlier works on quantum mechanics when there was assumed that electron in atom moves along a definite trajectory.

$$\frac{mv^2}{r} = \frac{e^2}{r^2}, \tag{3.7}$$

and using the relations (3.5), we find, that the total electron energy

$$E = \frac{mv^2}{2} - \frac{e^2}{r}, \tag{3.8}$$

equals to

$$E_n = -\frac{me^4}{2\hbar^2 n^2} \tag{3.9}$$

We see that the electron energy cannot be arbitrary. It takes a set of discrete values, which correspond to $n = 1,2..$ in formula (3.9). Negative values of energy mean that the electron is in a bounded state.

3.3 Discreteness versus continuity

Physical thinking was based on the supposition about continuity of all causal relations, since the creation of the analysis of infinitesimal quantities by Newton and Leibnitz. Therefore discreteness of quantum theory gave rise confusion among some physicists. Even creator of quantum mechanic Schrödinger said: "If we intend to preserve these damned quantum jumps, I feel sorry that I dealt with quantum theory"([39]).

Meanwhile the principle of continuity arose only after Newton. This principle was never stated explicitly but was often assumed tacitly.

However, discrete values are encountered not only in quantum mechanics but in classical physics also. For example, a string can

sustain oscillations not with arbitrary but only with the discrete set of frequency: ω, 2ω, 3ω,...etc., where ω is the basic frequency. This fact was revealed by Pythagoras. He decided that fundamental physical quantities are integral numbers. This conception contradicts the continuity principle.

Further, the hegemony of integral numbers was expanded on geometry: fundamental physical objects are simple and perfect. There are five regular polyhedrons: tetrahedron, cube, octahedron, dodecahedron and icosahedron.

Proceeding from this idea, Kepler revealed the "cosmographical mystery" [40], [41]: cube inscribed in the sphere with Saturn's orbit, is circumscribed over Jupiter's sphere. Tetrahedron inscribed in the Jupiter's sphere is circumscribed over Mars' sphere, Dodecahedron inscribed in the Mars' sphere is circumscribed over the Earth's sphere. Icosahedron inscribed in the Earth's sphere is circumscribed over the Venus' sphere. And finally, octahedron inscribed in the Venus' sphere, is circumscribed over the Mercury's sphere.

Now "cosmographical mystery" of Kepler is forgotten because it is based on old philosophy: the relative distance of planets from the Sun are discrete.

There are jumps in classical physics also. For example, water cannot be transformed continuously into steam. There is no halfwater-halfsteam. When temperature arises above $100^0 C$ water "jumps" into steam.

We see that discreteness is not a monopoly of quantum mechanics. It exists in classical physics also. Discrete quantities of quantum mechanics are in conflict not with common sense but only with the idolization of the continuity principle.

Chapter 4

Uncertainty relations

"After this Shendrikov's time came.

"Where do you serve?" an inspector addressed to him.

"I am an examiner in a local post. I was serving during twenty one years, Your Honour. Now information was demanded for presentation me to the rank of collegiate registrar. For this purpose I dare to undergo to the examination for the first class rank"...

Geography teacher "settled back and asked him "Well... say me what is a governing in Turkey?"

"It is well known... A Turkish one..."

"Hm!... It is an uncertain notion".

Anton Chehov. Examination for a rank

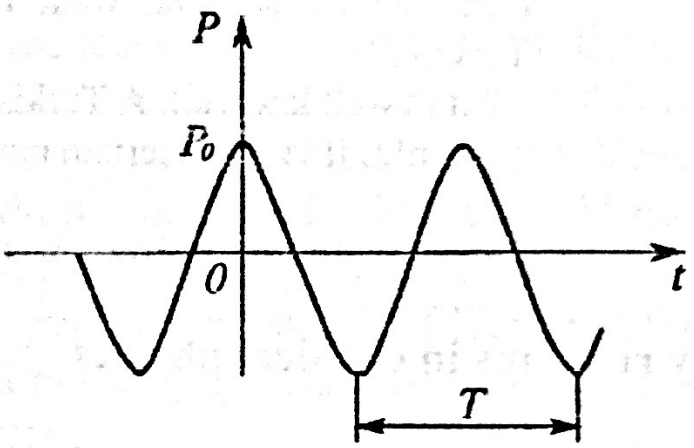
4.1 Uncertainty relations in classical physics

A further difference between a wave and a particle is that a harmonic wave extends to infinity, whereas a particle is localized within an infinitesimal portion of space Δx. However, this difference is unimportant because it is shown in the theory of Fourier integrals that any function that vanishes outside a finite interval Δx can be represented by a superposition (sum) of an infinite number of sinusoids with different wavelength λ and different amplitude. The wave amplitudes in this sum usually decrease rapidly with increasing difference between λ and some average wavelength λ_0. It can be said that the superposition of waves results in the wavelength λ being confined to the neighborhood of λ_0 defined by

$$\lambda_0 - \frac{\Delta\lambda}{2} \le \lambda \le \lambda_0 + \frac{\Delta\lambda}{2} \qquad (4.1)$$

where $\Delta\lambda$ may be looked upon as the uncertainty in the wavelength.

The resolution of sunlight into harmonic waves is performed in practice by, for example, a glass prism. White sunlight thus resolved into a rainbow of colors, namely, red, orange, yellow, green, blue, blue-violet, and violet. "There were numerous discussions in the nineteenth century about whether the monochromatic components of white light were there in the first place, i.e., in the incident beam, or whether the



Harmonic wave at a fixed point x at different time t:

$$P = P_0 \sin\left(\omega t + \pi/2\right) -$$

Excess of pressure above normal, P_0- amplitude, T – period

Fig 16.

---

components were produced by the prism. The question did not receive a satisfactory answer. In the final analysis, the most cautious position was: the monochromatic components are present in the incident light in a virtual, i.e., a potential state". ([42], 171)

Let us now return to the harmonic wave and introduce the wave number *k*,

$$k = 2\pi/\lambda \qquad (4.2)$$

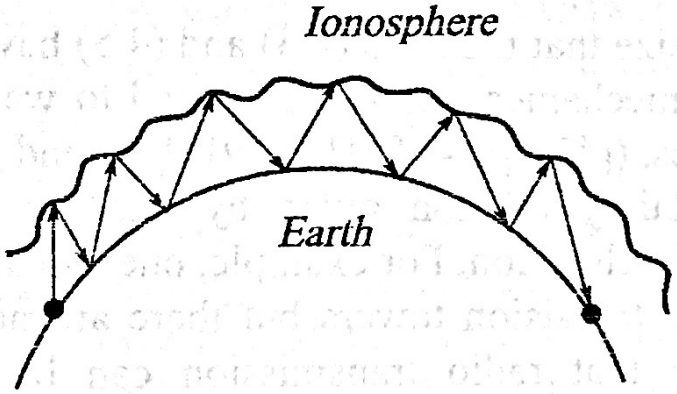
The uncertainty $\Delta\lambda$ then corresponds to the following uncertainty in the wave vector

$$\Delta k = \frac{2\pi\Delta\lambda}{\lambda^2}.$$

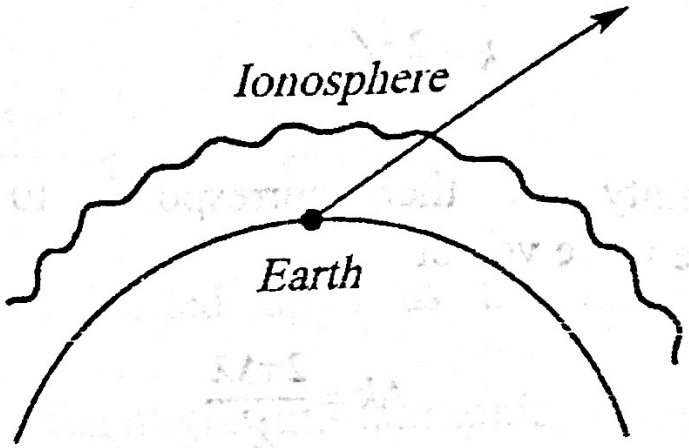
It will be shown in Sec.4.3, that the two uncertainties Δx and Δk are linked by uncertainty relation

$$\Delta x \Delta k \approx 1. \qquad (4.3)$$

In our discussion above, we considered a wave at different points x in space at given time t. We can also consider a wave at a fixed point x at a different time t (Fig.16). The wavelength λ is replaced by the oscillation period T, whereas the wave vector k is replaced by the frequency ω:



Radio-wave propagation
Fig 17.



Propagation of TV waves
Fig 18.

$$\omega = 2\pi/T. \tag{4.4}$$

In terms of these new quantities, the uncertainty relation becomes

$$\Delta\omega\Delta t \approx 1. \tag{4.5}$$

We emphasize that the laws (4.3) and (4.5) have nothing to do with quantum mechanics. They are applied to wave processes in classical physics. ([43], p.54; [44], p.191; [45] and [46])

The uncertainty relation given by (4.5) is encountered in connection with television. For example, one can ask why in many towns there are television towers but there are no radio towers? The answer is that radio transmission can be received from stations thousands of kilometers away whereas television transmissions come from neighborhood TV centers. This is so because radio waves have wavelengths ranging

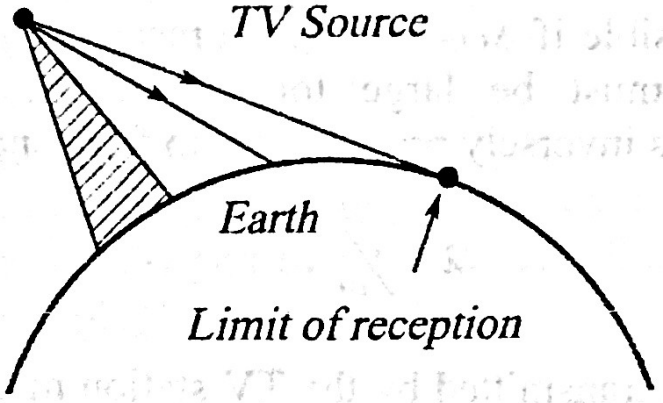


Fig 19.

from dozens of meters to several kilometers. They are reflected from the ionosphere and can therefore propagate to any point on the Earth's surface (Fig.17).

T.V. transmissions, on the other hand, employ ultra-short waves, whose wavelengths are of the order of a meter. Such short waves pass freely through the ionosphere (Fig.18), so that TV sets receive only transmissions from TV stations in their direct line of sight (Fig.19). We then ask again: why is it that TV transmissions cannot be made at longer wavelengths? The answer is that, when compared with the acoustic information carried by radio transmission, the rate of transmission of information for a TV set is enormous. The screen has a very large number of points, so that to ensure that the successive frames are received not as blips on the screen, but as a moving image, the entire picture must be changed completely at the rate of 24 per second. The duration Δt of each signal is therefore very short and it is clear from the uncertainty relation given by (4.5) that $\Delta\omega$ is then very large.

On the other hand, the TV receiver can cope with extremely weak signals. This is possible only because the natural frequency ω_0 of the television circuit is equal to the frequency ω of the transmitting station (resonance):

$$\omega_0 = \omega . \qquad (4.6)$$

This is possible if $\Delta\omega << \omega$, which means that, since $\Delta\omega$ must be large, ω must be large too. Since the electromagnetic wavelength λ is inversely proportional to frequency, i.e.

$$\lambda \approx 1/\omega \tag{4.7}$$

the wavelength transmitted by the TV station must be sufficiently short.

The uncertainty relation given by (4.3) also makes its appearance in the case of the brass band. Anyone who has seen a marching band will have noticed the "social inequality" of these people: the flautist carries a small instrument, but the tuba player has a large one. Why is it that the bass tuba cannot be made smaller? Low frequency (bass) sound corresponds to a large λ, i.e. small k and Δk. The uncertainty principle (4.3) then shows that the corresponding Δx (the size of the tube) must be sufficiently large.

The fundamental difference between classical waves and particles is that, in classical physics, waves are indefinitely divisible, i.e. there are no wave "atoms". Any classical waves, however small its amplitude, can be divided into two waves of even smaller amplitude.

In contrast to particles, classical waves are indistinguishable. For example, suppose that at the initial time t_0 the amplitude of the wave in water at a point A is 1 cm, whereas at a point B it is 3 cm. If at some subsequent time t_1 the wave amplitude at A becomes 3cm, we can say that the wave has travelled from B to A. Equally so, we are entitled to say that the waves have remained in place, but the wave amplitude at point A has increased.

4.2 Uncertainty relations in quantum mechanics

We can deduce the Heisenberg uncertainty relations (the uncertainty relations in quantum mechanics) by multiplying the uncertainty relations (4.3) and (4.5) by $\hbar$, and then, using (3.1) and (3.3), we obtain ([45]):

$$\Delta p \Delta x \approx \hbar \tag{4.8}$$

$$\Delta \varepsilon \, \Delta t \approx \hbar . \tag{4.9}$$

We shall not pause to consider the difficulties associated with the interpretation of the energy-time uncertainty relation (4.9) ([47];[48], p.103; [49]; [163]) and confine ourselves to two limiting cases of (4.8):

(1) $\Delta p = 0$, in which case $\Delta x = \infty$; this is a wave. It has a definite momentum, but occupies all space. We note that this case corresponds to an electron with definite velocity.

$$\upsilon = p/m \tag{4.10}$$

and therefore specific energy

$$\varepsilon = \frac{p^2}{2m} \tag{4.11}$$

(we limit ourselves to the nonrelativistic energies).

(2) $\Delta x = 0$, in which case $\Delta p = \infty$; this is a particle. The particle lies at a particular point in space, but its momentum is completely undetermined. This means that, according to quantum mechanics, the particle cannot be at rest.

We now turn to a more realistic situation in which Δx is different from zero, but is very small. For example, consider an electron in an atom. We then have

$$\Delta x \approx 10^{-8}\,\text{cm} \tag{4.12}$$

and

$$\Delta p \approx \hbar/\Delta x. \tag{4.13}$$

The uncertainty in velocity is more illustrative in this case

$$\Delta \upsilon = \Delta p/m. \tag{4.14}$$

Substituting in (4.13) and (4.14) $\hbar \approx 10^{-27} erg \cdot s$, $m \approx 10^{-27} g$., we obtain for an electron in an atom

$$\Delta \upsilon \approx 1000\ km/s. \tag{4.15}$$

We thus see that the velocity of an atomic electron is a random quantity ranging between 0 and 1000 km/s (values of υ much greater than 1000 km/s therefore have low probability).

Thus, if the electron is to fit into the volume of the atom, its velocity must be random and its typical value must exceed the velocity of a bullet by a factor of at least a thousand!

We note that a quantum object is a single entity that in one limiting case $\Delta x = 0$ behaves like a particle and in the other limiting case $(\Delta k = 0)$ behaves like a wave. However, in general $(\Delta x \neq 0, \Delta k \neq 0)$ the quantum object has the properties of both particles and waves.

The following thought experiment provides a very clear illustration of the Heisenberg uncertainty relations given by (4.8) and (4.9). To determine the position of an electron under a "microscope", we have to illuminate it. Since light is a wave, the uncertainty Δx in the position of the electron is of the order of the wavelength of light λ

$$\Delta x \approx \lambda\ . \tag{4.16}$$

If we reduce λ indefinitely, we increase without limit the precision with which the position of the electron is determined. However, the quantum of light - the photon - is also a particle with momentum given by (3.3). When an electron collides with a photon, it receives the additional momentum

$$\Delta p \approx \hbar/\lambda. \qquad (4.17)$$

By comparing (4.16) with (4.17), we obtain the Heisenberg relation (4.8). In other words, the more accurately we measure position, the more we disturb the original momentum. To put it another way, position and momentum cannot be measured simultaneously with absolute precision.

The above thought experiment serves as an illustration, but is not a proof. ([50], p.21). It is only examine for theory. Paraphrasing Spinoza, we may say that lack of skill to measure is not a proof.

The Heisenberg relations are not a conclusion drawn from the thought experiment but rather a mathematical theorem ([51]).

The above discussion does not therefore entirely remove the basic possibility that the position and momentum of an electron could be measured accurately by some other method. Moreover, many physical quantities have been obtained not by direct measurement, but by numerical calculation. For example, the temperature at the center of the Sun was determined not with a thermometer or a bolometer, but by computer calculation.

There are many examples in the history of physics in which a radical improvement in measurement technique resulted in the observation of fundamentally unobservable objects. For instance, prior to creation of *X*-ray microstructure analysis it was considered that an individual atom could not be observed. Here are a few lines from a letter written by E.S. Fedorov, the father of crystallography, to N. A. Morozov in 1912: "Dear Nikolai Aleksandrovich: You end your letter by saying that no man will

ever see an atom. But you wrote this more or less at a time when man had already seen the atom with his own eyes.; if not the atoms themselves, then photographic images of them, certainly" ([52], p.59). We can now see the atoms in crystals as the regularly distributed spots on an *X*-ray diffraction patterns.

The essence of the uncertainty relationships is not so much that we cannot simultaneously measure position and momentum, but that these quantities sometimes do not have exact values. We illustrate this by a simple example.

Let us get to know the distance between Kharkov and Moscow. In the railway station this distance pointed out as 781 km. In the bus station it is another: 773 km. This uncertainty arised not because measuring the distance between railway station, we disturb the distance between the bus stations. It arised also not because our measurement are nonperfect and true distance will be determined after use of more sophisticated measuring technique. The reason of this uncertainty lies in the fact that dimensions of Kharkov and Moscow are of order of some ten kilometers. Therefore the notion "the distance between Kharkov and Moscow" loses its sense, when it is measured with the accuracy of one kilometer. Different methods of measuring of this distance give different results.

The Heisenberg uncertainty relations is not a consequence of the fundamental imperfection of measuring devices, but a mathematical theorem ([53], p.67). "It is usually said that the uncertainty relation arises from the interaction between the measurer and the object being measured... The relation actually arises at the very beginning, well before there is any question of measurement" ([43], p.358).

The uncertainty relation for position and momentum is "a consequence of the formalism of quantum mechanics" ([54], p.13).

The uncertainty described by the Heisenberg relation arises because we are attempting to measure something that has no definite meaning. "If you ask a silly question, you get a silly

answer" ([55]). For example, according to the Einstein-de Broglie relation, in a state with definite momentum p, the electron has a precisely defined value k, i.e., it is a harmonic wave, and occupies all space. Its coordinates then are fully undetermined, Contrariwise, in a state with definite position coordinate $\vec{r}_0$, the momentum of the electron does not have a definite value.

If we do measure a quantity which have a definite value in a given state, we shall obtain a quite definite result. For instance, the energy of a hydrogen atom is quite definite and is expressed by the formula (3.9) in which n=1 must be put.

The quantum-mechanical unification of waves and particles is often exploited in classical physics too, e.g., in the analysis of wave interactions ([56], p.540, and [57]). Since quantum theory becomes identical with classical theory in the limit as $\hbar \to 0$, waves are regarded quantum mechanically as particles. The interaction between particles is mathematically simpler to describe than the interaction between waves. In the final formulas, Plank's constant cancels out in this case as $\hbar \to 0$.

4.3 Proof of the classical uncertainty relation

The relation

$$\Delta x \Delta k \approx 1 \tag{4.18}$$

can be obtained with the help of the Fourier transformation.

We confine our attention to a simple case in which the deviation of a field (for example, pressure) from a constant value is given by

$$u(x) = ReU(x), \tag{4.19}$$

$$U(x) = Ae^{ikx} exp\left[-x^2/(\Delta x)^2\right]; \tag{4.20}$$

where A is a complex constant. We consider the field at a fixed instant of time, which we take to be $t=0$.

The wave intensity at a point x is $|U(x)|^2$. If the exponential factor $exp\left[-x^2/(\Delta x)^2\right]$ were absent from (4.20), then the wave intensity would be the same at all points:

$$|U(x)|^2 = |A|^2 = const. \tag{4.21}$$

The uncertainty in the position coordinate would then be infinite:

$$\Delta x = \infty.$$

The factor $exp\left[-x^2/(\Delta x)^2\right]$ shows that the wave intensity decreases with x^2 and becomes infinitesimal when $x^2 >> (\Delta x)^2$. The wave described by (4.20) is therefore localized near the point $x=0$, and the uncertainty in the position coordinate is Δx.

On the other hand, the wave can be written as a superposition of plane waves $e^{ik'x}$ with different wave number k'

$$U(x) = \int_{-\infty}^{\infty} e^{ik'x}\, a(k')dk', \tag{4.22}$$

where $a(k')$ is the amplitude corresponding to wave vector k'. To find $a(k')$, we invert the Fourier transformation:

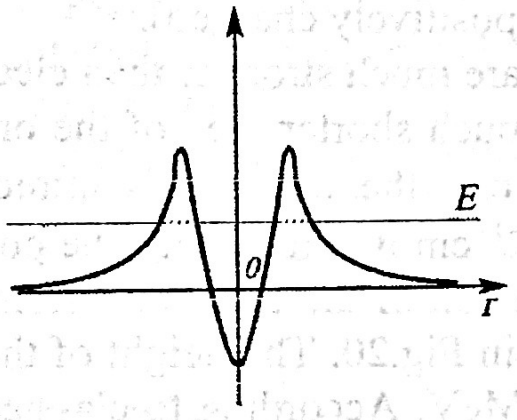
$$Ae^{ikx} exp\left[-x^2/(\Delta x)^2\right] = \int_{-\infty}^{\infty} e^{ik'x} a(k')dk' \tag{4.23}$$

We know that this inversion yields

$$a(k') = \frac{1}{2\pi}\int_{-\infty}^{\infty} Ae^{-i(k'-k)x} e^{-\frac{x^2}{\Delta x}} dx \tag{4.24}$$

which is readily evaluated:

$$a(k') = \frac{1}{2} A \frac{|\Delta x|}{\sqrt{\pi}} exp\left[-\frac{(\Delta x)^2 (\Delta k)^2}{4} \right] \qquad (4.25)$$



Potential energy of an α-particle in the field of a nucleus
Fig 20.

---

where

$$\Delta k = k' - k .$$

We thus see that the wave intensity decreases with increasing Δk and becomes infinitesimal when $(\Delta k)^2 >> 1/(\Delta x)^2$. Hence the uncertainties Δk and Δx are related by

$$|\Delta k| \cdot |\Delta x| \approx 1,$$

which was to be proved.

4.4 Penetration of a particle through a potential barrier

The decay of the radium -226 nucleus is accompanied by the emission of α-particles with an energy of $\mathcal{E} = 4.8\,\text{MeV}$[3] . We know that the α-particle is attracted to the nucleus by nuclear forces and is repelled by the electrostatic interaction (the α-particle and the nucleus being both positively charged).

Nuclear forces are much stronger than electrical ones, but their range of action is much shorter, i.e., of the order of 10^{-13} cm. For distances $r < 10^{-13}$ cm, the α particle is attracted to the nucleus, whereas for $r >> 10^{-13}$ cm it is repelled. The potential energy of the α-particle as a function of the distance r from the nucleus is thus of the form shown in Fig.20. The height of the potential barrier is of the order of 30 MeV. According to classical mechanics, a 4.8-MeV particle cannot cross a potential barrier of this height, since the kinetic energy inside the barrier would then be negative, which is impossible.Penetration of particles through potential barrier contains no contradiction from the point of view of quantum mechanics. Indeed, kinetic energy of particle K is determined by its momentum

$$K = \frac{p^2}{2m} \tag{4.26}$$

As to potential energy $V(\vec{r})$, it is determined by coordinates. If the particle has definite momentum, its coordinates, and thus its potential energy, is fully uncertain. Therefore, according to quantum mechanics, particles can penetrate potential barrier.

---

[3] MeV is the abbreviation for million electron volt $\left(1.6 \cdot 10^{-13} J\right)$ and is commonly used in nuclear physics.

Chapter 5

Genuine versus superficial randomness

Passenger: "What chaos! There are three pairs of clocks in this station and they all show different time".

Stationmaster: "What would be the sense of having three pairs of clocks in the same station if they all showed the same time?"

5.1 Randomness in classical physics

The unification of the two opposites-waves and particles-is possible because quantum mechanics describes not the established, but the potential, state of micro-objects. In other words, quantum mechanics contains the elements of randomness (i.e., it is statistical in character).

Random processes are described by the theory of probability. Before we consider such processes in quantum mechanics, we examine the more usual question of randomness in classical physics. We shall place particular emphasis on questions that are of special relevance to quantum mechanics, but are usually inadequately explored.

The possibility of a random event A is characterized by the probability $p(A)$ defined in the following way. When the total number N of trial is sufficiently large (more precisely, when $N \to \infty$), the ratio of the number of trials M in which A occurs to the total number of trials is given by

$$p(A) = \frac{M}{N}. \tag{5.1}$$

For example, suppose a factory has produced 10.000 radio components (N=10000) and 300 of them are rejected as faulty (M=300). The probability of a faulty component is then

$$p = \frac{300}{10000} = 0.03. \tag{5.2}$$

It may be expected that a batch of 20000 components will then contain 600 faulty ones.

If in a certain problem all events can be represented by a combination of equally possible events, then the probability can be calculated theoretically. The probability of an event A will then be

$$P(A) = \frac{m}{n}, \tag{5.3}$$

where n is the total number of equally possible events and m is the number of equally possible events in which A occurs.

For example, let us determine the probability that by throwing dice we obtain at least 5. We then have n=6 (the dice has six faces) and m=2 (the acceptable outcomes are 5 or 6). Hence

$$p = \frac{2}{6} = \frac{1}{3}. \tag{5.4}$$

Similarly, the probability of getting a head in a coin tossing session is

$$n = 2, \; m = 1, \; p = \frac{1}{2}$$

We note two special cases of (5.3). The first is the impossible event m=0 in which case p=0; the second is m=n (certainty) for which p=1. In general

$$0 \le m \le n \tag{5.5}$$

i.e.

$$0 \le p \le 1 \tag{5.6}$$

Thus, the probability of any event is nonnegative and does not exceed unity. This necessary condition is not satisfied by Wigner's hidden variables model (Section 9.7) in which certain values of hidden parameters have negative probability.

We have already discussed the probability of different events, but there were only two possibilities: the event either took place or it did hot.

We now turn to the probability of different values of a continuous variable, i.e., a quantity that can assume an infinite number of values. For example, let us consider the coordinates x of a point reached by an electron. We can define the probability density $f(x)$ such that $f(x)dx$ is the probability that the electron will fall into the interval $[x, x+dx]$. We note that the probability density can exceed unity, but cannot be negative.

Similarly, we can introduce a probability density in three-dimensional space: $f(\vec{r})d\vec{r}$ is the probability that a particle found near the point $\vec{r}$ will be in an infinitesimal volume $d\vec{r}$.

We emphasize that the probability density $f(\vec{r})$ is an objective characteristic of the classical particle, but is not a field. It describes the potential possibility that the particle will be found in a particular part of space, but the probability is not a form of matter.

The probability density contains t as a parameter. The time rate of change of the probability density is given by the transport equation

$$\frac{\partial f(\vec{r},t)}{\partial t} = \hat{K} f(\vec{r},t), \tag{5.7}$$

where $\hat{K}$ is an operator. For example, if $\hat{K}$ is the Laplace operator, the transport equation takes the form

$$\frac{\partial f}{\partial t} = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial f}{\partial z^2} \tag{5.8}$$

We emphasize that transport equation (5.7) describes an objective process that is independent of the state of our knowledge.

We note that the transport equation is a determinate equation, i.e., nonrandom, although it describes the evolution of a random process. This is so because randomness is the absence of regularity. On the other hand, mathematics is concerned with regularities and can operate with random variables only symbolically. For example, $Y=2X$. To obtain a result that could be compared with experiment, we must translate randomness into a deterministic language. A random event is then described by a deterministic number, i.e., its probability. A random quantity, on the other hand, is described by a determinate function, namely, the probability density. The evolution of a random variable is then described by a deterministic equation, i.e., the transport equation, given by (5.7).

We now turn to the description of two particles. We begin with determinate particles, i.e., with the case where the coordinates $\vec{r}_1$ and $\vec{r}_2$ are known precisely. When the particles do not interact, each of them travels independently in the same three-dimensional space.

On the other hand, if the particles do interact, then knowledge of the three-dimensional vector $\vec{r}_1$ will not be enough to enable us to determine the motion of the particle 1: we must also know the position $\vec{r}_2$ of the other particle. Hence the state of two interacting particles is described by the vector $(x_1, y_1, z_1, x_2, y_2, z_2)$ in six-dimensional space. This space is just a real as the familiar three-dimensional space.

Next, consider two particles with random coordinates. If the particles are mutually independent, the state of each of them is described by the probability density $f(\bar{r}) = f(x,y,z)$. On the other hand, if the probability of finding the particle in a certain volume depends on the position of the other particle, the probability density depends not on three but on six coordinates, i.e., $(x_1, y_1, z_1, x_2, y_2, z_2)$. This is the "multidimensional coordinate space" which Fock associated with "real physical space" (cf. Introduction).

In the discussion given below, we shall examine complex events. Let us suppose that, in an event C, at least one of two events A and B takes place. This complex event is called the sum of the two simple events and is denoted by

$$C = A + B \tag{5.9}$$

It is shown in probability theory that incompatible[4] events satisfy the following law of composition of probabilities:

$$p(A+B) = p(A) + p(B) \tag{5.10}$$

Let us now consider two types of randomness in classical physics. The fundamental laws of classical physics are determinate in character and randomness occurs for two reasons.

The first reason is: *uncontrollable interaction*, (including *uncontrollable initial state*). If we toss a coin, we get heads in some cases and tails in other cases. Here randomness arises because we give the coin different initial translational and angular velocities in each case.

The second reason is: *hidden parameters*. There is a certain probability that a particular person is color blind. There is nothing random about this: the retina simply has a congenital defect; the

[4] Two events are incompatible if occurrence of one excludes the occurrence of the other.

randomness is merely apparent. However, the defect is hidden from us and the randomness is actually a hidden regularity.

The question is: how can we distinguish between uncontrollable interaction and hidden parameters? The answer will depend on the outcome of repeated trials.

If we toss a certain number of coins and select those that show tails, then a repeated tossing of the chosen coins will produce a similarly random result, i.e., we again obtain heads or tails.

However, it we select people who are color blind and perform the examination again on the chosen set of people, we will again find that they are all color blind.

There are two approaches to probability: objective and subjective. The former was discussed above: the probability is the fraction of events in which we are interested among the total number of events.

However, the subjective approach is quite common and is concerned with "our degree of confidence". If this approach were to be correct, probability would only be used in logic, but not in physics in which we deal with objective processes that do not depend on whether the observer is confident about them or not.

5.2 Conditional probability

The concept of conditional probability plays an important part in our understanding of quantum mechanics. We shall illustrate this by considering again the example of a component in a radio set.

Suppose that the reject probability for components of a new design is not 0.03, but 0.01. It is then clear that we have to distinguish between two probabilities, namely, unconditional probability (p=0.03) and conditional probability (p=0.01). The conditional probability is relevant under certain conditions: in this example, the condition is that a new design is used. In other words, 0.03 is the fraction of faulty components among all

components, when 0.01-such fraction among components of new design.

The concept of conditional probability is often used as the basis for the subjective approach to probability, regarded as measure of our confidence. If we do not know the design of the radio components, then the supporters of the subjective approach would say that the reject probability is 0.03. On the other hand, if we know that we are dealing with a component of new design, the reject probability falls to 0.01. However, our knowledge is of little significance. Different probabilities were obtained not because we knew or did not know, but because we considered different sets of radio components ([58], p.10). In the above example, in the first case the batch consisted of components of different design, both old and new, whereas in the second case we had components of new design alone.

Despite the fact that the probability is an objective characteristic of the event, its dependence on the prevailing conditions introduces a subjective element into this concept, i.e., the selection of events satisfying particular conditions depends on the person making the selection.

"It was once decided to determine the average size of a family by asking people how many children their parents had. It is clear that this could not yield a true average because childless families were automatically excluded" ([43], p.355).

It is essential in each particular case to analyze the conditions under which the probabilities are obtained. Probabilities can depend on the prevailing conditions, on position, and on time. In the example of radio components, the reject probability can depend not only on the design, but also on other and often unexpected conditions. It may be found that the reject probability for components manufactured in Moscow and Kharkov is different. It may also be different for components manufactured at the end and at the beginning of a week.

We now turn to the delicate question of reduction of probability, which is important for the understanding of quantum mechanics.

Let us consider coin tossing again. The experiment is carried out in three different stages:

1. The coin has not been tossed; the probability of getting the tail is $\frac{1}{2}$;

2. The coin has been tossed; the result is the tail, but we have not looked at the coin and therefore believe that the probability of the tail is $\frac{1}{2}$, as before;

3. We have seen that the result is the tail; we can now usefully exploit this information to improve our knowledge of the state of the coin, since we are now sure that we have the tail and therefore the probability of this event has become equal to unity.

The transition from the second to the third stages, i.e., the transition from a definite, but unknown, state to a known state can be referred to by analogy with quantum mechanics as a reduction of probability.

The reduction of probability does not correspond to any objective process: it is a purely logical operation whereby we cross out probability and replace it with certainty.

Because of the reduction of probability we can say that "there are events in the physical world that cannot be regarded as occurring in space and time" ([59], p.276).

This also happens in the case of the reduction of a wave packet in quantum mechanics (see sec.6.4). However, because quantum-mechanical concepts are complex and unfamiliar, this process is sometimes treated in a mystical spirit.

5.3 Probabilistic interpretation of quantum mechanics

"Even if an atomic object is under fixed external conditions, the result of its interaction with an instrument is not in general unambiguous. This result cannot be foretold for certain using previous observations no matter how they were exact. Only the probability of the result is definite. The most complete expression of the results of a series of measurements is not the accurate value of the measured quantity, but the probability distribution obtained for it" ([11], p.467).

Randomness does occur in classical physics, but in quantum mechanics it has a totally different status: "Whilst all the great classical minds from Laplace to Poincare have always proclaimed that natural phenomena are always determinate and that probability, when it is introduced into scientific theories is a consequence of our lack of knowledge or our inability to understand the entire complexity of determinate phenomena, the situation in the currently accepted interpretation of quantum physics is that we are dealing with "pure probability", that does not appear to be a consequence of hidden determinism. In classical theories such as the kinetic theory of gases, probabilistic laws have regarded as a consequence of our lack of knowledge of the completely determinate, but disordered and complicated, motions of countless molecules of a gas: if we knew the positions and velocities of all the molecules then, in principle we could predict precisely the evolution of a gas. However, in practice, we do not know these hidden parameters and have to introduce probabilities. The pure probabilistic interpretation of wave mechanics rejects this interpretation of probabilistic laws" ([60], p.25).

The probabilistic laws of quantum mechanics are not due to our ignorance about some hidden parameters: there are in fact no such parameters (see sec. 12.3). Randomness in quantum mechanics is one of its postulates. "... the concept of probability is a primary concept in quantum physics in which it plays a

fundamental role. The quantum-mechanical concept of the state of an object is closely related to it" ([11], p.468).

The notion of probability is revival of a notion of possibility (potentia) in the Aristotle's philosophy: "... it is in some sense a transformation of old notion "potentia" from a qualitative notion into quantitative one" ([61], p.24). More detailed discussion about this see in ([62]; [166]).

The state of a quantum object is characterized by its wave function $\psi(\vec{r})$ which is not a determinate field, but a probability field. The probability dw of finding a particle near a point x, y, z in an infinitesimal parallelepiped with edges dx, dy, dz is proportional not to the function $\psi(x,y,z)$ but to the square of its modulus

$$dw = |\psi(x,y,z)|^2 dxdydz. \qquad (5.11)$$

In this discussion, we are treating the micro-object as a particle. To emphasize that the micro-object has wave properties as well, the function ψ is often referred to as the wave function.

"... the wave function of a particle describes the possibility of a subsequent observation" ([4], p.45).

For example, when an electron is in a state with a particular momentum $\vec{p}$, it is described by the following wave function (in the coordinate representation):

$$\psi = exp\left(\frac{i\vec{p}\vec{r}}{\hbar}\right). \qquad (5.12)$$

The momentum of the electron is precisely determined and is equal to the vector $\vec{p}$. On the other hand, the coordinates are completely indeterminate and can have any value with equal probability. The wave function (5.12) describes spatially infinite wave that has the same intensity at all points in space.

On the other hand, in a state with particular position vector $\vec{r}_0$, the electron is described by the wave function.

$$\psi = \delta(\vec{r} - \vec{r}_0) \tag{5.13}$$

The position of the electron is then determined precisely and is given by the vector $\vec{r}_0$ but its momentum is then totally undetermined and can assume any value with equal probability.

We now return to the unperturbed electron in the hydrogen atom. Its state is described by a wave function, which in terms of the polar coordinates r, ϑ, φ is

$$\psi(r,\vartheta,\varphi) = \frac{1}{(\pi a^3)^{1/2}} exp\left(-r/a\right) \tag{5.14}$$

where a is the Bohr radius, given by

$$a = \hbar^2/me^2 \tag{5.15}$$

The wave function (5.14) is independent of ϑ and φ, which means hat we are dealing with an isotropic situation. In this state, the coordinates of the electron are indeterminate.

The probability dw that the electron is in a cell $(r, r++dr),(\vartheta,\vartheta+d\vartheta),(\varphi,\varphi+d\varphi)$ is

$$dw = |\psi|^2 r^2 \sin\vartheta dr d\vartheta d\varphi = \frac{1}{\pi a^3} exp\left(\frac{-2r}{a}\right) r^2 \sin\vartheta dr d\vartheta d\varphi \tag{5.16}$$

The magnitude of the momentum in the state described by (5.14) is also indeterminate, but we shall not reproduce the expression for the probability of the different values of the momentum. As far as the energy ε of the electron is concerned, it is given by the following expression in the state described by the wave function (5.14)

$$\varepsilon = -\frac{me^4}{2\hbar^2} \tag{5.17}$$

We see, that in quantum mechanics total energy is in some cases strictly determined in spite of the fact that velocity and coordinate of particle are random.

"Understanding that angle of scattering is statistical, but conservation laws are not statistical, came not at once. In 1924 Bohr, Kramers and Slater proposed the theory, in which the law of conservation of energy and the law of conservation of momentum do not take place, are violated in individual acts of scattering. These laws take place in macroscopic physics only due to averaging over great number of elementary acts" ([63], p.162).

The evolution of the wave function in time is described by a determinate equation similar to the transport equation given by (5.7):

$$\frac{\partial \psi}{\partial t} = \frac{i}{\hbar}\hat{H}\psi \tag{5.18}$$

where $\hat{H}$ is the Hamilton operator obtained from the expression for the energy in which momentum is replaced with the differentiation operator

$$\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial \vec{r}} \tag{5.19}$$

The relation given by (5.18) is called the Schrödinger equation.

Sometimes the question arises: what a wave function characterizes-individual particle or ensemble of particles? We answer this question firstly in classical physics. Let the probability of occurring a head in tossing a coin equals to 1/2. What this probability characterizes: the individual coin or the ensemble of coins? It characterizes both the individual coin and the ensemble.

Probability 1/2 characterizes the individual coin in the sense that the coin is a symmetrical one. The same probability 1/2 characterizes great number of tosses in the sense that the head occurs in half of cases.

Analogously the wave function characterizes an individual particle, but the probability $|\psi|^2$ takes place only in great number of experiments [62].

5.4 A speck of dust at the back of beyond

We have written randomness in classical physics to be superficial. It is truth but it is not all truth.

Classical physics is deterministic if we consider macroscopic bodies. On the other hand, the motion of microscopic particles is deterministic only for isolated system.

It seems the system containing many particles to be isolated because external influence is a surface effect which may be neglected [64]. However gravity is a volume effect. This effect is fantastically small, but owing to the exponential instability of microscopic particle trajectories the gravity destroys the microisolation, while the macroisolation being conserved. Therefore description of microparticles must be statistical even in the frame of classical physics.

To illustrate this statement we give an example which belongs to Borel ([65], p.125).

We consider a gas under normal conditions (thermal speed of molecules is of order $10^3 m/sec$; the collision frequency ν is near $10^9\ sec^{-1}$; the ratio of mean free path l to the effective radius of molecule equals to 10).

Let a speck of dust with radius one micron to be placed one billion years ago at the distance of one billion light years. Owing to the gravitational attraction between gas molecules and the speck the trajectories of the gas molecules will change. The problem is:

after what elapse of time the gas molecules will totally change their motion? In other words after what time interval molecules velocity vectors will turn on angles of order 90^0 and thus the molecules will collide not with those ones as it would be without the perturbation?

The answer is quite striking: the elapse of time sought for equals to $10^{-6}\,sec!$ Therefore microscopically isolated systems exist almost never. This point of view while not generally accepted was expressed by a number of authors [66]-[72].

Turning to the proof of absence of isolation we use the law of gravitation. The force acting on the molecule owing to attraction to the speck is

$$f = \gamma \frac{\mu_1 \mu_2}{r^2}. \tag{5.20}$$

Here γ is the gravitational constant, $\gamma \cong 6{,}7 \cdot 10^{-11} m^3 / kg \cdot sec^2$, μ_1 is the speck mass, for the speck of size 1 micron and the mass density $10^3 kg/m^3$; $\mu_1 \cong 10^{-15} kg$; μ_2 is the mass of the molecule, r is the distance between them.

The acceleration α experienced by the molecule equals

$$\alpha = \frac{f}{\mu_2} \tag{5.21}$$

During the time of free flight τ between the subsequent collisions the molecule moves the distance

$$S_0 = \upsilon \cdot \tau, \tag{5.22}$$

where υ is the mean thermal speed of the molecule.

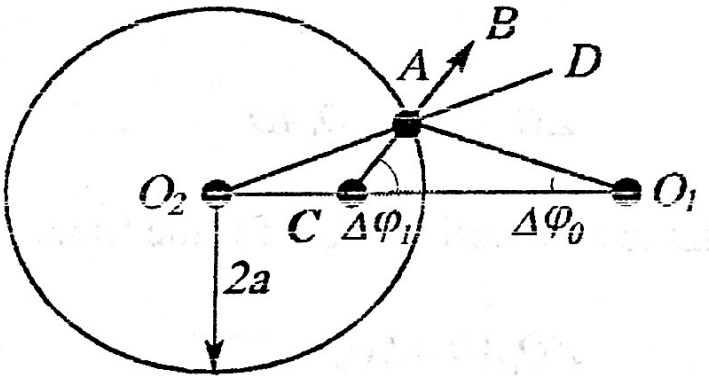
On the other hand the molecule deflects the distance S owing to gravitational attraction by the speck

$$S = \frac{\alpha\tau^2}{2} \tag{5.23}$$

The deviation angle $\Delta\varphi_0$ equals

$$\Delta\varphi_0 = \frac{S}{S_0} \tag{5.24}$$

Substituting in the formulae (5.21)-(5.24) $\tau = 10^{-9}\,sec$, $\upsilon = 10^3\,m/sec$, we get fantastically small value



The collision of two balls
Fig 21.

$$\Delta\varphi_0 = 10^{-600} \tag{5.25}$$

Now we determine the increase of the deflection angle $\Delta\varphi$ due to the collision between molecules of the gas. We treat the molecules as elastic balls. The collision of two balls of radii a is equivalent to the collision of zero radius ball with ball of radius $2a$. We refer to the figure 21. Let O_1 and O_2 be centers of the balls 1 and 2 respectively. We consider two trajectories of the ball 1.Let undisturbed trajectory be the line O_1O_2. The disturbed trajectory is the broken line O_1AB. The angle between both trajectories before

the collision is $\Delta\varphi_0 = \angle AO_1O_2$. The angle between those trajectories after collision is $\Delta\varphi_1 = \angle ACO_1$.

We will determine the ratio $\Delta\varphi_1 / \Delta\varphi_0$ i.e., the increase of deflection after one collision. We will neglect the term a relative to l $\left(a/l << 1\right)$. In the figure 21 $O_1O_2 = l$ and $O_2A = 2a$.

Assuming the collisions to be elastic we note that angle of fall equals to angle of reflection:

$$\angle O_1AD = \angle DAB \tag{5.26}$$

Therefore

$$\angle O_1AB = 2\angle O_1AD \tag{5.27}$$

Now $\angle O_1AD$ is an exterior angle for the triangle O_1O_2A:

$$\angle O_1AD = \Delta\varphi_0 + \angle O_1O_2A \tag{5.28}$$

Similarly $\angle O_1AB$ is the exterior angle for the triangle O_1CA:

$$\angle O_1AB = \Delta\varphi_1 + \Delta\varphi_0 \tag{5.29}$$

Further from the triangle O_1O_2A we find using the theorem of sinuses:

$$\frac{2a}{\sin\Delta\varphi_0} = \frac{O_1A}{\sin\angle O_1O_2A} \tag{5.30}$$

As we are interested only in the order of magnitude we may substitute $\sin\alpha$ by α.

Noting that $O_1A \approx l$, we have

$$\angle O_1 O_2 A = \frac{l}{2a}\Delta\varphi_0 \qquad (5.31)$$

Then combining the formulae (5.27), (5.28), (5.29), (5.31) and neglecting small terms we find the ratio sought for:

$$\frac{\Delta\varphi_1}{\Delta\varphi_0} = \frac{l}{a} \qquad (5.32)$$

In our example $l/a = 10$, so after n collision the angular spread $\Delta\varphi_n$ will increase in 10^n times

$$\Delta\varphi_n = \Delta\varphi_0 10^n \qquad (5.33)$$

During on microsecond the molecule suffers 1000 collisions, so

$$\Delta\varphi_{1000} = 10^{1000}\Delta\varphi_0 \qquad (5.34)$$

The number 10^{1000} is a tremendous number. It outweights the small value $\Delta\varphi_0 = 10^{-600}$. Thus after one microsecond the undisturbed state of the system is completely destroyed. "So the conception of a gas mass as a single model consisting of molecules, positions and velocities of which are strictly defined, is a pure abstract fiction. We may approach reality nearer only by considering a bundle of models, i.e. by diving to the initial data some indeterminacy" ([65], p. 124).

5.5 Zero uncertainty has zero probability in classical kinetics also

We saw in the preceding paragraph that isolated systems do not exist in classical kinetics. It is the reason to introduce some small uncertainty.

However, besides this external reason to introduce the uncertainty there is an internal reason too. Namely, without some initial uncertainty physical system does not exhibit any law of evolution. For example, in strictly deterministic system heat does not flow from a hot body to the cold one.

Indeed, the state of a dynamical system is described by a d-dimensional vector $\vec{x}$:

$$\vec{x} = (x_1, x_2, ..., x_d) \tag{5.35}$$

If there is no initial uncertainty, vector $\vec{x}$ changes erratically with the time and there are no laws of kinetics. In other words, the singular initial distribution

$$f(\vec{x}) = \delta(\vec{x} - \vec{x}_0), \tag{5.36}$$

which corresponds to the exactly determined initial state tends to nothing as $t \to \infty$.

In the ergodic theory that is the basis of classical kinetics the initial distribution is different from zero in some initial region Γ_i(initial indeterminacy).

In order to avoid pathological situations in statistical considerations (the ergodic theory) one neglects the sets of measure zero, i.e. one neglects isolated points in the phase space. In other words, one considers almost all sets in the phase space. In formulation of ergodic theory theorems there are words "almost everywhere" [73].

"Mathematicians who do not like the speculations in which the expressions "almost all" and "neglecting sets of zero measure" occur, may be objected that this is the only way to mathematical interpretation what "as a rule" takes place in the nature" [74].

Without initial uncertainty classical mechanics is possible, but kinetics and thermodynamics cannot exist.

Chapter 6

Reduction of a wave packet

Police Inspector Ochumelov crossed the marketplace... he saw the... individual..., who stood there with his right hand raised, displaying a bleeding finger... Ochumelov recognized in this individual Khryukin, the goldsmith.

In the very middle of the crowd sat... a white pup... "I was walking along, Your Honour",... began Khryukin... "and suddenly, for no reason whatever, that nuisance bit my finger"...

"H'm... well, well," said Ochumelov... I shan't leave it at this. I'll teach people to let dogs run about!

...And the dog must be exterminated... whose dog is it?"

"I think it belongs to General Zhigalov", said a voice from the crowd.

"General Zhigalov! H'm... One thing I don't understand-how did it happen to bite you? How could it have got at your finger?... it's a nice little doggie! Snap at his finger? Ha-ha-ha!"

Anton Chehov. "Khameleon".

6.1 Catastrophe in the micro-world

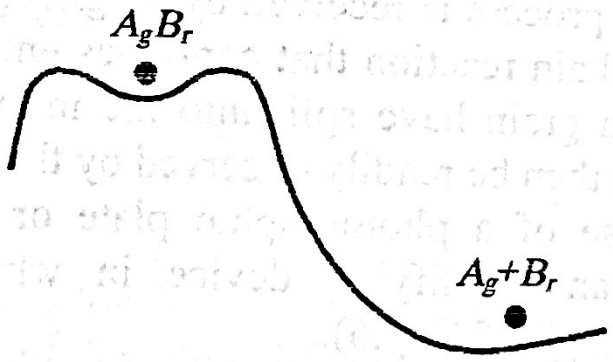
The fact that the momentum and the position of an electron cannot be measured, simultaneously with great precision is often said to be surprising. However, it is even more surprising that we can measure the position or momentum of an individual electron

separatcly with coarse macroscopic devices whose mass exceeds the mass of the electron by a factor of 10^{26}.

"...The macroscopic measuring device should be an *unstable system* (or more precisely an almost unstable system). It is only then that a micro-particle can change its state, and it is this change that is a macroscopic phenomenon. A micro-particle cannot affect an instrument in the form of a stable macroscopic system. It cannot "displace" its "pointer" from its zero position" ([5], p.120).

In other words, a measurement performed on a micro-object by a macroscopic device is a "catastrophe in the micro-world". It is precisely such catastrophes that enable us to perform measurements on individual micro-objects.

For example, let us consider how a Geiger counter records the position of an electron. The counter is a capacitor in which the space between the electrodes is filled



Schematic representation of a weakly-stable state of the silver bromide molecule

Fig 22.

---

with air. The voltage between the electrodes is low enough to avoid breakdown, but high enough to accelerate an electron to an energy that enables it to ionize the atoms of air by collision. This releases a number of electrons that are in turn accelerated by electric field in the capacitor and thus produce further ionization.

The result is a growing avalanche of free electrons. It causes electrical breakdown, which is readily recorded.

Another example is the detection of an electron by a photographic plate. The photographic emulsion contains silver bromide molecules. The state of the AgBr molecule is shown schematically in Fig.22 in which, for the sake of illustration, we have replaced the chemical bonding force by the more familiar gravitational one. The state of the molecule is represented by a ball rolling on the smooth surface. Gravity pulls the ball down and its state becomes stable when it reaches the bottom of the well. The potential, in which AgBr molecule finds itself, consists of a very shallow well and, next to it, a very deep well that corresponds to the slit of the molecule into the individual atoms of silver and bromine. Silver bromide is therefore stable, but it can be split by supplying to it a relatively small amount of energy. The energy released in this process is received by neighboring molecules, and the result is a chain reaction that continues until all the molecules in the emulsion grain have split into the individual atoms. Thus black grain can then be readily observed by the unaided eye.

"In the case of a photographic plate or a counter, we are dealing with an amplifying device in which avalanche-type processes develop" ([54], p.6).

"Measurements" in a nuclear reactor have a somewhat different character. The fission of the uranium-235 nucleus releases a few neutrons which results in a chain reaction. However, some of the neutrons are absorbed by other nuclei or leave the reactor. The relative number of neutrons captured by uranium-235 nuclei can be increased by exploiting the phenomenon of resonance. When it captures a neutron, the uranim-235 nucleus is raised to an excited state (before fission takes place) with low excitation energy ε_0. According to Planck's formula (3.1) the frequency corresponding to this energy is $\omega_0 = \varepsilon_0 / \hbar$.

Neutrons released in fission have high energy $\varepsilon >> \varepsilon_0$ which corresponds to a high frequency $\omega = (\varepsilon/\hbar >> \omega_0)$. If $\omega \approx \omega_0$ the resonance takes place and the neutrons are rapidly captured by the uranium-235 nuclei, which in turn leads to the fission of these nuclei. It means that for the chain reaction to take place a "moderator" must be introduced into the reactor. It is usually graphite which does not absorb neutrons, but does slow them down. Neutrons colliding with graphite nuclei lose some of their energy ε, so that the frequency ω decreases and the condition of resonance becomes fulfilled.

We note that, in this particular "measurement", the "macro-instrument" is a micro-object, namely, the uranium-235 nucleus. The quantity that is being measured is the neutron energy. Since the neutrons are located at large distances from the uranium-235 nucleus, their potential energy is zero, and their total energy is equal to their kinetic energy

$$\varepsilon = \frac{p^2}{2M} \tag{6.1}$$

where M is the neutron mass.

The magnitude of the momentum p of the neutron is thus accurately determined, so position remains undetermined. It is precisely this uncertainty in position coordinates enables the uranium-235 nucleus to interact with a large number neutrons at once.

In classical physics, measurement or observation do not usually affect the state of the object being examined. On the contrary, in quantum mechanics, measurement or observation of a micro-object is accompanied by the destruction of its previous state. For example, a Nicol prism is used to determine the polarization of light by allowing light to pass through it. Only those photons emerge from the prism for which the polarization

vector lies along a particular direction. Photons whose polarization vector is perpendicular to this direction are absorbed.

A more "humane" method of observation is the determination of the position of an electron with a Geiger counter. In this measurement, an initial electron with accurately known momentum, and thus unknown position, undergoes a transition to a different state. In this final state the coordinates of electron x, y at right angles to the direction of its motion have small uncertainty $\Delta x, \Delta y$ of the order of the transverse dimensions of the counter. Heisenberg's uncertainty relations then show that the uncertainties in the transverse components of the momentum $\Delta p_x, \Delta p_y$ arise.

We see that before measurement Δp_x and Δp_y were equal to zero while x and y were uncertain. After measuring x and y are definite while p_x and p_y become uncertain. "By suitably choosing a particular method of observation, we actually decide which properties of nature will be determined and which will be erased in the course of our observation. This distinguishes the smallest particles of matter from the range in which our sensory perception operates" ([75], p.68).

6.2 Superposition of states

We shall need in notions "superposition", "mixture", and "reduction of the wave packet" for more detail description of measurement process.

Quantum mechanics is "... a new set of accurate laws of nature ...One of the most fundamental and most drastic of there is the *Principle of Superposition of States*" ([76], p.4).

We shall illustrate the superposition principle by an example. Suppose that a beam of electrons is incident on a screen

containing two slits[5]. The state of the electron behind the screen will be described by the wave function ψ.

We now cover slit 2 so that only slit 1 is open. The electrons then pass through the slit 1 alone. Let the state of an electron in this case be denoted by ψ_1 and the state when slit 2 is open, but slit 1 is closed, by ψ_2. The principle of superposition then states that the wave function ψ is a linear combination ψ_1 and ψ_2:

$$\psi = c_1\psi_1 + c_2\psi_2 \tag{6.2}$$

where c_1 and c_2 are constants. This linear combination is called a superposition or a wave packet (to distinguish superposition from a mixture, we also refer to it as a "pure state"). The principle of superposition is a consequence of the linearity of the Shrödinger equation.

The quantity

$$A(C_1) = c_1\psi_1(\vec{r}) \tag{6.3}$$

is the probability amplitude that an electron which has crossed the first slit will reach a given point $\vec{r}$ on the photographic plate placed beyond the screen (event C_1). The corresponding probability will be denoted by $P(C_1)$.

One of the postulates of quantum mechanics is that probability is measured by the square of the modulus of the amplitude:

$$P(C_1) = |A(C_1)|^2 \tag{6.4}$$

Similarly, for the second slit

$$A(C_2) = c_2\psi_2(\vec{r}) \tag{6.5}$$

[5] The distance between the slits does not exceed in order of magnitude the wavelength of the electron λ given by (3.5).

and

$$P(C_2)=(|A(C_2)|)^2 \tag{6.6}$$

Let us now consider the composite event C_1+C_2 in which an electron reaches a given point on the photographic plate if both slits are open. The probability amplitude for this event will be denoted by $A(C_1+C_2)$. It is clear that the amplitude $A(C_1+C_2)$ is equal to the wave function

$$A(C_1+C_2)=\psi \tag{6.7}$$

From (6.2), (6.3) and (6.5) we then find that

$$A(C_1+C_2)=A(C_1)+A(C_2) \tag{6.8}$$

This means that the amplitude for the sum of the events is equal to the sum of their amplitudes.

The probability of the composite event C_1+C_2 will be denoted by $P(C_1+C_2)$. According to (6.4) and (6.8), we have

$$\begin{aligned} P(C_1+C_2)&=|A(C_1+C_2)|^2=|A(C_1)+A(C_2)|^2= \\ &=P(C_1)+P(C_2)+A(C_1)A^*(C_2)+A^*(C_1)A(C_2) \end{aligned} \tag{6.9}$$

We see that interference takes place.

Therefore, the superposition of two events results in the addition of the probability amplitudes, but not of the probability themselves.

We note that the representation of the wave function ψ by the superposition (6.2) is natural, but not unique. For example, instead of the function ψ_1 and ψ_2 in (6.2) we can take their linear combinations

$$\psi_1' = \frac{\psi_1 + \psi_2}{\sqrt{2}}, \psi_2' = \frac{\psi_2 - \psi_1}{\sqrt{2}} \tag{6.10}$$

The formula given by (6.2) then takes the form

$$\psi = c_1'\psi_1' + c_2'\psi_2' \tag{6.11}$$

where

$$c_1' = \frac{c_1 + c_2}{\sqrt{2}}, c_2' = \frac{c_2 - c_1}{\sqrt{2}} \tag{6.12}$$

For better understanding of the notion of "superposition" we avail ourselves of an analogy.

Let us pour into a glass 100 *g* of water and then pour 100 *g* once more. As result both portions of water lose their individuality inter-flowing into a single portion of 200*g* of water:

$$200=100+100 \tag{6.13}$$

We could obtain 200 *g*. of water by another way, pouring into the glass firstly 150 *g* and then some more 50 *g*. of water:

$$200=150+50 \tag{6.14}$$

The distinction between two expressions (6.2) and (6.10) for superpositions has the same meaning as the difference between the equations (6.13) and (6.14)

We have already noted that quantum-mechanical randomness arises when we try to find something that does not exist. In the above example, randomness arises because we try to determine which particular slit was traversed by the electron whereas the state ψ in (6.2) describes the passage of an electron through both slits.

paragraph. The probability that the state of an electron is described by ψ_1 is

$$P_1 = |c_1|^2 \tag{6.15}$$

and, similarly, the probability of the state ψ_2 is

$$P_1 = |c_1|^2 \tag{6.16}$$

The state of the electron is now no longer described by the single wave function, but requires two wave functions ψ_1 and ψ_2 and their probabilities P_1 and P_2. This type of states is called a mixed state (mixture of ψ_1 and ψ_2).

We note that, in contrast to the superposition of states, the decomposition of a wave function ψ into the two wave functions ψ_1 and ψ_2 in the case of a mixed state is unique, i.e., the basis functions ψ_1 and ψ_2 are the eigenfunctions of the operator $\hat{A}$ (see 7.5) that corresponds to the measured quantity a.

For a mixed state, the law of composition of probabilities is

$$P(C_1 + C_2) = P(C_1) + P(C_2), \tag{6.17}$$

i.e., there is no interference.

We note that the concepts of *superposition and mixed state* are not specifically quantum mechanical. They are also encountered in classical theory. In 2.2, we considered the water wave passing through two slits as a *superposition* of two waves. On the other hand, a stream of bullets crossing two slits is a mixture of the two currents emerging from the slits.

On the other hand, if we look for something that does exist, we find there is no randomness. In particular, it we measure the momentum of the electron in state (5.12), we obtain a perfectly definite value for p.

It can be shown that any superposition corresponds to a precise value of a particular physical quantity. This statement will be illustrated by an example in 7.5. Randomness arises only when we measure a physical quantity that does not have a particular value in a given state.

6.3 Mixture of states

To obtain a mixture of states, we place Geiger counter behind of each of the slits, so that we can detect electrons from each slit separately. As noted above, the operation of measurement or detection is not as innocent in quantum mechanics as it is in classical physics. Thus, in classical physics, we can record an event without affecting it appreciably. In quantum mechanics, on the other hand, the situation is totally different because the process of measurement is accompanied by a significant change in the state of the micro-system.

"The first step in measuring process consists of an external influence on a system. This influence is physically real and it changes course of events ... This influence leads to passing on the observable system to the ``mixture" of states" ([50], p.50).

The more detailed discussion on the problem of measurement in quantum mechanics see in [46, 77, 78, 79].

When an electron interacts with a Geiger counter, the unconditional probability described by the wave function ψ is replaced by a conditional probability. This is described mathematically by saying that the original wave function ψ no longer characterizes the state of the electron and is replaced by two new wave functions ψ_1 and ψ_2 defined in the last (Section)

6.4 Use of additional information

When a single electron is incident on the two slits, it is recorded by only one Geiger counter. The process of measuring (or, more strictly in this case, of observation) of electron passing though two slits, has three stages:

1) Electron state before interaction with the slits is described by a wave function, which corresponds to definite electron momentum.

2) The electron have passed through the slits. Apparatus have recorded passing of the electron through one of the slits. The state of the electron have changed. It is an objective physical process which occurs in definite place and lasts definite time. However, we have not looked at the apparatus yet. We cannot say which slit have been crossed. All we can do is to specify the probabilities P_1 and P_2 corresponding to the passage of the electron through the slit 1 or the slit 2, respectively. The previous state ψ of the electron is converted into the mixture of the two wave functions ψ_1 and ψ_2 with probabilities P_1 and P_2.

3) We have looked at the apparatus. Now the state of the electron is described by one of the wave function ψ_1 or ψ_2 which corresponds to crossing by the electron the slit 1 or 2, rather than by the mixture. When the observer recognizes that the counter behind of slit 1 has recorded the electron he knows that there is no point in describing the state of the electron by the mixture. Using the new information, he then replaces the unconditional probability by the conditional one. To describe this state of the electron, the observer therefore replaces the mixture of ψ_1 and ψ_2 with the single wave function ψ_1. This process is called a reduction of wave packet.

Heisenberg stated: "...the act of recording, on the other hand, which leads to the reduction of the state, is not a physical, but rather, so to say, a mathematical process. With the sudden change

of our knowledge also the mathematical presentation of our knowledge undergoes, of course, a sudden change" [80].

Thus the reduction of the wave packet is not a physical process that occurs in space and requires a certain interval of time for its completion. Wave packet reduction is a change in the method of description-a purely logical process [81].

Wave-packet reduction is a transition to conditional probability, as in the case of coin tossing. This transition, i.e. the recognition of the results of measurements, is "familiar even in classical theory" ([82], p.50).

Chapter 7

An abstract space

Answering the question of his younger brother: "What is algebra?", Albert Einstein said: "It is arithmetics for lazy persons".

7.1 Multidimensional space

Sometimes people see paradoxality of quantum mechanics in the fact that in the case of a complicated system, which contains many particles, the wave function depends on all degrees of freedom rather than on three space coordinates. It is a function of point "in multidimensional configuration space and not in the real physical space". At this point we merely mention that the real physical space is not always three-dimensional, even in classical physics, when randomness is absent. For example, a table tennis player must take into account not only the three coordinates of the ball, but also the three components of its translational velocity and three components of its angular velocity. Hence the "real physical space" involved in the game of table tennis is a nine-dimensional configuration space and not the familiar three-dimensional one. Unfamiliar does not mean unreal.

Besides, in the case of several particles "the real physical space" is three-dimensional only if there is no interaction between the particles. The space of n interacting particles is $3n$-dimensional.

If we take into account, that in classical physics the state of a particle is characterized not only by three coordinates, but also by three components of momentum, then the system, consisting of n particles, is represented by a point in $6n$-dimensional space.

Multidimensional space is no less real than atoms and molecules.

As far as the term "abstract" (as applied to space) is concerned, it by no means denotes "unreal", i.e. existing only inside a human head. There are many mathematical books devoted to abstract multidimensional space. These books imply many concrete physics applications of abstract spaces, but their authors restrict themselves only to their general properties, leaving aside the specific character of each particular space. Therefore the expression "the wave function is defined in a multidimensional abstract space" means only that the concept of the space of a wave function is a special case of a more general mathematical notion (Hilbert space).

7.2 Spin

The spin wave function is the simplest one. We shall often use it to illustrate different aspects of quantum mechanics.

Spin is a nondimensional quantity which is proportional to magnetic moment of a system.

A graphic, but very crude, notion of spin is an absolute value of angular velocity of a charged particle around its axis. (Magnetic moment arises only in the case, when the rotating particle is charged).

In classical physics the absolute value of the angular velocity of the particle can have any positive value from zero to infinity. According to this *projection* of the angular velocity on arbitrary axis can assume any real value from $-\infty$ to $+\infty$.

In quantum mechanics, on the other hand, spin can have only integral or hall-integral values. Each particle (or micro-system in a particular state) can have only one value of spin. For example, the spin of the electron is $S=1/2$. The helium atom has $S=0$ in the singlet state and $S=1$ in the triplet state. The spin of ^{7}Li is $S=3/2$.

The projection of spin onto a given axis, say, the z axis, can assume $2S+1$ possible values (the values are numbered in descending order):

$$S_z^{(1)} = S, S_z^{(2)} = (S-1), \ldots S_z^{(2S+1)} = -S \tag{7.1}$$

In particular, the z-component of the spin of the electron is either 1/2 or -1/2.

The spatial state of a particle was discussed in some details on the previous pages. It is described by the wave function $\psi(\vec{r})$. The probability of finding a particle near a point $\vec{r}$ in an infinitesimal volume $d\vec{r}$ was taken to be

$$|\psi(\vec{r})|^2 d\vec{r} \tag{7.2}$$

The spin state is described in a somewhat different way. Thus, the spin state of an electron is characterized by the two quantities ψ_1 and ψ_2, written in the form of column ψ, given by

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{7.3}$$

This column can be treated as the wave function ψ whose argument is not the position vector $\vec{r}$, but the spin index j that can assume two values, namely, 1 or 2.

The wave function ψ does not predict the value of S_z obtained by measurement. The above column merely gives the probability of different values of S_z, namely, the probability P_1 that measurement will yield $S_z = 1/2$ is

$$P_1 = |\psi_1|^2 \tag{7.4}$$

and the probability P_2 that measurement will yield $S_2 = -\frac{1}{2}$ is

$$P_2 = |\psi_2|^2 \tag{7.5}$$

Similarly, the state of a particle with spin S is characterized by a column consisting of $2S+1$ components

$$\psi = \begin{bmatrix} \psi_1 \\ \psi_2 \\ \vdots \\ \psi_{2S+1} \end{bmatrix} \tag{7.6}$$

The probability P_1 that measurement of S_z will yield S is

$$P_1 = |\psi_1|^2 \tag{7.7}$$

and the probability P_2 that the result will be $S_z = S-1$ is

$$P_2 = |\psi_2|^2 \tag{7.8}$$

and so on.

Since the particle must be in one the possible spin states defined by (7.1), the probabilities $|\psi_j|^2$ must satisfy the normalization condition

$$\sum_{j=1}^{2S+1} |\psi_j|^2 = 1 \tag{7.9}$$

7.3 Eigenvalues and eigenvectors

The mathematical formalism of quantum mechanics makes use of the concept of Hilbert space. This space is abstract, which

gives rise to difficulties in understanding of quantum mechanics. It is therefore useful to begin by illustrating this mathematical scheme by a simple model involving electrical conductivity in two-dimensional space.

We know that an electric field $\vec{E}$ applied to a medium produces a current density $\vec{j}$. These two quantities are related by Ohm's law

$$\vec{j} = \sigma \vec{E} \tag{7.10}$$

where σ is the electrical conductivity of the medium.

The conductivity σ is a number in the simple case where the properties of the medium are the same in all direction (isotropic medium). However, in crystals, the conductivity is a function of direction (anisotropy). The electric field E_1 pointing along the x_1 axis then produces not only a current j_1 along this axis, but also a current j_2 along another axis x_2 perpendicular to x_1. Similarly, a field E_2 pointing along the x_2 axis produces current components j_1 and j_2.

For small values of $\vec{E} = (E_1, E_2)$ the dependence of $\vec{j} = (j_1, j_2)$ on $\vec{E}$ is linear, i.e.

$$j_1 = \sigma_{11} E_1 + \sigma_{12} E_2 \tag{7.11}$$

$$j_2 = \sigma_{21} E_1 + \sigma_{22} E_2 \tag{7.12}$$

These two equations can be written in the compact form

$$\vec{j} = \hat{\sigma} \vec{E} \tag{7.13}$$

where $\hat{\sigma}$ is the conductivity operator acting on the vector $\vec{E}$ and is actually a matrix:

condition for a nontrivial solution is that the determinant of (7.17) must be zero:

$$\begin{vmatrix} \sigma_{11}-\delta & \sigma_{12} \\ \sigma_{21} & \sigma_{22}-\delta \end{vmatrix} = 0 \tag{7.18}$$

This second-order algebraic equation defines two eigenvalues δ_1 and δ_2. When $\delta = \delta_1$ or, $\delta = \delta_2$ one of the equations in (7.17) is a consequence of the other. For example, if we take the first equation and put $\delta = \delta_1$ we obtain the eigenvector $\vec{E}^{(1)} = \left(E_1^{(1)}, E_2^{(1)}\right)$:

$$E_1^{(1)} = \sigma_{12}, \quad E_2^{(1)} = \delta_1 - \sigma_{11} \tag{7.19}$$

The eigenvector $\vec{E}^{(1)}$ is defined to within an arbitrary factor c: if $\vec{E}^{(1)}$ is an eigenvector then $c\vec{E}^{(1)}$ is an eigenvector also.

Similarly, we can define a second eigenvector $\vec{E}^{(2)}$ corresponding to the eigenvalue δ_2.

In the above discussion, we have tacitly assumed that all the quantities were real. However. we often have to deal with an electric field that varies in accordance with the harmonic law

$$\vec{E} = \vec{E}_0 e^{-i\omega t}, \tag{7.20}$$

in which case $\vec{E}, \vec{j}$, and $\hat{\sigma}$ are all complex.

We note that, according to the well-known Onsager relations[83], we have.

$$\sigma_{12} = \sigma_{21}^*, \quad \sigma_{11} = \sigma_{11}^*, \quad \sigma_{22} = \sigma_{22}^*, \tag{7.21}$$

where the asterisk represents the complex conjugate. This type of matrix is called Hermitian (or self-adjoint).

$$\hat{\sigma} = \begin{pmatrix} \sigma_{11} & \sigma_{12} \\ \sigma_{21} & \sigma_{22} \end{pmatrix} \tag{7.14}$$

The two vectors $\vec{j}$ and $\vec{E}$ in (7.13) are column vectors:

$$\vec{j} = \begin{pmatrix} j_1 \\ j_2 \end{pmatrix}, \quad \vec{E} = \begin{pmatrix} E_1 \\ E_2 \end{pmatrix} \tag{7.15}$$

There are directions in a crystal in which the current is parallel to the electric field:

$$\vec{j} = \delta \vec{E} \tag{7.16}$$

where δ is a number and not a matrix. The vector $\vec{E}$ corresponding to this direction is called an eigenvector and the number δ is an eigenvalue of the operator $\hat{\sigma}$. The eigenvector and eigenvalue of the operator $\hat{\sigma}$ can be found from (7.11), (7.12) and (7.16):

$$\begin{aligned} (\sigma_{11} - \delta)E_1 + \sigma_{12}E_2 &= 0 \\ \sigma_{21}E_1 + (\sigma_{22} - \delta)E_2 &= 0. \end{aligned} \tag{7.17}$$

This set of two linear homogeneous equations always has a zero solution:

$$E_1 = E_2 = 0$$

This solution corresponds to the trivial case that, when there is no electric field, there is no current also.

On the other hand, the electrical conductivity describes the nontrivial solution for which there is a nonzero current. The

Vectors usually have three or more components. For example, the position of two particles with coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) is characterized by the six-dimensional vector $\vec{l} = (x_1, y_1, z_1, x_2, y_2, z_2)$. The set of such vectors is called a six-dimensional space. More complicated physical systems are described in a space of a larger number of dimensions.

The relations given by (7.11), (7.12), (7.16)-(7.18) can be directly generalized to the case of n-dimensional space:

$$m_i = \sigma_{ij} l_j \tag{7.22}$$

$$\vec{m} = \hat{\sigma}\vec{l} \tag{7.23}$$

$$\left(\hat{\sigma} - \delta\hat{I}\right)\vec{l} = 0 \tag{7.24}$$

$$det\left(\hat{\sigma} - \delta\hat{I}\right) = 0 \tag{7.25}$$

where $\vec{m} = (m_1, m_2, ...m_n)$, $\hat{\sigma}$ is an n- dimensional matrix with elements σ_{ij} and $\hat{I}$ is a unit matrix (a matrix with units along the main diagonal and all other elements equal to zero); repeated indices indicate summation between 1 and n.

7.4 Hilbert space

Hilbert space is an infinite-dimensional space in which we can define the scalar product of two vectors[6].

In finite-dimensional space, the scalar product of two vectors $\vec{l}$ and $\vec{m}$ is defined by

$$\left(\vec{l}, \vec{m}\right) = l_i^* m_i \tag{7.26}$$

[6] We omit some of the mathematical details ([84], Chap.2)

(repeated indices indicate summation). This formula is a direct generalization of the scalar product in ordinary Euclidean space.

The scalar square of a vector defines its norm (length):

$$\|\vec{l}\|^2 = l_i^* l_i = \sum_{i=1}^{n} |l_i|^2 \tag{7.27}$$

We note that the norm of any nonzero vector is positive. It is the reason for the complex conjugation symbol introduced in (7.26). A vector of unit length is said to be normalized. To normalize an arbitrary vector $\vec{l}$ we divide it by its norm. This means that the vector $\vec{l}/\|\vec{l}\|$ has unit length.

The orthogonality (i.e. perpendicularity) of two vectors is another important concept that can be expressed in terms of the scalar product. In particular, two vectors $\vec{l}$ and $\vec{m}$ are called orthogonal if their scalar product is zero

$$(\vec{l}, \vec{m}) = 0 \tag{7.28}$$

The Hermitian property can be directly generalized to the case of n-dimensional space:

$$\sigma_{ij} = \sigma_{ji}^* \tag{7.29}$$

It can be shown that all eigenvalues of Hermitian matrix are real, and different eigenvectors are orthogonal in pairs.

The latter property enables us to write any vector $\vec{q}$ in the form of a superposition (sum) of eigenvectors $\vec{l}^{(i)}$ of an arbitrary Hermitian operator $\hat{\sigma}$

$$\vec{q} = c_i \vec{l}^{(i)} \tag{7.30}$$

The eigenvectors $\vec{l}^{(i)}$ can be considered to be normalized.

The orthogonality relations ensure that the coefficients c_i have a very simple form [85]:

$$c_i = \left(\vec{l}^{(i)}, \vec{q}\right) \tag{7.31}$$

In quantum mechanics, we consider more complicated objects such as the functions $\psi(\vec{r})$ where $\vec{r}$ is a vector in ordinary three-dimensional space. This function is an infinite-dimensional vector because its description involves an infinite number of values (components of the vector) $\psi_1(\vec{r}), \psi_2(\vec{r}), \ldots$

On the other hand, the component l_i of a vector $\vec{l}$ can be regarded as a function of the index i. That is why there is no difference between vectors and functions in the theory of Hilbert space.

In an infinite-dimensional space, a scalar product such as (7.26) contains an infinite number of terms. Since an integral is defined by

$$\int \varphi(t)dt = \lim_{\Delta t \to 0} \sum_i \varphi(t_i)\Delta t_i \tag{7.32}$$

it is natural to define a scalar product of two functions in the form

$$(\varphi, \phi) = \int \varphi^*(\vec{r})\phi(\vec{r})d\vec{r} \tag{7.33}$$

Formulas (7.27) and (7.28) can be directly generalized to the case of Hilbert, i.e., infinite-dimensional, space.

7.5 Possible results of measurement

The shortest exposition of quantum mechanics is as follows. Let us measure some quantity. In quantum mechanics with every physical quantity a we associate an operator $\hat{A}$. When quantity a is

measured, the result can be only one of the eigenvalues a_i of the operator $\hat{A}$.

The probability $P(a_i)$ that a measurement will result in a value a_i, is expressed by the formula

$$P(a_i) = \left|\left(\psi^{(i)}, \psi\right)\right|^2 \tag{7.34}$$

Here $\psi^{(i)}$ is the eigenvertor corresponding to the eigenvalue a_i and ψ is state of the system being measured. In this paragraph we examine (7.34) in some details.

We shall now illustrate a measurement scheme in quantum mechanics by considering the measurement of electron spin.

The components of an electron spin along the x, y, z axes are associated with following operators [86]:

$$\hat{S}_x = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \tag{7.35}$$

$$\hat{S}_y = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \tag{7.36}$$

$$\hat{S}_z = \frac{1}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \tag{7.37}$$

Using (7.18) and (7.37), we obtain

$$\begin{vmatrix} \frac{1}{2} - S_z & 0 \\ 0 & -\frac{1}{2} - S_z \end{vmatrix} = 0 \tag{7.38}$$

Hence the eigenvalues of the operator $\hat{S}_z$ are

$$S_z = \pm\frac{1}{2} \tag{7.39}$$

Equations similar to (7.17) then show that the eigenfunction corresponding to $S_z = \frac{1}{2}$ is

$$\psi^{(1)} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{7.40}$$

whereas the eigenfunction

$$\psi^{(2)} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{7.41}$$

corresponds to the eigenvalue $S_z = -\frac{1}{2}$. The arbitrary constants in (7.40) and (7.41) were chosen to ensure that the vector eigenfunctions were normalized:

$$\left\|\psi^{(1)}\right\| = 1, \quad \left\|\psi^{(2)}\right\| = 1$$

The physical meaning of the eigenfunction $\psi^{(1)}$ is that the electron spin component S_z in this state is determinate and equals to $+1/2$. Similarly, the magnitude of S_z in the state $\psi^{(2)}$ is $-1/2$.

Quantum mechanical randomness arises only in those cases when we measure a quantity that does not have a definite value. For example, suppose we measure the projection S_z of the spin of the electron in the state $\psi^{(\delta)}$ in which the projection $\hat{S}_{\vec{\delta}}$ on the $\vec{\delta}$ axis has a definite value. It can be shown that the operator $\hat{S}_{\vec{\delta}}$ takes the form

$$\hat{S}_{\vec{\delta}} = \delta_x \hat{S}_x + \delta_y \hat{S}_y + \delta_z \hat{S}_z, \tag{7.42}$$

where $\delta_x, \delta_y, \delta_z$ are the components of the unit vector $\vec{\delta}$ on the coordinate axes.

We now choose the coordinate frame in such a way that the vector $\vec{\delta}$ lies in the x,z plane and makes an angle φ to the z axis. We then have $\delta_x = \sin\varphi$, $\delta_y = 0$, $\delta_z = \cos\varphi$ and

$$\hat{S}_{\vec{\delta}} = \frac{1}{2}\begin{pmatrix} \cos\varphi & \sin\varphi \\ \sin\varphi & -\cos\varphi \end{pmatrix} \tag{7.43}$$

One of the eigenvalues $\hat{S}_{\vec{\delta}}$ is $\hat{S}_{\vec{\delta}} = \frac{1}{2}$. The corresponding eigenvector is

$$\psi^{(\delta)} = \begin{pmatrix} \cos\frac{\varphi}{2} \\ \sin\frac{\varphi}{2} \end{pmatrix} \tag{7.44}$$

When we measure S_z in the state $\Psi^{(\delta)}$, the only possible outcomes are $S_z = \frac{1}{2}$ and $S_z = -\frac{1}{2}$. The result of the measurement of S_z is random. The only determined quantity is the probability of

the values S_z. To determine this probability, we write vector $\psi^{(\delta)}$ in the form of superposition of the vectors $\psi^{(1)}$ and $\psi^{(2)}$

$$\psi(\delta) = c_1\psi^{(1)} + c_2\psi^{(2)} \tag{7.45}$$

The probability of the result $S_z = \frac{1}{2}$ in state $\psi^{(\delta)}$ is $|c_1|^2$ and the probability of $S_z = -\frac{1}{2}$ is $|c_2|^2$. Equations (7.40), (7.41) and (7.44) then yield

$$\psi^{(\delta)} = \psi^{(1)} cos\frac{\varphi}{2} + \psi^{(2)} sin\frac{\varphi}{2} \tag{7.46}$$

Hence the probability of obtaining $S_z = \frac{1}{2}$ in the state $\psi^{(\delta)}$ is $cos^2\frac{\varphi}{2}$ and probability of $S_z = -\frac{1}{2}$ in $\psi^{(\delta)}$ is $\sin^2\frac{\varphi}{2}$.

In the special case $\varphi = \frac{\pi}{2}$, i.e., when the vector $\vec{\delta}$ points along the x axis, we have

$$\psi^{(\delta)} = \frac{1}{\sqrt{2}}\psi^{(1)} + \frac{1}{\sqrt{2}}\psi^{(2)} \tag{7.47}$$

When the spin projection along the z axis is measured, we obtain the value 1/2 with 50% probability and analogously for the value -1/2 we have 50% probability also. If, on the other hand, we measure a quantity, which has defined value in this state, i.e., the spin projection along the x axis, we find there is no randomness and the result is always

$$S_x = \frac{1}{2}$$

Let us now consider the general case. Suppose we measure a quantity a in the state ψ. We associate with a the corresponding operator $\hat{A}$. This operator has eigenvectors $\psi^{[1]}, \psi^{(2)}, \ldots$ and the corresponding eigenvalues are a_1, $a_2 \ldots$ Measurement of a can result only in one of eigenvalues a_i of operator $\hat{A}$.

The eigenvalues of an operator are frequently discrete. This occurs, for example, in the case of the energy operator of an electron in an atom. The electron energy, which in classical physics can assume any of a continuous set of values, then takes on only certain definite discrete values in quantum mechanics.

To find the probability of a value a_i, we write the state vector ψ in the form of the superposition of the eigenvectors $\psi^{[1]}, \psi^{(2)}, \ldots$:

$$\psi = c_1\psi^{(1)} + c_2\psi^{(2)} + \ldots \tag{7.48}$$

The probability that a_i will be obtained is $|c_i|^2$. If we now use (7.31), we obtain (7.34).

7.6 Reality of abstractions

The opinion, that abstractions exist only in human head, and they do not exist objectively is widespread. For example, Berkley wrote, that "triangle, in general, is neither oblique, nor rectanguilateral, nor equilateral, nor isosceles, nor nonequilateral, but that is at the same time both anything and nothing of them" [87].

Abstrations indeed do not exist in the form of separate realities. But they do exist in the form of sets (We remind that a set is defined by properties which have elements belonging to it). In the example above this property consists in the fact that a polygon has three angles. It is what we mean when we say about

abstraction of triangle: the set of all oblique, rectangular, equilateral, isosceles and other triangles.

"Realism is not restricted to physics. There is also a mathematical realism ..." [81].

Two of the most general abstractions are the physical space and the physical time. The question is: whether they exist objectively outside any thinking being, or they are only mental constructions? The answer is: they exist objectively, but neither as "spatial fluid" nor as "temporal fluid". The physical space and time are not substances but universal objective relations between various particles, fields and phenomena.

Chapter 8

The "Paradoxes" of Quantum Mechanics

"If quantum theory does not disturb on first acquaintance, it could not have been properly understood".

Niels Bohr, [28].

8.1 Imaginary numbers and operators

Quantum mechanics is a logically consistent theory. This means that, strictly speaking, it does not involve any paradoxes. However, our intuition is based on everyday macroscopic experience which corresponds to classical mechanics. When quantum mechanics is subsequently encountered, people subconsciously tend to replace quantum concepts with classical ones, which results in apparent paradoxes. We shall discuss the best known of them.

Some authors see a "paradox" in the fact that the basic formula of quantum mechanics, i.e., Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t}=-\frac{\hbar^2}{2m}\Delta\psi+V\psi \tag{8.1}$$

contains the imaginary quantity $i=\sqrt{-1}$. However, the imaginary unit i is not actually a symbol of anything from other world. The imaginary number in the Schrödinger equation means that the wave function ψ is a complex quantity

$$\psi=\psi_r+i\psi_i \tag{8.2}$$

i.e., that it consists of two parts: real part ψ_r and imaginary part ψ_i: The terms "real" part and "imaginary" part must not be understood in the same sense as the terms "real apple" and "imaginary apple". They mean that single equation (8.1) for complex quantity ψ splits into two equations

$$\begin{aligned} \hbar\frac{\partial\psi_r}{\partial t} &= \left(V-\frac{\hbar^2}{2m}\Delta\right)\psi_i \\ -\hbar\frac{\partial\psi_i}{\partial t} &= \left(V-\frac{\hbar^2}{2m}\Delta\right)\psi_r \end{aligned} \tag{8.3}$$

for two real quantities ψ_r and ψ_i: We note that equation (8.1) is not merely a shortened expression for the two equations (8.3). Complex quantities permit easier treatment of various transformations.

For instance the equation (8.1) is invariant under the transformation

$$\psi \to \psi \exp(ia) \tag{8.4}$$

In the case of two equations (8.3) the equivalent transformation has cumbersome form

$$\begin{aligned} \psi_r &\to \psi_r \cos(\alpha) - \psi_i \sin(\alpha) \\ \psi_i &\to \psi_i \cos(\alpha) - \psi_r \sin(\alpha) \end{aligned} \tag{8.5}$$

The splitting of the complex wave function ψ into two real variables in another way is more natural. Namely, the wave function may be split into modulas $|\psi|$ and argumentum $\arg\psi \equiv Im\ln\psi$. The evolution of ψ is determined by both quantities $|\psi|$ and $\arg\psi$, whereas only $|\psi|$ is observable. Complex variables

are very handy mathematical tools for reflection these properties of the function.

The measurement of any physical quantity always produces a real number. In the above mathematical formalism, it is guaranteed by the fact that physical quantities are always represented by Hermitian operators (sec. 7.3), whose eigenvalues are always real.

We now pass onto operators. "An essential feature of the new theory is that physical quantities or, in Dirac's terminology, observables (momentum, particle energy, field components, and so on) are represented not by variables, but by symbols with a noncommutative multiplication law or, to be specific, operators" ([45], p. 105).

For example, the momentum p of particle, is expressed not by a number, but by the differentiation operator

$$\hat{p} = \frac{\hbar}{i}\frac{\partial}{\partial x} \tag{8.6}$$

where, for simplicity, we have confined ourselves to the one-dimensional case.

Quantum-mechanical operators are related to one another moreover by the same expressions as corresponding quantities in classical mechanics. In particular, if external field is absent, the energy of a classical particle is given by

$$\varepsilon = \frac{p^2}{2m} \tag{8.7}$$

According to (8.6), energy is represented in quantum mechanics by the operator

$$\varepsilon = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} \tag{8.8}$$

We note that, strictly speaking, even in classical mechanics, physical quantities are expressed not by numbers, but also by operators. For example, when we say that a car is travelling with a speed of 60km/h, the number 60 does not mean "sixty pieces", but signifies an operator producing an expansion by a factor of 60 when applied to the speed 1km/h. In precisely the same way, a temperature of 100^0C does not mean 100 temperature of 1^0C ([82], p.33).

The difference between classical and quantum mechanics, in this sense, is that, in classical mechanics, we use only simple operators such as operators producing the expansion or addition of physical quantities, whereas in quantum mechanics we use more complicated operators. Expansion and addition operators always commute with one another, whereas quantum-mechanical operators often do not.

8.2 The particles identity paradox

The indistinguishability of waves leads in quantum mechanics to the indistinguishability, or more precisely, the identity of particles.\footnote[7]. For example, all the electrons in an atom are absolutely identical. Even if at the initial time t=0 we label all the electrons, we have no way of telling which is which at the subsequent time $t > 0$ because the concept of a particle trajectory is meaningless for an electron. For instance, there is no change in any physical phenomenon when two electrons are interchanged. This property seems strange in classical physics. For example, the planets in the solar system are all different which means, in particular, that if we were to interchange the Earth and Mercury, we would soon notice a difference.

When a lady buys a classical object, for example, a blouse, she always feels the quality of the material. However, when she buys a

[7] We note that any two particles with sufficiently small difference of masses within a sufficiently short interval of time behave as if they would be identical [88].

gold ring, she is interested only in its quantitative content, i.e., its weight and the number of carets. She's not normally interested in the quality of the gold. Gold atoms are quantum objects and are therefore identical. In contrast to a fabric, there is no such thing as "inferior" gold.

The atoms of chemical elements are so small that they exhibit quantum- mechanical effects such as indistinguishability of particles of the same kind. For atoms other than gold, this property is masked by the fact that different atoms enter different chemical compounds and thus become distinguishable. Gold, however, is a noble metal and does not readily enter chemical reactions, so that gold atoms cannot be distinguished from one another. The existence of the same, immutable gold is a quantum effect that cannot be explained in terms of classical physics. We have become accustomed to the fact that gold remains unaltered even after it has been exposed to a huge number of external factors.

However, the property of identity is so strange to us that even the founding fathers of quantum mechanics have been known to be wrong. For instance Dirac writes:

"... the wave function gives information about the probability of one photon being in a particular place and not the probable number of photons in that place. The importance of the distinction can be made clear in the following way. Suppose we have a beam of light consisting of a large number of photons split up into two components of equal intensity. On the assumption that the intensity of a beam is connected with the probable number of photons in it, we should have half the total number of photons going into each component. If the two components are now made to interfere, we should require a photon in one component to be able to interfere with one in the other. Sometimes these two photons would have to annihilate one another and other times they would have to produce four photons. This would contradict the conservation of energy. The new theory, which connects the wave function with probabilities for one photon, gets over the difficulty by making each photon go partly into each of the two components.

Each photon then interferes only with itself. Interference between two different photons never occurs" ([76], p.9).

The fact is that two photons of the same frequency cannot be different. Two photons from different components of the original beam do not differ in any way from one photon that belongs "partly to each of the two components". The reference to the conservation of energy is also inconsistent because the violation of the law is only apparent and merely a manifestation of interference (see 2.2).

Recent experiments have shown that photons from two different lasers do interfere with each other [88,89,90]. In contrast to thermal sources of photons, an individual emission event in a laser takes a relatively long interval of time, so that interference between photons from statistically independent lasers can be observed [92].

It is irrelevant whether there are two photons from one source or from different sources. Because of the identity of photons, all sources throughout the universe must be looked upon as a single source. An observed photon can be related to a particular source only if the probability of arrival of a photon from all other sources at a given point is negligible [90].

In the case of electrons, one of the manifestations of the principle of indistinguishability is Pauli's exclusion principle: no two electrons can be found in the same state. This principle is a revival at a higher level of the ancient principle of impenetrability of matter: two different bodies cannot occupy simultaneously the same position.

The principle of impenetrability of matter has already been violated in classical physics. When the inventor of radio Alexander Popov demonstrated the transmission of radio-waves the Peterburg physicists were puzzled not only by the fact that the radio-waves propagate without any wires but also by the fact that they penetrate thorough closed windows. Radio-waves pass freely through all nonconducting bodies.

8.3 Schrödinger's cat

In one of his papers on quantum mechanics Schrödinger produces an example of paradoxical situation. Suppose that a chamber contains a speck of radium, a Geiger counter, a glass vial containing prussic acid, and a cat. The decay of a radium nucleus causes the emission of an alpha particle which crosses the Geiger counter. The counter produces a pulse which is used to initiate a mechanical device that breaks the vial and releases the prussic acid, which kills the cat.

Since the decay of radium is random, we have a superposition of two quantum states, namely, the live cat and the dead cat, and the two states can interfere. This interference means that the cat does not occupy one particular state (dead or alive), but is half dead and half alive, which is absurd.

The true situation is different. The discussion that we have just given not take into account the fact that the operation of the counter is a catastrophe in the micro-world which converts a superposition into a mixture. The state of the cat is therefore described not by the single wave function

$$\psi_m = c_1\psi_1 + c_2\psi_2$$

but by two wave functions, namely, ψ_1 with probability $|c_1|^2$ and ψ_2 with probability, $|c_2|^2$ and interference between the two state is not possible.

This situation is consistent with the classical theory of probability. For example, let 48% of all people are men. It means that a person selected at random has a probability of 0.48 of being a man. This is not a paradox; we understand the result.

However, the same fact can be formulated in a mystical, paradoxical form: "each person is 48% man and 52% woman".

8.4 The Einstein-Podolsky-Rosen paradox

Einstein, Podolsky and Rosen devised an example of a physical situation which, in their opinion, demonstrated the incompleteness of quantum mechanics. The incompleteness was understood in the sense that there were some hidden parameters which, when discovered, would show that quantum mechanics was in fact a deterministic theory. Einstein, Podolsky and Rosen maintained that the denial of the existence of such parameters leads to a paradox, i.e. a logical inconsistency. They considered the measurement of the position and momentum of an electron.

We shall discuss a simple modification of this thought experiment, due to Bohm, which involves the measurement not of the position and momentum, but of a component of the spin of the electron.

Let us consider two electrons with zero resultant spin. For example, this can be the atomic shall of an atom of helium in the singlet state. When a neutron knocks out the nucleus from the helium atom, the two electrons fly apart because of the Coulomb repulsion between them. The projection of the spin of one of them on an arbitrary axis, say the x axis, is a random quantity equal to 1/2 or $-1/2$. The projection of the spin of the other electron on the same axis is also random and equal to $\pm\frac{1}{2}$.

Since momentum has to be conserved, the resultant of the two electrons must be zero. Let us now determine the projection of the spin of one of the electrons along the x axis. Suppose that the resultant of this measurement is $S_x^{(1)} = +\frac{1}{2}$. The projection of the spin of the other electron onto the x axis must then be $S_x^{(2)} = -\frac{1}{2}$. We thus see that the state of the second electron has changed instantaneously: if prior to the measurement on the first electron, $S_x^{(2)}$ could be +1/ 2 or -1/ 2 with equal probability, then after the

measurement $S_x^{(2)} = -\frac{1}{2}$. However, the electrons can be at an arbitrary large distance. For example, one electron could be in Paris and the other electron in Peking. This means that the measurement of the projection of the spin of the Paris electron could not possibly affect the Peking electron. This instantaneous reaction between electrons separated by an enormous distance is the Einstein- Podolsky-Rosen-Rosen paradox. Einstein considered that the paradox could be regarded as an evidence for the incompleteness of quantum mechanics.

In reality, the Einstein-Podolsky-Rosen paradox does not contain a logical inconsistency. Prior to measurement, the two electrons were not localized, and each of them was potentially both in Paris and in Peking [92]. Hence, during the measurement of the spin of the Paris electron there is an instantaneous change not in the state of the Peking electron, but in the probability of its state. Such an instantaneous change in probability is not specific to quantum mechanics: it is also encountered in classical physics ([4], p.96).

For example, let's consider two rooms, in one of which there is a princess and in the other a tiger. The two rooms are a great distance apart. A slave can, at his wish, open the door of one of the two rooms whereupon he either marries the princess or is torn to pieces by the tiger. Thus, by opening the door of one of the rooms he knows immediately who is in the other. This does not involve a paradox, but it does involve a hidden parameter ξ. For example, $\xi = 1$ if a given room contains the princess and $\xi = 0$ if it contains the tiger. We shall see in Chap.9 that hidden parameters are impossible in quantum mechanics. The Einstein-Podolsky-Rosen effect is therefore in conflict with common sense. It has given rise to doubts about the validity of quantum mechanics in the Einstein-Podolsky-Rosen situation. However, experiments (Refs.92-94) have revealed no evidence for a deviation from the predictions of quantum theory.

Nevertheless, it seems appropriate to recall Mach's words: "The history of science teaches us that experiments with a negative result must never be regarded as conclusive. Hooke did not succeed with his balance to demonstrate the effect of distance from the Earth on the weight of a body, but it presents no peculiar difficulty to the more sensitive modern balances" ([3], p.219).

A detailed discussion of the Einstein-Podolsky-Rosen paradox is given in [95] and [96].

8.5 Aharonov-Bohm paradox

We shall illustrate this paradox by an example. Let us consider a particle carrying an electric charge e and traveling in a region with constant potential φ. The total energy of the particle is

$$H = \frac{p^2}{2m} + e\varphi \tag{8.9}$$

We know that the potential φ has no direct physical meaning. The physical situation does not change when we add an arbitrary constant C to φ

$$\varphi \to \varphi + C$$

It is only the electric field

$$E_d = -\frac{\partial \varphi}{\partial x} \tag{8.10}$$

that has a direct physical meaning (for the sake of simplicity we confine ourselves to the one-dimensional case).

In classical physics, the fact that the physical picture is independent of C means that C does not appear in Hamilton's equations

$$\frac{dx}{dt} = \frac{\partial H}{\partial p} = \frac{p}{m} \tag{8.11}$$

$$\frac{dp}{dx} = -\frac{\partial H}{\partial x} = -e\frac{\partial \varphi}{\partial x} \tag{8.12}$$

In quantum mechanics, the Schrödinger equation has the form

$$i\hbar\frac{\partial \psi}{\partial t} = -\frac{\hbar^2}{2m}\frac{\partial^2 \psi}{\partial x^2} + e\varphi\psi \tag{8.13}$$

The solution of this equation for a particle with momentum p is

$$\psi = A\exp\left\{\frac{i}{\hbar}\left[px - \left(\frac{p^2}{2m} + e\varphi\right)t\right]\right\} \tag{8.14}$$

where A is a constant of integration.

In contrast to classical mechanics, the state ψ of a particle in quantum mechanics depends directly on the potential φ. This can be demonstrated experimentally by means of interference. For example, a particle beam can be divided into two parts by two slits in a screen. One part is sent through a region with potential φ and the other thorough a region with zero potential. When the two parts of the beam are recombined, and interference pattern is observed because a path difference has been introduced between them.

The paradox is that we can experimentally detect [97] the potential that contains the arbitrary term C.

In reality, there is no paradox [98]. We cannot say in quantum theory that one particle crosses the region of potential φ and the

other the region with zero potential. Each electron can be present in both regions. Hence the interference pattern reflects not the potential φ itself, but the difference between φ and 0, which means that the arbitrary constant C eliminated.

A more detailed discussion of the Aharonov-Bohm effect may be found in [92].

Chapter 9

The impossibility of hidden-variable models

> "... When the mayor's clerk, report in hand, entered the mayor's office in the morning, he faced a curious sight: the mayoral body, dressed in uniform, was seated behind a deck, and on a pile of records of unpaid taxes there lay, like a foppish paperweight, the totally empty mayoral head... The town's leading physician was summoned, and three questions were put to him: (1) could the mayoral head have separated from the mayoral trunk without any blood being spilled? (2) Could it possibly have happened that the mayor removed and emptied his head himself? And (3) could it be assumed that the mayoral head having been removed could be reinstated later by some as yet unknown process? The medic thought long and hard, and murmured something about a "mayoral substance" that allegedly issued from the mayoral body ..."
>
> (M. E. Saltykov-Shchedrin. History of a town)

9.1 The problem of hidden variables

Quantum mechanics rejects the determinism of Newtonian mechanics. Many physicists consider this unacceptable. Their point of view has been articulated by David Bohm: "The usual interpretation of quantum theory, which is internally closed, nevertheless includes the assumption that the most complete description the state of an individual system is achieved by using the wave function that determines only the probable results of actual measurement process. The only way of verifying the

validity of this proposition is to try to find some other interpretation of quantum theory in terms of what are still "hidden parameters", but which in principle would determine exactly the behavior of the individual systems; measurements that are practicable at present constitute averages over these parameters" ([99], p.34).

In other words, supporters of the hypothesis of hidden parameters state that nondeterministic quantum mechanics is only the visible part of the complete edifice. This visible part rests on an invisible foundation, i.e., some deeper deterministic theory, created in the spirit of classical theory.

Supporters of hidden parameters assume that the situation in quantum mechanics is the same as in classical kinetic theory. For example, the blue color of the sky is a consequence of the scattering of sunlight by random pulsations in the density of air, which have a certain particular size. Randomness is then only apparent because the air molecules follow determined motion and randomness arises because we do not know the positions and velocities of the individual molecules.

Many hidden-parameter models have been proposed. To illustrate the situation, we shall consider three of them: the model of subquantal particles (9.2), the model of the subquantal fluid (9.3), and the model of the subquantal wave function (9.4). These models explain only some of the quantum effects, and are in conflict with others. The assumption, on which hidden-parameter models rely are therefore wrong.

The founding fathers of quantum mechanics knew that reasonable hidden-parameter models were impossible in principle. However, a rigorous proof of this proposition was lacking, and the speculations remained unpublished.

The first rigorous mathematical proof that hidden parameters could not be introduced without radical change to quantum mechanics was provided by von Neuman ([84], pp. 234-244). The proof is based on certain postulates, one of which is that the equations of the subquantum (i.e., more fundamental) theory

should be linear in the same way the Schrödinger's equation is linear. Supporters of hidden parameters objected to this and noted that classical mechanics is nonlinear, so that the linearity postulate was inadmissible.

The first proof that it is impossible to construct a hidden-parameter model, without assuming the linearity of the equations of this model, was given by Bell (Refs.91,93,99). A simple proof was given soon after by Kochen and Specker [101]; see about it 9.5. These proofs are based on the idea, that, in quantum mechanics, randomness combines with necessity in such a way that it is impossible to reduce randomness to a set of hidden parameters.

A very short and simple proof of the impossibility of hidden-parameter models has been given by Turner [102], but his proof relies on familiarity with quantum logic. These questions are discussed in Chap.10. and a detailed discussion of the problem of hidden parameters is given in Refs. 92-94, 102, 103 and 104.

We note that Maxwell's electromagnetic field theory was initially regarded as unsatisfactory because it described the behavior of the abstract vectors $\vec{E}$ and $\vec{H}$ and not the motion of matter. "Many models were proposed to overcome this difficulty. They were based on the behavior of a fictitious continuous medium, called the "aether", which was capable of transmitting action from point to point. Unfortunately, calculations and experiments showed that the existence of the aether could not be proved for the electromagnetic field, and even a description of it could not be provided" ([94], p.147).

9.2 Model of subquantal particles

The fact that the α-particle does cross the potential barrier, and the quantum-mechanical randomness of the process, could be explained classically by assumption that the α-particle is collided with some as yet unknown small subquantal particles ("zerons",

[106]) or vacuum fluctuations (Refs.106-108). This was suggested by the fact that equation (9.5), which is a consequence of Shrödinger's equation, has the same structure as the diffusion equation (Refs.109,110), namely, it contains the first derivative with respect to time and the second derivatives with respect to the coordinates.

However, if this were so, the α-particles crossing the potential barrier would have random energies, ranging from zero to the height of the barrier, whereas all the α-particles leaving the nucleus are known to have an energy of exactly 4.8 Mev. The subquantal particle model is therefore wrong.

9.3 The subquantal fluid model

In order to obtain the subquantal fluid model, we write the wave function ψ in the form of

$$\psi = R \exp\left(\frac{i}{\hbar} S\right), \tag{9.1}$$

where R and S are real. Denoting

$$\rho = R^2 \tag{9.2}$$

and

$$\vec{v} = \frac{1}{m} \nabla S \tag{9.3}$$

we obtain the real continuity equation

$$\frac{\partial \rho}{\partial t} + div \rho \vec{v} = 0 \tag{9.4}$$

and the real equation of motion (Bohm, Refs.98,111)

$$m\frac{\partial \vec{v}}{\partial t}+m(\vec{v}\nabla)\vec{v}=-\nabla V+\frac{\hbar^2}{2m}\nabla\left(\frac{\Delta\rho^{1/2}}{\rho^{1/2}}\right) \tag{9.5}$$

Equation (9.4) may be looked upon as the equation of continuity (Ref. 37) of a (subquantal) fluid, whereas (9.5) is the equation of motion of a particle that experiences both the classical potential V and the "quantum potential" (Refs.98,111)

$$-\frac{\hbar^2}{2m}\left(\frac{\Delta\rho^{1/2}}{\rho^{1/2}}\right) \tag{9.6}$$

The "quantum potential" keeps the electron on a quantized orbit around the nucleus and not at an arbitrary distance, which would be the case if there were an analogy with the planets in the solar system.

The velocity $\vec{v}$ of a particle in the subquantal fluid model is interpreted as a hidden parameter, and it is assumed that, after measurement, the particle momentum $\vec{p}$ is different from the "true momentum" $m\vec{v}$ that is the value prior to measurement. On the other hand, the measured position is the same as the true position.

Bohm has examined a number of simple measurement processes that could lead to agreement with the predictions of quantum theory.

However, the "wildness" of quantum mechanics lies not so much in the fact that it predicts some special effects that cannot be explained in terms of classical theory as in the existence of effects every of which may be explained classically, but this explanation are in conflict with one another. In particular, the laws of quantum mechanics are symmetrical under the replacing coordinates by momentums and momentums by coordinates.

The subquantal fluid model does not meet this requirement and cannot therefore explain the results of more complicated

experiments. We shall not pause to examine these experiments because we shall show in (9.5) that hidden parameters cannot, in principle, be introduced into quantum mechanics. Thus subquantal fluid model is flimsy also.

We note, that by writing two real equations in the form of a single complex Schrödinger equation we ensure not only that the expression is compact, but also that the Schrödinger equation is linear whereas the two equations (9.4) and (9.5) are nonlinear. The advantage of having a linear equation is that there is a well established mathematical formalism for solving such equations. This relationship between linear and nonlinear equation is used for the precise analytic solution of nonlinear problems. In particular, the real nonlinear equation is treated as a component of a quantum-mechanical particle linear problem whose solution can be derived in an explicit form. The solution found in this way is then translated into the language of the nonlinear problem thus obtaining a solution of the original nonlinear equation [113].

9.4 Subquantal wave function model

In the model proposed by Wiener and Siegel [114], the state of a micro-system is described by two wave functions, namely, the usual quantum-mechanical wave function ψ and the "hidden" wave function ξ. The latter is introduced to ensure that we can accurately predict which of the eigenvalues of the observed variable is obtained by measurement. We shall illustrate the Wiener-Siegel model by considering the measurement of the projection of the electron spin.

Wiener and Siegel assumed that, in addition to the explicit electron wave vector

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{9.7}$$

there was a further "hidden" vector

$$\xi = \begin{pmatrix} \xi_1 \\ \xi_2 \end{pmatrix} \tag{9.8}$$

which predetermines the result of any measurement of S_z. In particular, when

$$\frac{|\psi_1|}{|\xi_1|} > \frac{|\psi_2|}{|\xi_2|} \tag{9.9}$$

we have $S_z = \frac{1}{2}$ whereas if

$$\frac{|\psi_1|}{|\xi_1|} < \frac{|\psi_2|}{|\xi_2|} \tag{9.10}$$

we have $S_z = -\frac{1}{2}$. In contrast to ψ, the vector ξ is not normalized: $|\xi_1|^2 + |\xi_2|^2$ is not necessarily equal to unity.

To ensure that the hidden-parameter model provides the foundation for quantum mechanics it must lead to the quantum-mechanical postulates. In particular, the Wiener-Siegel model must lead to probabilities (7.4) and (7.5). It can be shown that these probabilities are obtained if it is assumed that the "hidden" vector ξ is random and that the quantities $|\xi_1|$ and $|\xi_2|$ are independent and lie between zero and infinity with the distribution

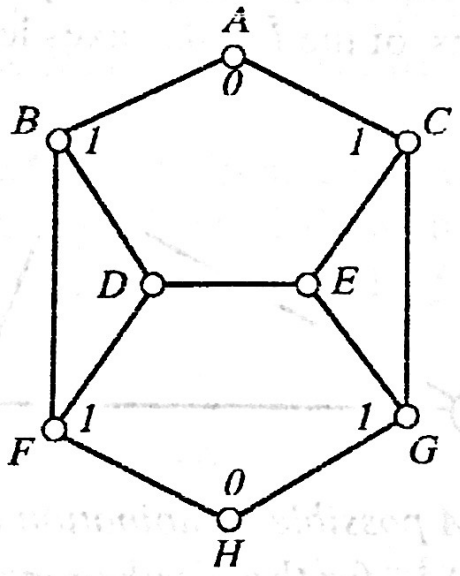
$$f(|\xi_j|) = |\xi_j| exp\left(-|\xi_j|^2/2\right), \quad (j=1, 2) \tag{9.11}$$

This model can be directly generalized to the case of arbitrary spin.

The Wiener-Siegel model explains the Single measurement of spin projection, but cannot explain more complicated sets of several measurements (see 9.5).

9.5 The Kochen-Specker proof

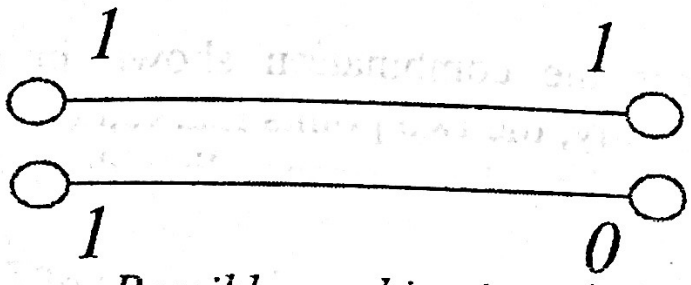
We have already noted that quantum mechanics is nonclassical not so much because it involves randomness as because the randomness combines in a strange way with necessity. The random results of simple measurements could be explained by hidden parameters, but the random results of more complicated measurements are subject to essential constrains (correlations) which, as we shall show, exclude the possibility of hidden parameters.



Hidden-parameter models cannot be constructed. Orthogonal directions correspond to points Joined directly by straight lines

Fig 23.

To prove that hidden parameters cannot be introduced into quantum mechanics, we need only find onc example that cannot be explained by the existence of such parameters.



Possible combination of S_l^2 and S_m^2, for two orthogonal directions

Fig 25.

Whenever the directions of $\vec{l}$ and $\vec{m}$ are orthogonal (i.e., perpendicular), the corresponding points L and M are joined by a line. On the other hand, whenever two directions are not orthogonal, the corresponding points are not joined by a line. For example, in Fig.23, we show the eight directions A, B, C, D, E, F, G, H. Directions A and B are orthogonal, whereas directions A and D are not.

The question is: what is difference between orthogonality and nonorthogonality of axes? We shall show in 9.6 that, when $\vec{l}$ and $\vec{m}$ are orthogonal, the values of $S_{\vec{l}}^2$ and $S_{\vec{m}}^2$ are compatible. This means that, when

$$\vec{l}\vec{m} = 0 \tag{9.14}$$

there is a quantum state in which the quantities $S_{\vec{l}}^2$ or $S_{\vec{m}}^2$ are simultaneously determined.

When the three axes $\vec{l},\vec{m},\vec{n}$ are orthogonal in pairs, then according to 9.6, the values of $S_{\vec{l}}^2, S_{\vec{m}}^2, S_{\vec{n}}^2$ are compatible and two of them are equal to unity whereas the third is zero. In other words, for three directions that are orthogonal in pairs, the only possibility is the combinations of zero and units illustrated in Fig 24. Hence it follows that, for two mutually orthogonal directions, the only possible combinations of zeros and units are those shown

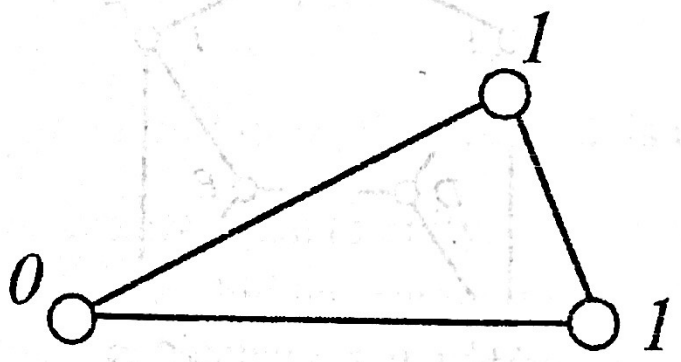
We shall take this example to be the measurement of the square of spin projection for a particle with unit spin

$$S = 1. \tag{9.12}$$

It follows from 7.2, that the square of the spin projection $S_{\vec{l}}^2$ on an arbitrary axis $\vec{l}$ can assume two values, namely

$$S_{\vec{l}}^2 = 0 \quad \text{or} \quad S_{\vec{l}}^2 = 1 \tag{9.13}$$

Suppose that the $\vec{l},\vec{m},\vec{n},...$ axes emerge from the same point O, and let us construct sphere of arbitrary radius centered on O. The sphere cuts $\vec{l},\vec{m},\vec{n},...$ axes at points $L,M,N,....$ We now map topologically the sphere on a plane that coincides with the plane of the drawing. Each point $L,M,N,...$ on the plane is then associated with its own directions of the $\vec{l},\vec{m},\vec{n},...$ axes leaving the point O.



A possible combination of
S_l^2, S_{ml}^2, S_n^2, *for three orthogonal directions*

Fig 24.

in Fig.25. whereas the combination shown in Fig.26. is not possible. Consequently, the two points marked O in Fig.23 cannot be joined by a line, i.e., the corresponding directions cannot be orthogonal.

Now consider the case when the directions of $\vec{l}$ and $\vec{m}$ are not orthogonal:

$$\vec{l}\vec{m} \neq 0 \tag{9.15}$$

We show in 9.6 that, when this is so, the values of $S_{\vec{l}}^2$ and $S_{\vec{m}}^2$ are incompatible. This means that, for any particular value of $S_{\vec{l}}^2$, the quantity $S_{\vec{m}}^2$ is always random and can assume two values, 0 or 1.

For example, when

$$S_{\vec{a}}^2 = 0$$

(see Fig.23), $S_{\vec{h}}^2$ can be 0 or 1. If hidden parameters were to exist, $S_{\vec{h}}^2$ would be equal to 0 for some and to 1 for other such parameters.

We shall show now that the experimental situation illustrated in Fig.23 is inconsistent with any hidden-parameter model. We can prove this by **reductio ad absurdum**.

If hidden parameters were to exist, then quantities $S_{\vec{a}}^2$ and $S_{\vec{h}}^2$ would be simultaneously equal to 0 for some such parameters. It is clear from Fig.25 that $S_{\vec{a}}^2 = 0$ leads to

$$S_{\vec{b}}^2 = 1$$

Similarly

$$S_{\vec{c}}^2 = 1$$

In precisely the same way

$$S_{\vec{h}}^2 = 0$$

leads to

$$S_{\vec{f}}^2 = 1, \quad S_{\vec{g}}^2 = 1$$

It follows from the triangle *BFD* of Fig.24 that

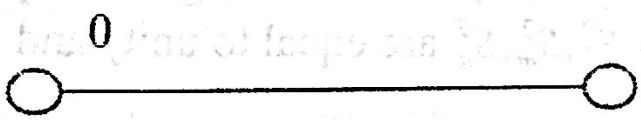
$$S_{\vec{d}}^2 = 0$$

Similarly, it follows from the triangle *CEG* that

$$S_{\vec{e}}^2 = 0$$

We thus have obtained zero values for the square of the projection of the spin for the two mutually orthogonal directions $\vec{d}$ and $\vec{e}$, i.e., we have the impossible situation illustrated in Fig.26.

Hence, the assumption that hidden parameters exist leads to the contradiction. In other words, hidden parameters cannot be introduced into quantum mechanics.



The impossible combination of
S_l^2 *and* S_m^2, *for two orthogonal directions*

Fig 26.

$$\hat{S}_{\vec{m}} = m_x \hat{S}_x + m_y \hat{S}_y + z_z \hat{S}_z \tag{9.19}$$

and the three operators $\hat{S}_x, \hat{S}_y, \hat{S}_z$ are given by the matrices [86]

$$\hat{S}_x = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

$$\hat{S}_y = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix}$$

$$\hat{S}_z = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \tag{9.20}$$

Since the z axis is not special in any way, we may suppose that it lies along the direction of $\vec{m}$, so $\vec{m} = (0,0,1)$. From (9.18), (9.19) and (9.20) we find that

$$\left[\hat{S}_{\vec{l}}^2, \hat{S}_{\vec{m}}^2\right] = \begin{bmatrix} 0 & -\frac{l_z}{\sqrt{2}}(l_x - il_y) & 0 \\ \frac{l_z}{\sqrt{2}}(l_x + il_y) & 0 & -\frac{l_z}{\sqrt{2}}(l_x - il_y) \\ 0 & \frac{l_z}{\sqrt{2}}(l_x + il_y) & 0 \end{bmatrix} \tag{9.21}$$

For nonparallel $\vec{l}$ and $\vec{m}$, i.e., when l_x and l_y are not simultaneously equal to zero, the matrix given by (9.21) equal to zero if and only if $l_z = 0$.

Hence, the quantities $S_{\vec{l}}^2$ and $S_{\vec{m}}^2$ are compatible if and only if the directions of $\vec{l}$ and $\vec{m}$ are orthogonal.

If is tacitly assumed in the above proof that the configuration shown in Fig.23 is realistic. This follows from the following example:

$$\vec{a}=\vec{i}-\vec{j}+\vec{k},\ \ \vec{b}=\vec{j}+\vec{k},\ \ \vec{c}=\vec{i}+\vec{j},\ \ \vec{d}=\vec{i},$$
$$\vec{e}=\vec{k},\ \ \vec{f}=\vec{j}-\vec{k},\ \ \vec{g}=\vec{i}-\vec{j},\ \ \vec{h}=\vec{i}+\vec{j}+\vec{k} \qquad (9.16)$$

where $\vec{i},\vec{j},\vec{k}$ are three mutually orthogonal unit vectors.

The above proof of the impossibility of hidden parameters in quantum mechanics is due to Kochen and Specker [101], modified in [93].

9.6 Missing points in the Kochen-Specker proof

The proof that hidden parameters cannot be introduced into quantum mechanics, given in 9.5, makes use of the following propositions:

1. The squares of the projections of the unit spin (S=1) on the $\vec{l}$ and $\vec{m}$ axes, i.e., $S_{\vec{l}}^2$ and $S_{\vec{m}}^2$ are compatible if and only if the two axes are orthogonal.
2. If the three axes $\vec{l},\vec{m},\vec{n}$ are orthogonal in pairs, then two of the values $S_{\vec{l}}^2, S_{\vec{m}}^2, S_{\vec{n}}^2$ are equal to unity and the third is zero.

We shall prove now the first of these two propositions. The compatibility condition for $S_{\vec{l}}^2$ and $S_{\vec{m}}^2$ is

$$\left[\hat{S}_{\vec{l}}^2,\hat{S}_{\vec{m}}^2\right]\equiv\hat{S}_{\vec{l}}^2\hat{S}_{\vec{m}}^2-\hat{S}_{\vec{m}}^2\hat{S}_{\vec{l}}^2=0 \qquad (9.17)$$

where

$$\hat{S}_{\vec{l}}=l_x\hat{S}_x+l_y\hat{S}_y+l_z\hat{S}_z \qquad (9.18)$$

We now turn to the proof of the second proposition. To do it, we must calculate the operator representing the square of the spin:

$$\hat{S}^2 = \hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2 \tag{9.22}$$

It follows from (9.20) that

$$\hat{S}_x^2 + \hat{S}_y^2 + \hat{S}_z^2 = 2\hat{I} \tag{9.23}$$

where $\hat{I}$ is the unit matrix. Since the values of S_x^2. S_y^2 and S_z^2 are compatible, they must be related by

$$S_x^2 + S_y^2 + S_z^2 = 2 \tag{9.24}$$

As S_x^2, S_y^2, S_z^2 can only be equal to unity or zero, we find that two of them are equal to unity and the third to zero, which was to be proved.

We note, by the way, that (9.24) is a further manifestation of the quantum nature of spin. The classical unit vector would be the subject to the three dimensional Pythagorean theorem

$$S_x^2 + S_y^2 + S_z^2 = 1 \tag{9.25}$$

9.7 Negative probabilities

In the proof given in 9.5, we confined our attention to "reasonable" models. However, it we now turn to "strange" models, we find that the hidden parameters are possible.

A "strange" model of this kind was proposed by Wigner ([115]); see also ([116]-[124]), who introduced the following joint probability density for position coordinate x and momentum p:

$$f(x,p)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\psi^{*}\left(x+\frac{h\tau}{2}\right)e^{ip\tau}\psi\left(x-\frac{h\tau}{2}\right)d\tau, \tag{9.26}$$

where $\psi(x)$ is the wave function and $h=2\pi\hbar$.

This function $f(x,p)$ can be used to obtain the probability $d\omega_x(x)$ of finding a particle in the range $(x,x+dx)$, which is given by the following expression:

$$d\omega_x(x)=dx\int f(x,p)dp \tag{9.27}$$

Similarly, the probability that the particle momentum lies in the range $(p,p+dp)$ is given by

$$d\omega_p(p)=dp\int f(x,p)dx \tag{9.28}$$

However, the expression given by (9.26) does not actually signify that the particle has simultaneously determined position and momentum because the function $f(x,p)$ can assume negative values, which is inadmissible (see 5.1).

It is important to note that the utility of the Wigner distribution (9.26) lies not only in introducing hidden parameters into quantum mechanics, but also in that it is convenient for the evaluation of different quantum effects [125], [126, [127]].

At this point, it is appropriate to recall a simple fact of life: a working man is rarely a conman. Our usual understanding of probability as a nonnegative quantity may mean that our interpretation of the concept of probability is too narrow [128], [129].

Here is a historical analogy. In the sixteenth century, Cardano derived the formula

$$x=\sqrt[3]{-\frac{q}{2}+\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}}+\sqrt[3]{-\frac{q}{2}-\sqrt{\left(\frac{q}{2}\right)^2+\left(\frac{p}{3}\right)^3}} \tag{9.29}$$

for the solution of the cubic equation

$$x^3+px+q=0 \tag{9.30}$$

This solution often involves a negative expression under the square root, i.e., it is a complex number, which was regarded as inadmissible at the time. However, it we formally treat imaginary numbers in the same way as real ones, the imaginary parts eventually disappear and we obtain real roots. For instance, for

$$x^3-3x+\sqrt{2}=0$$

we have

$$x=\sqrt[3]{-\frac{\sqrt{2}}{2}+i\frac{\sqrt{2}}{2}}+\sqrt[3]{-\frac{\sqrt{2}}{2}-i\frac{\sqrt{2}}{2}}.$$

It we write this expression in trigonometric form, i.e.,

$$x=\sqrt[3]{cos135^0+isin135^0}+\sqrt[3]{cos135^0-isin135^0}$$

and use the de Moivre formula

$$(cos\alpha+isin\alpha)^n=cos\,n\alpha+i\,sin\,n\alpha$$

with $n=1/3$, we obtain

$$x=2cos45^0=\sqrt{2}$$

9.8. Bell's inequalities.

Bell's inequalities [168, 170] and their verification [169] are one of the proofs of impossibility to introduce into quantum mechanics local hidden parameters. One form of Bell's inequalities is as follows:

1. Let the object, characterized by three magnitudes $\hat{A}, \hat{B}, \hat{C}$ taking values ± 1 be. In quantum theory noncommuting operators can correspond to those values.

Suppose, however, that particle has simultaneously A, B, C. Having examined ensemble of identical particles, indicating A^+ case, when A takes on value +1 (the same for B and C), A if A takes on value −1 (the same for B and C), we obtain evident equality:

$$N(A^+B^-) = N(A^+B^-C^+) + N(A^+B^-C^-) \tag{9.31}$$

where N – number of articles with corresponding characteristics. From equalities:

$$N(B^-C^+) = N(A^+B^-C^+) + N(A^-B^-C^+) \tag{9.32}$$
$$N(A^+C^-) = N(A^+B^+C^-) + N(A^+B^-C^-) \tag{9.33}$$

it obviously follows:

$$N(A^+B^-) \leq N(B^-C^+) + N(A^+C^-) \tag{9.34}$$

This is exactly one of the Bell's inequalities.

Let operators $\hat{A}$ and $\hat{B}$ not commute. Then, if it is to consider, that A and B properties exist as elements of physical reality, that only interference of device prevents to define them

simultaneously, we thereby suppose the existence of hidden parameters.

So, testing Bell's inequality experimentally, we check if the particle actually "has" A, B, C or they appear when measured as corresponding relations between particle and device. If A, B, C exist simultaneously (though $\hat{A}, \hat{B}, \hat{C}$ don't commute), this is to signify incompleteness of quantum mechanics and necessity to introduce hidden parameters into quantum theory.

Experimental testing of Bell's inequalities [169] shows, that they are invalid and thereby correctness of quantum theory was proved. A, B, C characteristics, corresponding to noncommuting operators don't exist simultaneously.

Chapter 10

Quantum logic versus classical logic

A mathematician could not find his glasses. After a long and fruitless search he called logic to his aid: "I had spectacles, i.c., I had poor vision. But since I can see that they are nowhere to be found, this must mean that I am wearing them". And then by touching the bridge of his noise he verified that he had not taken his glasses off.

10.1 Classical logic

The above proof that hidden-parameter models are impossible, i.e., that quantum mechanics cannot be reduced to classical physics, has all the appearances artificiality. There is a very simple and natural proof [102] that is based on the incompatibility of the logical structures of quantum and classical physics. However, to understand this proof we have to be familiar with classical and quantum logics.

"Classical logic" and "quantum logic" are generally accepted but somewhat infelicitous, phrases. Logic is the science of general laws of thinking. The laws of thinking in quantum mechanics are not different from the laws of thinking in classical physics, in the same way that high-temperature plasma logic is not different from the low-temperature plasma logic.

Quantum logic is the phrase usually applied to mathematical logic augmented by the postulate of superposition (see 6.2) In contrast, mathematical logic without any additional postulates is called classical logic. In other words, classical logic is simply the algebra of statements.

The word "algebra" means that logical operations are denoted by mathematical symbols of addition and multiplication, and the operation upon them constitute a special algebra. (We set forth only that laws, which are necessary for the proof of impossibility of introducing of hidden parameters into quantum mechanics. One can be acquainted with algebra of statements in more detail in [130], [131]).

To each statement A there corresponds a certain set Ω_A of points in phase space in which the statement is true. The set Ω_A is called the support of the statement A. For example, the support Ω_A of the statement $x^2 + p^2 < 1$ is a circle of unit radius on the x, p plane with center at the origin.

If we have two statements A and B, and the statement C is that at least one of the two statements A or B is valid, we say that statement C is the sum of A and B, and we write this in the form of the equation

$$C = A + B \tag{10.1}$$

If the statement C is that both A and B are true, then we say that statement C is the product of these two statements and we write this in the form of the equation

$$C = AB \tag{10.2}$$

Colloquially, logical addition corresponds to the union "or" and logical multiplication corresponds to the union "and".

The operations of addition and multiplication constitute, in classical logic, the set-theoretical addition and multiplication of the supports of the corresponding statements:

$$\Omega_{A+B} = \Omega_A + \Omega_B, \quad \Omega_{AB} = \Omega_A \Omega_B, \tag{10.3}$$

Between some pairs of statements A, B, C, ... we can establish a cause and effect relation

$$A \rightarrow B \tag{10.4}$$

which means that if statement A is true then statement B is also true. In other words, statement B is a consequence of statement A.

The relation $A \rightarrow B$ means that the support Ω_A is subset of the support Ω_B of statement B:

$$\Omega_A \subset \Omega_B \tag{10.5}$$

For example, if statement B is $p>0$ and is a consequence of statement C which states that $p>1$, we have

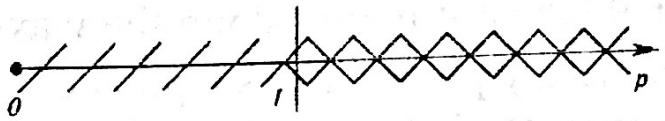
$$C \rightarrow B \tag{10.6}$$

Fig.27 shows the domain Ω_B by the oblique shading whereas the domain Ω_C is shown by the cross hatching. It is clear that

$$\Omega_C \subset \Omega_B \tag{10.7}$$

We note that the cause-and-effect relation does not apply to every pair of statements. For example, it cannot be valid for $p>0$ and $p<0$ because neither is consequence of the other.

The above notation provides us with a compact way of writing down complicated logical structures. For example, consider the statement: "If one of his colleagues was



Cause-and-effect relation in classical physics

Fig 27.

late for prayers, or rumours of a trick played by some school-boys reached his ears, if a dame de classe was seen late at night in the company of an officer, he would be profoundly agitated, repeating constantly that he was afraid it would lead to no good" (Anton Chekhov "The Man who lived in a shell"). It can be written in a compact form:

$$A + B + C \rightarrow DE, \tag{10.8}$$

where A represent "one of his colleaques was late for prayers", B stands for "rumours of a trick played by some school-boys reached his ears", C is "a dame de classe was seen late at night in the company of an officer", D is "he would be profoundly agitated" and E stands for "repeating constantly that he was afraid it would lead to no good".

The cause-and-effect relation clearly involves the operations of addition and multiplications of statements:

$$A \rightarrow A + B, \quad AB \rightarrow A \tag{10.9}$$

called laws of implications, and also the laws

$$\textit{if} \quad A \rightarrow B, \textit{ then } \quad A+B=B \quad \textit{and} \quad AB=A \tag{10.10}$$

called laws of absorption.

We shall illustrate these properties of statements by an example. Suppose that $\vec{\mu}$ is the magnetic moment of atom. Statement A is that this magnetic moment points along the x axis, whereas statement B is that it points along the y axis; statement C states that it lies in the x, y plane. The supports of statements A, B, C are: Ω_A is the x axis, Ω_B is the y axis, and Ω_C is the x, y plane. Next, statement $A+B$ is that the vector $\vec{\mu}$ lies either along the x axis or along the y axis. The support of this statement Ω_{A+B} is the

set of two straight lines x and y. Statement AB is that the vector $\vec{\mu}$ points along the x axis and along the y axis, which is impossible. This is an absurd statement and does not therefore have any support. In mathematically language, the support of this statement is an empty set. Obviously,

$$\Omega_A \subset \Omega_C \tag{10.11}$$

so that

$$A \to C \tag{10.12}$$

10.2 Quantum logic

The structure of phase space in quantum mechanics is quite different from that in classical physics. In classical physics, the support Ω_A of statement A can be any region of phase space. On the other hand, in quantum mechanics, because of the superposition principle, the state of a system described by a wave function ψ is also described by the wave function

$$\Psi = c\psi, \quad \text{(c=const)}. \tag{10.13}$$

We are assumed in this paragraph that the wave functions are not normalized.

Statement (10.13) signifies that the support of statement A, namely, "the state of the system is described by wave function ψ" is not a point in the phase space of ψ, but the straight line L_A, described by (10.13), where the constant c is arbitrary.

Next, if the system may be in both states ψ_1 and ψ_2, it can also be in any state

$$\Psi = c_1\psi_1 + c_2\psi_2 \tag{10.14}$$

In other words, the support of this statement is the plane (10.14) spanning the vectors ψ_1 and ψ_2. In quantum logic, as in classical logic (see (10.1)), we can construct an algebra of statements based on a operations of addition, multiplication, and the cause-and-effect relation [132]-[134]. The operation of multiplication and the cause-and-effect relation induce the same relations between the supports of statements, just as in classical logic:

$$L_{AB} = L_A L_B; \tag{10.15}$$

$$\textit{if} \quad A \to B, \quad \textit{then} \quad L_A \subset L_B \tag{10.16}$$

As far as the operation of addition is concerned, this corresponds not to the set-theoretical sum $L_A + L_B$ of the supports of the individual terms L_A and L_B, but to the set of all the possible sums of vectors $x+y$, where $x \in L_A$ and $y \in L_B$. This set of vector sums is called the direct sum of supports L_A and L_B, and is written as follows:

$$L_{A+B} = L_A \oplus L_B \neq L_A + L_B \tag{10.17}$$

Suppose, for example, that statement A is that the vector $\vec{\mu}$ points along the x axis and statement B is that it points along the y axis, whereas statement C is that it lies in the x, y plane. Statement $A+B$ is then that the vector $\vec{\mu}$ has the form

$$\vec{\mu} = c_1 \vec{\mu}_1 + c_2 \vec{\mu}_2 \tag{10.18}$$

where $\vec{\mu}_1$, and $\vec{\mu}_2$ point along the x and y axes, respectively, and c_1 and c_2 are constants. In other words, statement $A+B=C$ is that the vector $\vec{\mu}$ lies in the x, y plane. (We recall that, in classical logic, proposition $A+B$ is that the vector $\vec{\mu}$ points either along the x or along the y axis.)

10.3 Impossibility of imbedding of quantum mechanics into any classical theory

We shall now show that it is impossible to introduce hidden parameters into quantum mechanics [102]. The proof is based on the fact that the cause-and-effect relation $A \to B$ may be violated as we pass from the classical logic to the quantum one. It is not admissible if we demand that some new classical theory should be the foundation of quantum mechanics.

We shall now prove the impossibility of hidden parameters by **reducto ad absurdum**. Let us suppose that quantum mechanics has some classical foundation. For any statement A in quantum theory there will then be several statements $A(\xi)$ in classical theory, where ξ is the value of some hidden parameter that uniquely determines the results of arbitrary measurements.

In quantum phase space L, statement A corresponds to a set of vectors (L_A is the support of the statement). In classical phase space Ω, a set of vectors corresponds to the same statement ($\Omega_{A(\xi)}$ is the support of statement $A(\xi)$). The hidden parameter ξ then runs through all values compatible with the quantum statement A.

The cause-and-effect relation $A \to B$ means in quantum theory that the support L_A of statement A is the subset of the support L_B of statement B:

$$L_A \subset L_B \tag{10.19}$$

Similarly, in the proposed classical theory with hidden parameter ξ, which is the foundation of quantum mechanics, the cause-and-effect relation $A(\xi) \to B(\xi)$ means that the support $\Omega_{A(\xi)}$ of the statement $A(\xi)$ is a subset of the support $\Omega_{B(\xi)}$ of statement $B(\xi)$

$$\Omega_{A(\xi)} \subset \Omega_{B(\xi)} \tag{10.20}$$

The cause-and-effect relation $A \to B$ then means that (10.19) and (10.20) are equivalent.

To refute the hidden-parameter hypothesis, it is sufficient to provide at least one example, in which (10.19) and (10.20) are not equivalent. For this purpose we consider three vectors $\vec{\mu}_1, \vec{\mu}_2, \vec{\mu}_3$, which lie in the same plane. Suppose that statements A, B, C are that the system is in state $\vec{\mu}_1, \vec{\mu}_2, \vec{\mu}_3$, respectively. Speculating as in 10.2, we then find

$$L_C \subset L_A \oplus L_B \equiv L_{AB} \tag{10.21}$$

Hence

$$C \to A + B \tag{10.22}$$

On the other hand, in the classical logic the corresponding formula would be

$$\Omega_C \subset \Omega_{A+B} \tag{10.23}$$

It is invalid as it was proved in 10.1.

We see that cause-and-effect relations can be violated, so that there is no classical theory, that could serve as the foundation for quantum mechanics, i, e., hidden parameters are impossible.

Chapter 11

The principle of causality

"Almost every philosopher and Scientist uses his own definition of causality, even if he has not succeeded to formulate it clearly".

Mario Bunge, ([135], p.46)

11.1 Manifestations of causality

The principle of causality has many manifestations:

1. Determinism: cause uniquely determines effect.

2. Materiality of cause: cause must be material.

3. No action at a distance: cause has a direct effect only on objects in close proximity. Interactions propagate with finite speed.

4. Asymmetry of time: cause always precedes effect.

5. Materiality of memory: the Past influences on the Future only through the Present.

6. Ultimate goal: nature achieves its goals in the shortest way.

7. Anthropic principle: if world constants would have other magnitudes the existence of intellect beings would be impossible.

8. Le Chatalier-Braun principle: existing physical systems are stable against external disturbances.

9. Inexhaustibility of matter: any physical law is consequence of some deeper law.

11.2 Determinism

Determinism prevails in classical mechanics: "... The laws governing the external would were considered complete in the following sense: if the state of objects at a given time is known completely, then their state at some subsequent time is completely determined by the laws of nature. It is this that we have in mind when we speak of causality" ([17], vol.4, p.317).

The situation is quite different in quantum mechanics: "When an observation is made on any atomic system that is in a given state, in general the result will not be determinate, i.e., if the experiment is repeated several times under identical conditions several different results may be obtained" ([76], p.13). For example, when an electron having a definite momentum passes through an aperture in a screen, it can reach any point on a photographic plate placed beyond the screen. Quantum mechanics provides us with only the probability of finding the electron at different points on the photographic plate.

However, there is a fear of admitting the absence of determinism in quantum mechanics because it somehow identified with absence of physical law.

It is then said that the principle of determinism is valid in quantum mechanics, but it is quantum mechanical, or probabilistic determinism. In other words, the absence of determinism is renamed "quantum mechanical determinism". It is the same as to say that every man is bald, however, in some cases the bald spot is covered with hair.

In our view, there is little point in reversing the meaning of determinism since this word has a history stretching over millennia.

Of course, certain elements of determinism remain even in quantum mechanics.

The variation of the wave function with time is determinate. However, the wave function gives only probabilistic predictions for the behavior of a micro-object.

It is precisely in this sense that we speak of the absence of determinism in quantum mechanics.

"... Despite the successful application of quantum mechanics to many practical problems, there are still serious doubts (and not only in philosophy!) about the final significance and self-consistency of the quantum mechanical formalism. These doubts are serious enough for some physicists to consider that, eventually, a new and intuitively more acceptable picture of the world will replace quantum theory which will come to seen as a set of recipes capable of yielding the correct answer under the experimental conditions attainable in the twentieth century" ([136], p.671),

Einstein considered quantum mechanics to be an incomplete and temporary theory because the basic laws of quantum mechanics included randomness. This is particularly surprising because Einstein himself did introduce randomness into quantum mechanics in his paper on the quantum theory of radiation. "The most important point in this paper by Einstein is the introduction of probability into the description of a micro-object. In addition to the probabilities of spontaneous and stimulated emission, it is necessary to assume a random direction of emission of a photon by the molecule, i.e. the direction of emission cannot be predicted" ([4], p.30).

"My scientific instinct, - wrote Einstein, - drives me against this type of departure from strict causality" ([17], vol.4, p.108).

We note that the elevation of determinism to the status of an absolute principle is in conflict with the principle of causality even in classical physics. It is referred to as Laplace determinism: "All phenomenon, even those that because of their relative insignificance do not appear to depend on the major laws of

nature, are in fact consequences of these laws, just as unavoidable as the periodicity of the Sun ..., all phenomena are related to the past by the obvious principle whereby no phenomenon can arise without its generating cause... We must therefore consider the present state of the universe as the effect of the preceding state and as the cause of the next state... A mind possessing the knowledge of all the forces existing in nature at a given time, and the relative motion of all its components, would, if it were powerful enough to subject these data to analysis, be able to combine in a single formula the motion of all the major bodies of the universe as well as the motion of the smallest atoms: nothing would be uncertain for him; the future as well as the past would be present to his eyes" ([137], p.10-11).

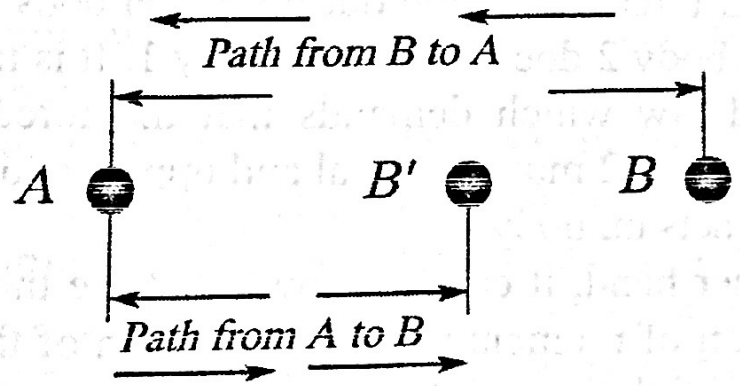
The Laplace determinism is often looked upon as a triumph of science or, more precisely, a triumph of the principle of causality. Actually, it is a rejection of the principle of causality because the concept of a cause includes the possibility of the absence of a cause. If all causes are inevitable they cease to be causes. The entire scenario of the world is then subject to predestination. We then have neither cause nor effect, but merely a rigid sequence of events, one after another. The analog of this in the cinema is the sequence of frames on film which do not cause one another, but merely constitute a series of takes that are photographed strictly in accordance with the script, and are independent. The death of a hero can be photographed before his birth.

We note that determinism has assumed the status of an all-pervading philosophical principle because of the phenomenal success of Newtonian mechanics. Some philosophers rejected the principle of determinism before Newton, and considered it not only false, but actually amoral because it could be used as a justification for practically anything, including crime.

11.3 Materiality of cause and absence of action at a distance

Newtonian mechanics does not require the principle of materiality of cause and allows action at a distance. Actually, the law of universal gravitation states that any motion of a body is transmitted by vacuum and that it instantaneously affects other bodies however distant. In other words, the gravitational interaction propagates through vacuum and does so with infinite speed, which actually constitutes a violation of the principle of causality. That is why the mechanics of Newton was rejected by Leibnitz: "Some men begin to revive under the specious influence of forces, the **occult qualities** of Scholasticism: but they bring us back again into the kingdom of Darkness" ([59], p.272-273).

It was well understood by Newton himself: "That one body may act upon another at a distance through a vacuum, without the mediation of anything else, by and through which their action and force may be conveyed from one to another, is to me so great an absurdity, that I believe no man, who has in philosophical matters a competent manner of thinking, can ever fall into it" [161].



Interaction of two bodies
Fig 28.

It is difficult to understand how such a considerable force can be transmitted through vacuum. If the Earth were to be held in its orbit not by the force of attraction to the Sun but by a steel cable,

the diameter of the latter would have to be greater than the diameter of the Earth.

It would appear that the conflict with the principle that there is no action at a distance can be avoided by saying that gravitation propagates with finite but very high velocity, which only seems to us be infinite.

Newton maintains that his "... law must be regarded not as a final explanation, but as a rule deduced from experiment" ([17], vol.3, p.86). However, in reality, the infinite speed of propagation of interactions adopted in Newton's mechanics is not so much a generalization from observations but rather a philosophical principle.

Indeed, suppose that a body 1 is at a point A and is at rest, whereas a body 2 travels from a point B in the direction of A (Fig.28). During the time that the gravity wave leaving the point B at the initial time takes to reach the stationary body 1, body 2 reaches the point B'. Since the distance AB is greater than AB' the gravity wave propagating from A to B' will arrive before the wave emitted at B at the same moment of time arrives at A. It means that there will be an interval of time during which body 1 already acts on body 2, but body 2 does not act on body 1. It is in conflict with Newton's third law which demands that the force with which body 1 acts on body 2 must be equal and opposite to the force with which body 2 acts on body 1.

On the other hand, it can be shown that the third law implies the conservation of momentum. Any violation of this Law would be a violation of the principle that motion cannot be created or destroyed. Hence an infinite speed of propagation of interaction is, in Newtonian mechanics, a consequence of the philosophical principle of conservation of motion. It means that the speed of propagation of interaction in Newtonian mechanics must in principle be infinite.

We note that "when we use the phrase "in principle" we have in mind a particular theory and its principles that allow some things and forbid others" ([5], p.140).

We recall that, in relativity theory, gravity propagates with finite speed, which equal to the speed of light. This manifestation of the principle of causality is therefore preserved in relativistic mechanics in the sense that there is no interaction at a distance. However, we have just shown that the speed of propagation of interaction must in principle be infinite. Does this mean that our demonstration contains an error? This is indeed the case. The error lies in the fact that we have implicitly assumed that only particles, but not field, can have momentum, so that the finite speed of propagation of gravitation that appears in relativity is not in conflict with the principle of conservation of momentum.

It is therefore clear that the theory of relativity is in better agreement with the principle of causality than the Newtonian mechanics.

11.4 Asymmetry of time

"It is obvious to everybody that the phenomena of the would are evidently irreversible... You drop a cup and it breaks, and you can sit there a long time waiting for the pieces to come together and jump back into your hand... .

The demonstration of this in lectures is usually made by having a section of moving picture in which you take a number of phenomena, and run the film backwards, and then wait for all the laughter. The laughter just means that it would not happen in the real world. But actually is a rather weak way to put something which is as obvious and as deep as the difference between the past and the future... we feel that we can do something to affect the future but none of us or very few of us believe there is anything we can do to affect the past... the most obvious interpretation of this evident distinction between past and future and this irreversibility of all phenomena would be that some laws, some of the motion laws of the atoms are going one way-that the atom laws are not such that they can go either way. There should be

something in the world some kind of a principle that uxles only make wuxles, and never vice versa... but we have not found this principle yet. That is, in all the laws of physics we have found so far there does not seen to be any distinction between the past and the future" ([34], p.96-97).

The asymmetry of time does not appear in the equations of either classical or quantum mechanics. Indeed, Newton's equations of motion are unaffected by time reversal. On the other hand, time reversal makes cause and effect change places. Classical mechanics enable us not only predict the solar eclipses but also to determine the time and the place of previous eclipses (for example, it is possible to deduce a more accurate date and place for the solar eclipse described in **The Lay of Prince Igor**). The requirement that cause must always precede effect is used as a boundary condition for the differential equations of classical mechanics.

The asymmetry of time has the same status in quantum mechanics[8] We note at this junction that the role of time asymmetry is quite different from the role of the above manifestations of the principle of causality. Determinism, materiality of cause, and action at a distance are either already incorporated in the postulates of the theory or are in conflict with the theory (we have shut our eyes to this difficulty). As far as time asymmetry is concerned, this must be formulated explicitly when the corresponding differential equations are solved. This is the reason why physicists usually understand the principle of causality as only asymmetry of time: "... we must also satisfy the principle of causality which demands that any event that has occurred in the system can influence the evolution of the system only in the future, but cannot effect its behavior in the past" ([139], p.192).

[8] In quantum mechanics, the replacement of t with $-t$ means that the wave function ψ must be replaced with the complex conjugate ψ^*, but this has no observable effect.

11.5 Materiality of memory

Very briefly this principle may be formulated as follows: the Past influences on the Future only through the Present. We illustrate it by an example: properties of a solid body depend on its history. Elasticity and plasticity of a sample, for example, of steel, are different for cast steel, chilled steel, tempered steel, cold-rolled steel, and so on. Exactly speaking, properties of the steel depend on its present micro-structural state rather than on its past history. The past history influences on properties of the sample through its present microstructure.

In Newtonian mechanics the equation of motion has the form

$$\frac{d^2\vec{r}}{dt^2} = \vec{F}(\vec{r},t). \tag{11.1}$$

It is the differential equation of the second order. Therefore its solution at $t>0$ depends not only on the $\vec{r}$ at $t=0$ but also on the $\frac{d\vec{r}}{dt}$ at $t=0$. Remembering that

$$\frac{d\vec{r}}{dt} = \lim_{\Delta t \to 0} \frac{\vec{r}(t) - \vec{r}(t-\Delta t)}{\Delta t} \tag{11.2}$$

we see that movement of the particle depends not only on the value of $\vec{r}$ at $t=0$, but also at the time $t = -\Delta t$.

This apparent conflict with the principle of materiality of memory was removed by Hamilton. He considered momentum $\vec{p} = m\vec{v}$ as characteristics of the present state of the particle. In the Hamilton's formulation of Newtonian mechanics the coordinate $\vec{r}$ and the momentum $\vec{p}$ are treated on the same footing. The Hamilton equation

$$\frac{d\vec{r}}{dt} = \frac{\partial H}{\partial \vec{p}}$$
$$\frac{d\vec{p}}{dt} = -\frac{\partial H}{\partial \vec{r}} \qquad (11.3)$$

contain $\vec{r}$ and $\vec{p}$ in the same way.

They may be even interchanged by transforming $\vec{r}$ into $\vec{p}$ and $\vec{p}$ into $-\vec{r}$.

We may say that according to materiality of memory evolution of any physical system must be governed by a differential equation of the first order

$$\frac{\partial f}{\partial t} = \Phi(f) \qquad (11.4)$$

Here f is a function which determines the state of the system.

If the governing equation has the form

$$\frac{\partial^2 f}{\partial t^2} = \Psi(f) \qquad (11.5)$$

then f is an unsufficient characteristic of the system. It must be supplemented by some additional variables.

11.6 Ultimate goal

Enumerating different manifestations of causality Aristotle mentioned expediency: why does a calf have horns? In order to defend himself from wolves. Here a cause is inverted in time: Present depends on the Future rather than on the Past. In reality, of course, the Present depends not on the Future, but on the Past. Such expediency was explained by Darwin: only that beings survive which is adjusted to the environment.

In the Middle Ages the ideas of Aristotle were reduced to absurd by scholastics: "A ran exists to be boiled in a soup; a donkey has long ears to be distinguished from a man; finally, the man exists to eat ran soup and to be different from donkey" (Holbach, teleology).

In the 17-th century the principle of expediency was revived in the form of "Principle of thrift" [140] - "Nature achieves its goals with the simplest means" [141] (Variational Principles of Mechanics).

The simplest example of variational principle occurs when a material point moves inertially being attached to a sphere. In this case the point moves along major arc of the sphere, i.e. along the shortest way between two points.

The variational principles produced a rough discussion of philosophical character. "This interest to the principle of minimum was based on the metaphysical idea, that supremacy of Deity is revealed in Nature. Therefore in the base of every process in nature an intention lies to achieve a definite goal. This goal is achieved in the shortest way, by the simplest means" [142].

In reality a stationary point takes place rather than minimum. So the point which moves along major arc of the sphere moves by the longest way if this way exceeds half of the length of the major arc "Consequently, divine foresight could not act beyond half of a major circle" [142].

11.7 Anthropic principle

The ultimate goal was revived last time in the form of anthropic principle. For instance, the question is: why electron charge equals to $e = 1.6 \times 10^{-19}$ Coulomb? The answer is: if electron charge would be more by an order of magnitude, then the mass of star M, which is necessary to maintain self-sustaining thermonuclear reactions, would increase by two orders of magnitude. In this case the stars with the mass of the order of Sun

mass would not exist at the present time. The stars with such mass existed earlier would have collapsed forming white-dwarf and pulsars [143], [144].

Sometimes the stronger principle is used [87]-world constants must have such magnitude that existence of an intellect beings would be possible: "... "we know in our bones" that the lifetime of the proton must be greater than about 10^{16} yr otherwise we would not survive the ionizing particles produced by proton decay in our own bodies" [145].

11.8 Le Chatelier-Braun principle

One of the varieties of ultimate goal is the Le Chatalier-Braun principle. We illustrate this principle by an example: if a rubber plait is stretched it heats himself. On the contrary, if the plait is heated it shortens ([146], p.162).

In general case the Le Chatalier-Braun principle runs as follows: if one factor in an equilibrium changes, the equilibrium shifts in such a way as tend to annul the effect of the change.

In electrodynamics the Le Chatelier-Braun principle takes form of the Lenz rule: all induction effects are in a direction to oppose the cause producing them. For instance, if a magnet moves into a solenoid the current is excited in the last which creates magnetic field with opposite direction.

The Le Chatelier-Braun principle expresses the condition of stability of the system. For instance, if stretched rubber plait would heat itself whereas heated plait would stretch itself, then an arbitrary fluctuation of the plait length would increase indefinitely.

11.9 Inexhaustibility of matter

We now turn to the principle of inexhaustibility of matter which states that any physical law must have its own cause, i.e., a

deeper law.

Every theory is based on some postulates, i.e., on propositions that cannot be explained by this theory. It means that, all theories violate the principle of causality. Newton's contemporaries rejected his mechanics because it was based on "strange" postulates that had no explanation and involved the causeless motion of an isolated body traveling with constant velocity, and the causeless mutual attraction of all bodies: "... why do bodies continue moving once they start, and what is the origin of the law of gravitation-all it was unknown" ([12], vol.1, p.40).

The principle of inexhaustibility of matter means that the subdivision of physical theories into fundamental (microscopic) and phenomenological (macroscopic, i.e., consequences of fundamental) is temporary in character. A theory can be fundamental only at a given level of development of science. When a more fundamental theory subsequently appears, the earlier fundamental theory becomes phenomenological. However, the new fundamental theory always contains postulates that are its phenomenological elements.

For example, Planck's constant is introduced into quantum mechanics phenomenologically i.e., without any explanation. The main point is, however, that the basis of quantum mechanics-its probabilistic character-is postulated; in other words, the random character of its laws has no cause ([147], p.46-47).

The postulates of quantum mechanics should have a cause: "We cannot be sure that the equal charges of electrons are the result of pure coincidence: this fact should be fundamental in the natural scheme and should have a cause" ([59], p.278).

"Our picture of physical reality" - wrote Einstein - "can never be final. We must always be ready to change it, i.e., to change the axiomatic basis of physics" ([17], vol.4, p.136). It applies to all physical theories, including both the theory of relativity and quantum mechanics which are often treated as absolutely true.

"According to modern thought, even the best of physical laws do not assert an absolute truth, but rather an approximation to the

truth. A physical law is regarded as a model for a certain part of nature, asserting not so much what nature is but rather what it is like" [148].

"... It was perfectly sensible for the classical physicists to go happily along and suppose that the concept of position - which obviously means something for a baseball-meant something also for an electron... Today we know that the law of relativity is supposed to be true at all energies, but sometime somebody may come along and say how stupid we were. We do not know where we are "stupid" until "we outgrow ourselves" ([12], vol.3, p.234).

Let us now consider an alternative point of view, in which the number of laws of nature is finite. It is based on the analogy with geography. The middle ages produced great geographical discoveries: new continents, seas, and even oceans were found for the first time. However, by now, the surface of the Earth is fully explored, except for a few particularly inaccessible areas. A new continent will never be discovered again.

The entire body of modern physics can now be reduced to four types of interaction, namely, electromagnetic, strong, weak, and gravitational. A unified field theory is being created at present and will combine all four interactions. Opponents of the principle of inexhaustibility of matter say that the development of the unified theory will spell the end of physical science. Physics will then develop, they say, exclusively by expanding its application. Fundamentally new theories will no longer appear.

Pascal assumed that infinity small and infinity gross have the same structure and differ only by scales: "From this point of view we must encounter in all levels the same aspects of reality to which the same notion may be applied.

Thus Universe may be like to those dolls, which are put one into another, - absolutely identical, but diminishing in sizes more and more" (Paul Langevin, [149], p.361).

Here is an illustration of this point of view by a verse after Valerii Bryusov [150]:

"We may assume that electrons
Are tiny worlds with dolts and genii,
The worlds with arts, with wars and thrones,
With memory of four millennia".

We believe this point of view to be wrong. The more the scale of phenomenon differs from dimension of our body, the more its laws differ from the habitual laws of classical physics.

Opponents of the principle of inexhaustibility of matter say that the world must not be similar to a matryoshka-one of those multiple Russian dolls, nestled one inside the other. However, this is not an argument but mere sophistry. One could just as well object to a spherical Earth on the ground that it should not be similar to a Watermelon.

We suggest that matter is similar to a matryoshka not in the geometric, but in the causal sense. We do not say that all particles consist of a smaller particles and the latter in turn consist of still smaller particles, and so on ad infinitum. We merely say that any physical law is a consequence of more fundamental laws.

We have shown in 9 and 10 that it is impossible to construct a determinate theory that leads to the same observational results as quantum mechanics. A more deeper determinate theory should therefore be able to predict other experimental results. At the same time, for the parameter values accessible to modern science, the more fundamental theory should pass over to quantum mechanics. This more fundamental future theory will violate other philosophical principles because it will not be fundamental either.

The more fundamental theory will be even "stranger" than quantum mechanics because it will be even more remote from our everyday experience. Niels Bohr was once asked how he saw a hypothesis that was claiming to lead to a fundamental theory. He replied: "It is not lunatic enough for the purpose".

The theory of relativity has now invalidated Newton's action at a distance which is in conflict with at the principle of causality. However, in Newton's time, attempts to reconcile action at a distance with causality were just as hopeless as any attempts today to reconcile quantum mechanics with determinism.

Chapter 12

Conceptual aspects of quantum mechanics

"You have written that a man derives from some kinds of monkeys such as marmosets, orangutans, and similar creatures. Please, forgive the old man, but I cannot agree with you to this important point of view and, indeed, I should ask a question in return. If a man, the dominant figure in the world, the most intelligent of all mammals, has descended from a stupid and ignorant ape, then he should have a tail and an uncultivated voice. You have written and published in your erudite essay... that even the largest star, the Sun, has black spots upon it. This can not be, because it can never happen... for what is the purpose of such spots if there is no need of them?"

Anton Chechov, "A letter to a learned neighbour".

12.1 The micro-world versus its macroscopic description.

"Bohr's suggestion that the quantum-mechanical description of the properties of an atomic object must be combined with the classical description of the means of observation (i.e., the apparatus) has played an essential role ... in the interpretation of quantum mechanics. In his papers devoted to fundamental questions in quantum mechanics, Bohr insisted that it was essential to consider the experiment as a whole, and to extend its description to include instrumental readings" ([11], p.463).

Thus, in his discussion of the Einstein-Podolsky-Rosen paradox, Bohr considered the passage of an electron through an aperture and wrote: "... We start by assuming that our screen with a slit cut in it, the second screen with several slits parallel to the

first, and the photographic plate are initially rigidly coupled to a heavy base... But we could have used a different apparatus, in which the first screen was not rigidly coupled to the rest of the apparatus" ([13], p.183).

If the screen is rigidly coupled to the base, the transverse coordinate of electron passing through its slit is determined, but the transverse component of its momentum is not. On the other hand, if the screen is "not coupled to the rest of apparatus" the transverse momentum component of the electron crossing it is zero while its transverse coordinate is undetermined. In the first case we have an electron with a particular position coordinate, but undetermined momentum, whereas in the second we have an electron with a determined momentum, but undetermined position.

"The recording of observations", wrote Bohr, "reduces in the final analysis to the creation of the stable marks on measuring devises, e.g., spots on a photographic plate, produced by an incident photon or electron" ([13], p.603).

This feature of quantum mechanics is often made absolute. Quantum mechanics is then treated as a science limited to the study of the interaction between a micro-object and a macroscopic device, and no attempt is made to examine the micro-objects itself.

"The "output" of any instrument always presents a macroscopic phenomenon: the rotation of a pointer, the formation of droplets in the Wilson cloud chamber, the blackening of a photographic emulsion, and so on... . It is therefore correct to say that quantum mechanics investigates the micro- world in so far it relates to the macro-world. Macroscopic (classical) instruments present us with reference systems in which the state of micro-systems is defined in quantum theory" ([5], p.84).

The analogous point of view is encountered in mathematics. It is said that "we cannot understand infinity because our brain is finite". However, if we were to argue this way, we would conclude that we are unable to recognize rams because we are not rams ourselves.

Indeed, the statement, that a micro-objects must always be treated in relation to a particular measuring instrument is confined to philosophical publications which Bohr himself referred to as "pseudorealistic" ([13], p.414).

Descriptions of real experiments that involve the micro- world often employ expressions such as "a 100 MeV proton" or "a hydrogen atom in the S-state". The associated macroscopic apparatus is very rarely described, i.e., little reference is made to the way in which these states are produced and measured.

An electron with momentum $\vec{p}$ is described by the wave function

$$\psi = exp(i\vec{p}\vec{r}/\hbar)$$

and not in terms of the readings of macroscopic instruments. Indeed, this formula is valid for any method of observation.

We are forced, say the supporters of the absolute interpretation of the role of macroscopic instruments, to describe quantum-mechanical objects in the language of classical physics, which is the language of our instruments and also the language in which we think. This again is incorrect. Indeed, the language of instruments is the language of milliamperes, the number of counts, the blackening of photographic plates, and so on. However, we think in the nonclassical language of quantum states, Pauli's principle, and so on. It is only in the above philosophical paper by Bohr ([13], p.414) that he describes the "first screen" and the method in which it is supported when the state of the electron is measured.

The recording of the experimental results in the form of instrumental readings or spots on a photographic plate is the function of a laboratory assistant and not of the research scientists. The latter is more concerned with the interpretation of the experiment, i.e., with drawing of certain conclusions about the properties of the microobject "as such". No journal will accept for publication a report confined to the description of "spots on a

photographic plate". Potential authors are warned about this, for example, in the editorial note in Physical Review Letters.

Of course, the readings of macroscopic instruments constitute the final stage of any experiment, but this does not mean that the physics can be reduced to instrument readings. Indeed, every experimenter has not only a pair of eyes but a head too!

Similarly, algebraic derivations end in formulas that relate Latin letters such as a,b,c, and so on, but this does not mean that algebra is reduced to Latin.

"...Excessively underlining of the role of apparatus gives occasion to reproach Bohr in underestimating necessity of abstractions and in forgetting that there are investigated properties of microobject rather than the reading of a measuring device in physics. The properties of atomic objects such as charge, mass and spin, the form of the energy operator, and the law describing the interaction of particles with an external field, are, on the one hand, completely objective and can be treated separately from the means observation, and, on the other hand, they require new quantum-mechanical concepts for their formulation. This applies in particular to the formulation of the many-body problem" [V.A.Fock ([11]. P.463)].

Feynman was also against attaching absolute significance to macroscopic instruments: "It is not true that science can be constructed by using only concepts that are directly related to experiment. Indeed, even quantum mechanics operates with both the amplitude of the wave function and the potential, as well as other mental constructs that cannot be measured directly" ([12], vol.3, p.233).

"Every observation", writes Heisenberg, "leads to a certain discontinuous change in the mathematical quantities characterizing the atomic process and, consequently, to an abrupt change in the physical phenomenon itself... For heavy bodies, for example, planets revolving around the Sun, the pressure due to sunlight which is reflected from their surface and which is necessary for observation plays no part in this; however, for the

very small particles of matter, each observation does affect their physical behavior because their mass is so small" ([151], p.27-28).

Here we encounter two different entities, namely, "observation" and "physical process", which are in conflict with the alleged absolute role of macroscopic description. These two different entities correspond to two different mathematical formalisms. Thus, the Schrödinger equation describes the evolution of micro-system as such, and without interaction with an observer or a macro-object. On the other hand, the reduction of the wave packet describes the process of measurement.

12.2 Is the wave function only an information in the observer's possession?

It is sometimes said that the wave function is only an information about a objects. Of course, any record constitutes information. More than that, any additional information about a micro-object forces us to modify the wave function (6.3 and 6.4). However, the supporters of the "informational interpretation of the wave function" would make us believe something more: they maintain that it is meaningless to speak of the state of a micro-object and that we can only say something about information. The question is: information about what? If this is information about the micro-object then the micro-object must objectively exist because... information is a reflection of the objective laws of nature as represented by modern science" ([152]).

On the other hand, if this is abstract information, then it is totally unrelated to quantum mechanics. Abstract information can describe practically anything, from the result of a football match to the evolution of the universe.

"Of course, we can examine the observer's knowledge of physics (but not the physics itself), which is not our purpose here. For example, the observer's knowledge of a particular system may radically change when he is hit on the head and loses his memory,

or when he receives new information. Subjectivists tend to ignore the former and emphasize the latter ([103], p.371-372).

The wave function is not only information: it has an objective meaning too. It describes the motion of micro-particles in an external field ([4], p.129).

A common objection to the objectivity of the wave function ψ is that it is not uniquely defined. The physically meaningful quantity is $|\psi|^2$ and not ψ itself. The same observational results are obtained when we multiply all wave functions by i or -1. However, this is not a real objection because most mathematical objects are not uniquely defined. For example, nothing changes if we write $\frac{1}{2}$ instead of $\frac{3}{6}$ or $\frac{5}{10}$.

12.3 Physics and philosophical principles

"The interpretation of Bohr's ideas in the spirit of positivism, performed by some of his successors, naturally gave rise to a reaction that resulted in the rejection of the new ideas in the name of materializm (de Broglie, Vigier, and so on). The principal factor that led these scientists to reject the usual probabilistic interpretation of quantum mechanics is the erroneous belief that the probabilistic interpretation constitutes a rejection of the objectivity of the micro-world and its laws..." ([11], p.464).

Einstein did not accept quantum mechanics although he was himself responsible for supplying its fundamental stone (the theory of the photoelectric effect): "Quantum mechanics is the last, highly successful creation of theoretical physics... . The quantities that appear in its laws do not claim to represent physical reality itself; they provide only the corresponding probabilities... . I am nevertheless inclined to think that physicists will not for a long time be limited to such an indirect description of reality" ([153], pp.243-247).

Einstein was unwilling to accept quantum mechanics because he believed that the fundamental laws of nature should be deterministic and not random. However, we noted in 11.9, that the existence of fundamental laws of nature would be in conflict with the principle of causality because a fundamental law is a law without cause. On the other hand a fundamental that can be explained ceases to be fundamental. We can illustrate this by the law governing the free fall of a body on the Earth. Aristotle explained this by saying that a body raised above ground tends to return to its natural position, i.e., to the Earth's surface. However, this does not explain anything because it is not clear why a body raised above ground is in an unnatural position whereas a body resting on the ground is in a natural state. Such explanations were very popular among scholastics. They were ridiculed by Molière: in one of his plays, a scientist says: "Opium makes you sleepy because it has a soporific effect".

Actually, the free fall of bodies is a consequence of Newton's law of universal gravitation. It can be shown that, according to this law, not all bodies fall to the Earth: the Moon does not, nor do satellites. When its velocity is high enough (much higher than was possible in Aristotle's time), a body raised above ground may actually leave the Earth. Thus, having explained the law of free fall for all bodies, we have shown that this law is generally invalid because it turns out that not all bodies undergo free fall.

No individual physical theory can be absolutely true. All such theories have a limited range of validity. "I suggest that, strictly speaking, and with the exception of mathematics, there are no inviolable principles" (M. Born, [45]).

All theories involve a degree of distortion of reality. Every physical theory is in conflict with some experiment (possible one that has not yet been performed). Moreover, every physical theory disrupts some existing connections and thus contradicts certain philosophical principles ([154]). On the other hand, "all principles known to us are mutual incompatible, so that some things have to be rejected" ([34], p.147).

While building a new fundamental theory one cannot base himself upon old physical laws. Therefore the new fundamental theory cannot be created without guidance of some philosophical ideas. But "we can't be terrorized by the verdict "philosophically false". It means only that the new physical laws which appear now disguised as philosophical principles with pretensions of eternal validity" ([155], p.175).

"The test of all knowledge is an experiment" ([12], vol.1, p.47).

Philosophical principles change themselves with elapse of time. They often transform themselves into contrasts. For instance, at the time of Aristotle and later till Newton there was belief that two kinds of physical laws existed --- celestial laws and terrestrial laws: "The heavenly bodies and their motions were contrasted, as incorruptible and immutable , with the corrupt and ever-varying things of earth" ([91], p.32).

Only Newton rejected the principle of distinction between terrestrial and celestial laws. Even Galileo wrote about two laws of inertia: 1) terrestrial bodies move uniformly and rectilinearly; 2) heavenly bodies move according to inertia along closed paths.

On the contrary, now it is assumed that all physical laws acting on the Earth are true in arbitrary point of Universe (the Cosmological Principle).

Quantum mechanics is in conflict with determinism: single events are not determined. Newtonian mechanics was in conflict with the asymmetry of time, the materiality of cause, and the contact interaction.

Heisenberg considered quantum theory to be unsatisfactory because in this theory the philosophical principle of nonlinearity is violated. According to this principle all linear laws are only the first approximation in description of nature: "As nonlinearity plays so important role in nature it is possible that we shall be forced to replace even such essentially linear theory, as quantum mechanics, by nonlinear one" ([156]).

However, if every physical theory can contradict philosophical principles, is there some limitation on a physical theory that follows from philosophy?

We think that sole obligatory philosophical principle is the principle of objective reality. External to human being reality is assumed to exist, whether or not it is observed by someone.

12.4 Statements accepted as a true without proof.

All explanations that are based on some initial assumptions are believed to be correct, i.e., they are based on postulates. The conditions that such postulates must need are radically different for laymen and for scientists.

For a layman, a postulate must be obvious and readily visualized. Moreover, the layman is not bothered by the fact that different postulates are required to explain different effects.

On the contrary, in science, we should be able to explain all known effects in terms of small number of postulates. It is, however, practically impossible to ensure that the postulates are obvious or readily visualized. The further we are from our everyday experience, the stranger the postulates are.

The merit of Newton consists not only in formulation of fundamental laws of physics and creation of higher mathematics. Newton was the first to carry out an axiomatic construction of a physical theory such as in geometry of Eukleidës (Eukleidës made a distinction between postulates and axioms. Postulates are initial statements which are specific for the theory. For instance, "through two points always single straight line passes". On the contrary, axioms are logical rules for deduction. For example, "if equal quantities are added to equal ones, the result will be equal". We do not make distinction between postulates and axioms).

The character of axioms in physics is opposite to their character in geometry of Eukleidës. Namely, in Eukleidian

geometry an axiom is a truth which is so evident that no proof is need.

In physics axioms do constitute a basis of theory. They are far from immediate perception and therefore they seem to be wild.

The axiomatic character of Newton's physics was criticized sharply by both philosophers and physicists. For instance, philosopher Berkley wrote: "Can conclusions be scientific when principles are not evident? And can principles be evident if they cannot be understood?" ([155], p.172).

On the other hand, physicist Leibnitz also denied the axiomatic construction of theory, because it contradicts to the causality principle: "Some men begin to revive, under specious influence of **force, the occult qualities** of Scholasticism; but they bring us back again into the Kingdom of Darkness ". (see [59]). However, it is impossible to construct any strict theory without introducing of axioms, i.e., without disruption of the cause-effect chain.

Axiomatic construction of theory makes it to be abstract: "Contemporary and subsequent criticism of Newtonian mechanics (including criticism by Huygens and Leibnitz) was largely concerned with its abstract construction, just as Maxwell's electrodynamics, Einstein's theory of relativity, and, especially, quantum mechanics were subsequently criticized for their high degree of abstraction, and were regarded as difficult to visualize..." ([157], p.55).

12.5 An obvious theory versus a deep theory

"In the beginning of the history of experimental observation,... or any other kind of observation on scientific things, it is intuition, which is really based on simple experience with everyday objects, that suggests reasonable explanations for things. But as we try to widen and make more consistent the description of what we see, as it gets wider and wider and we see a greater range of

phenomena, the explanations become what we call laws instead of simple explanations. One odd characteristic is that they often seem to become more and more unreasonable and more and more intuitively far from obvious...

There is no reason why we should expect things to be otherwise, because the things of everyday experience... involve... conditions that are special and represent, in fact, a limited experience with nature. It is a small section only of natural phenomena that one gets from direct experience. It is only through refined measurements and careful experimentation that we can have a wider vision. And then we see unexpected things: we see things that are far from what we would guess - far from what we could have imagined. Our imagination is stretched to the utmost, not, as in fiction, to imagine things that are not really there, but just to comprehend those things that are there" ([34], p.115-116).

In the seventeenth and eighteenth centuries, two theories were put forward to explain the motion of celestial bodies, namely the theory of Descartes and the theory of Newton ([158], p.93-95).

Descartes considered that the necessary postulates had to be visualized and obvious. Similar demands were introduced at the beginning of the Renaissance: the starting point should be reason and not religious dogma.

The requirement that the postulates had to be obvious and readily visualized was a step forward as compared with blind faith. However, it had to be regarded as idealistic because thought and not experiment was regarded as primary. Moreover, the principal instrument of cognition was considered to be intuition.

Descartes derived the properties of nature by reasoning. For example, nature's abhorrence of vacuum was justified as follows. Matter is extension, i. e., space. Vacuum is impossible because one cannot imagine a place in the universe in which there is no space with length, depth, and width.

Descartes considered that matter was inert and passive. The followers of Descartes rejected inertial motion and attraction at a distance because they considered them to be causeless.

Descartes thought that natural motion is not rectilinear (which could not be directly observed) but circular because it could be seen on the celestial sphere. According to him, all planets are brought into motion by vortices in the ether.

Descartes confined himself to a qualitative explanation of the motion of celestial bodies and made no attempt to explain quantitative relationship, such as, for example, Kepler's laws of motion.

In contrast to Descartes, Newton considered that experiment was the exclusive source of our knowledge. Theory arises only as a generalization of individual uncoordinated facts.

According to Newton, the rotation of planets around the Sun are due to the laws of inertia and the force of universal gravitation. This force acts instantaneously and always points in the radial direction, i. e., at right angels to the orbit, which roughly speaking, can be regarded as circular.

Descartes' postulates seemed natural because they involved interaction by contact. Uniform motion required the constant application of a force pointing along the trajectory. On the other hand, Newton's law of inertia seemed strange because motion with constant velocity required no cause.

Next, Newton's second law made use of previously unknown and strange idea, namely, that of acceleration, i.e., the derivative of velocity or, in other words, the second derivative of position. To formulate his second law, Newton had to create the differential calculus - a new branch of mathematics. Integral calculus also had to be created by him to solve the equations of motion that arose in this way. The differential calculus and the integral calculus seemed to Newton's contemporaries to be so complicated that they were combined under the respectful title of "higher mathematics".

"In his researches, Newton always employed his new mathematics, but when he presented his results he often used the old synthetic method of presentation in order to avoid placing technical complexities in the way of an appreciation of his results" ([157], p.61).

Many of Newton's contemporaries rejected instantaneous attraction at a distance. "The advance of science", wrote Mach, "would undoubtedly be impeded if we were to abandon the assumption of action at a distance because we have no true or even apparent explanation of it" ([3], p.245).

The advantage of Newtonian mechanics was that it led directly to the three laws of Kepler. However, the force acting along the radius produced circular motion. This conclusion was obtained by long abstract calculations and was therefore unconvincing. The fact that all planets revolve in the same sense was obvious in Descartes theory, but required the application of an abstract theory in Newton's case.

Newtonian mechanics seems to us understandable and natural because we first encounter it in our childhood, when we are prone to instinctive imitation and our critical faculties are not fully developed. Newton's contemporaries, on the other hand, gave his theory a hostile reception.

Today, the remnant of the lively disputes between Newton's and Descartes' supporters is the unit of length, i.e., the meter which was defined as $\frac{1}{40000000}$th part of the length of the Paris meridian. However, this unit of length was very inconvenient in practical applications and was soon replaced by the distance between two marks on a platinum-irridium rod used as standard. The length of the Paris meridian is then no longer equal to exactly $4*10^7$m.

The question is why was it not clear at the very outset that the length of the Paris meridian would be very inconvenient as a standard?

The answer is that it was clear. So why was the meter defined in this way? The answer is that Laplace, who was the chairman of the metric commission, wished to determine whether Newtonian or Cartesian mechanics was valid.

According to Newton, the terrestrial globe is slightly stretched at the equator by centrifugal forces, whereas Descartes proposed

that it is slightly compressed along the equator by the action of the ether. Two very expensive expeditions, one to the equator and the other to the polar region, were proposed as means of verifying who was right. To organize these expeditions, Laplace proposed that the unnatural unit of length commonly used at the time, namely, the length of Charlemagne's foot, should be replaced with a natural standard, namely the length of the circumference of the Earth. Expeditions were dispatched to Brazil and Finland, and confirmed that the Earth was stretched at the equator, thus verifying the validity of Newton's theory.

However, while Newton tried to understand the motion of celestial bodies, assuming it as given, Descartes speculated how the universe evolved to its contemporary form and structure, having originally arisen in accordance with natural laws.

Since Newtonian mechanics has now been replaced by the more rigorous relativistic theory, in which interactions propagate with finite speed and the universe is nonstationary, we can say that Descartes and not Newton was right. However, this is like saying that a nonworking clock shows the right time twice a day.

Descartes' theory was obvious and readily visualized, but qualitative in character. It contained no quantitative laws that could be used in an experimental verification. Descartes incorrectly guessed the laws of motion, so that his theory is fruitless.

Newton's strange and abstract theory, on the other hand, provided a highway for the development of science, by which humanity has arrived at its present stage of civilization.

Our common sense has now reached a stage where Newtonian mechanics seems obvious and readily visualized. Quantum mechanics, on the other hand, is still natural only for well-prepared scientists.

"... the main object of physical science is not the provision of pictures, but is the formulation of laws governing phenomena and the application of these laws to the discovery of new phenomena. If a picture exists, so much the better; but whether a picture exists

or not is a matter of only secondary importance" (Dirac P., [76], p.10).

Conclusion

Quantum mechanics contradicts to some philosophical principles and to the common sense. But it is the fate of every deep theory.

Quantum mechanics is "the only thing that provides a satisfactory logical explanation of the dual (corpuscular and wave) properties of matter" ([17], vol.3, p.295-296). The predictions of quantum mechanics have been confirmed experimentally for a huge number of physical systems ranging from nuclear reactors to biological molecules. ([136], p.671).

Bibliography

[1] Einstein A. and Infeld L. The Evolution of Physics: The Growth of Ideas from Early Concepts to Relativity and Quantan, Cambridge University, Press., Cambridge, 1961. [Russ. Transl. Fizmatgiz., M., 1965]

[2] Heisenberg. W." The meaning of value and beauty", Voprosy Filosofii, №12, 1979 [Engl. Transl. in: Across the Frontiers., Harper, N.Y., 1974, p.166]

[3] Mach E., Knowledge and Error [Russ. transl.], Izd. Skirmunta, M., 1909.

[4] Migdal A.B. Quantum Physics for Adults and Children. [in Russian], Nauka, M., 1989.

[5] Blokhintsev D.I. Fundamental Problems in Quantum Mechanics. [in Russian], Nauka, 1987.

[6] Heisenberg W., Physics and Philosophy: Part and Whole [Russ. Transl,], Fizmatlit, M., 1989 [Engl. Transl., Penguin, London, 1989].

[7] Dewitt B.S. and Graham R.N., Am. J. Phys. V.39, p.724, 1971.

[8] Deitz D. Cultural Values of Physics, Journ. Appl. Phys., 1939, v.10, №2, p.86-95.

[9] Stratton J.A. Electromagnetic theory, McGrow-Hill book Co., New York, 1941.

[10] Richtmyer F.K., Kennard E.H., Introduction to Modern Physics, New York, McGrow-Hill book Co., 1942.

[11] Fock V.A., Usp. Fiz. Nauk, 1957,v.62, p.461.

[12] Feynman R.P., Leighton R.B. and Sands M., Feynman Lectures on Physics, Addison-Wesley, Reading Mass., 1964 [Russ. Transl., Mir, M., 1967].

[13] Bohr N., Collected Papers [Russ. Transl.], 2, Nauka, M., 1971.

[14] Gribanov D.P., "Philosophical views of Albert Einstein" Voprosy Filosofii, 1979, №2, pp15-17.

[15] Laitko Kh. and Khoffman D., "Ernst Mach: 150th anniversary of his birth", Voprosy Istorii Estestvoznaniya i Tekhniki, 1988, №4, 45-47.

[16] Mach E., The Principle of Conservation of Work - Its History and Roots [Russ. Transl.], SPb, 1909.

[17] Einstein A., Collected Scientific Papers [Russ. Transl.], v.3 and v.4, Nauka,M., 1966 and 1967.

[18] Cited in: Gribanov D.P., Philosophical Views of Einstein and the Development of the Theory of Relativity [in Russian], Nauka, M., 1987.

[19] Cited in [3], p.242.

[20] Berkley G., Treatise concerning the Principles of Human Knowledge, vol.1 of Works, edited by A. Fraser, Oxford, 1871.

[21] Heisenberg W., "Developments of concepts in 20th century physics", Voprosy Filosofii, 1975, №1. pp.79-88.

[22] Cited in [25], p.18.

[23] Sudbery A., Quantum Mechanics And the Particles of Nature: An Outline for Mathematicians, Cambridge University Press, Cambridge 1986 [Russ. Transl., Mir, M., 1989].

[24] Brillouin L., Am. J. Phys. v.29, p. 318, 1961.

[25] Rumer Yu.B., The Birth of Matrix Mechanics, In: 50 Years of Quantum Mechanics [in Russian], Nauka, M., 1979, pp. 3-21.

[26] M.A. Markov, Thinking about Physics [in Russian], Nauka, M., 1988.

[27] Feynnberg E.L. In: Problems of Theoretical Physics and Astrophysics. The volume devoted to the 70th anniversary of V.L. Ginzburg, Nauka, M., 1989.

[28] Cited in [6], p.318.

[29] Sommerfeld A. Vorlesungen über theoretische Physik. Band IV, Optik. Wiesbaden, 1950, paragraph 1.

[30] Bridgman P.W., The Logic of Modern Physics, N.Y., 1958.

[31] Barthel R., Am. J. Phys.,v.24, pp.268-272, 1956.

[32] Hill T.I., Contemporary Theories of Cognition [in Russian], Progress, M., 1965, Ch.4.

[33] Clemence G.M., Rev. Mod. Phys., v.29, p.2, 1957.

[34] Feynman R.P., The Character of Physical Law, M.I.T.Press, Cambridge Mass., 1965 [Russ. Transl., Fizmatlit, M., 1987].

[35] Tisza L., Rev. Mod. Phys., v.35, p.151, 1963.

[36] Cited in [35].

[37] Landau L.D. and Lifshitz E.M., Fluid Mechanics, Pergamon Pres, Oxford, 1984 [Russ. Original, Fizmatlit, M., 1986].

[38] Einstein's views cited in [c4], p. 24.

[39] Cited by [159], p.29.

[40] Belyi Yu.A. Johannes Kepler, Nauka, M., 1971 [in Russian].

[41] Danilov Yu.A., Smorodinskii Ya.A., Uspekhi fizicheskih nauk, 1973, v.109, pp.175-209.

[42] L. de Broglie, Revolution in Physics (New Physics and Quanta) [Russ. Transl., Atomizdat, M., 1965].

[43] Mandel'shtam L.I., Complite Collected Paper [in Russian] AN USSR, M.-L., 1950, Ch 5.

[44] Landau L.D. and Lifshitz E.M., Classical Theory of Fields, Pergamon Press, Oxford, 1969 [Russian original, Fizmatlit, M., 1967].

[45] Born M., "The state of ideas in physics and perspectives for their development", in Problems of Causality in Quantum Mechanics [Russ. Trunsl.], I.L., M., 1955, pp.103-121.

[46] Loinger A., Nucl. Phys., 1968, v.A.108. p.245.

[47] Dodonov V.V. and Man'ko V.I., " Generalization of the Uncertainty relations in quantum mechanics", Trudy FIAN,

v.183, p.5, 1987. [Proc.(Tr) P.N. Lebedev Phys. Inst. Acad. Sci. USSR, v.183, 1987].

[48] Pauli W., Principles of Wave Mechanics [Russ. Transl.], Gostechizdat, M.-L., 1947.

[49] Gislason E.A, Sabelli N.H., J.W. Wood J.W. Phys. Rev. - 1985, -V.A31, N 4. - P. 2079-2081.

[50] Heisenberg W. Physical Principles of Quantum Theory [Russ. transl.] - M.-L.: GTTI. - 1932.

[51] Weil H. The theory of groups and quantum mechanics - Dover Publications inc. - 1931. Chap.2, paragraph 7.

[52] Cited in C. Kittel, Introduction to Solid State Physics - N.Y. Wiley. - 1971. [Russ. Transl., Fizmatlit, M., 1978].

[53] Landau L.D. and Lifshitz L.D. Quantum Mechanics. (Non-relativistic Theory) - Oxvord: Pergamon Press. - 1977. [Russ. Original, Fizmatgiz, M., 1974].

[54] Fock V.A. Usp. Fiz. Nauk. - 1951. - V.45. - P.3. (in Russian).

[55] Feshbach H. and Weisskopf V. Ask a foolish question... Phys. Today. - 1988. V.41. P.9.

[56] Ginzburg V.L. Usp. Fiz. Nauk. - 1959. - V.69. - P.537. (in Russian).

[57] Tsytovich V.N. Nonlinear Effects in Plasmas - Nauka, M., 1962 (in Russian).

[70] Frenkel' Ya.I. Statistical Physics - M. - L.: - 1948. Izd. AN USSR. (in Russian).

[71] Green H.S. Proc. Roy. Soc. - 1947. - V.A189, N 1016. P.103.

[72] Silin V.P. Introduction to the kinetic theory of gases - M.: Fizmatlit. - 1971. (in Russian).

[73] Billingsley P. Ergodic Theory and Information - New York: Joghn Wiley and Sons. - 1965.

[74] Hopf E. Ergoden Theorie Ergebnisse der Mathematik und inner Grenzgebiete - Berlin: Springer. - 1937. - V.2.

[75] Heiseberg W. Philosophical Problems of Atomic Physics - I.L. - M.: - 1953. (in Russian).

[76] Dirac P.A.M. Principles of quantum Mechanics - Oxford: Clarendon Press. - 1958. [Russ. transl., Fizmatlit, M., 1960].

[77] Jauch J.M. Helv. Phys. - 1964. Acta 37, p. 293.

[78] Jauch J.M., Winger E.P. and Yanase M.M. Nuovo Cimento. - 1967. - V.48. P.144.

[79] Fukuda R. Phys. Rev. A. - V.35, N 1. P.8-13.

[80] See 159, p.650.

[81]Omnes R. Rev. Mod. Phys. - 1992. - V.64, N 2. pp.339-382.

[82] Krichevskii I.R. Concepts and Fundamentals of Thermodynamics - M.: Khimiya. - 1970. [in Russian].

[83] Landau L.D. and Lifshitz E.M. Statistical Physics - Oxford. Pergamon Press. - 1962. [Russ. Original, Nauka, M., 1964].

[84] J. von Neumann Mathematical Foundations of Quantum Mechanics - Princeton: Princeton University Press. - 1955. [Russ. transl., Nauka, M., 1964].

[85] Shilov E.G. Mathematical Analysis, Finite-dimensional Linear Spaces - M.: Nauka. - 1969. (in Russian).

[86] Schiff L. Quantum Mechanics - McGraw-Hill. - 1957. [Russ. transl., IL., M., 1959].

[87] The Works of George Berkley Bishop of Cloyne. - London: Ed. Lucea A.A., Jessop T.E. - 1949. V.2. P.30-31.

[88] Lyuboshits V.L. and Podgoretskii M.I. Zh. Eksp. Teor. Fiz. - 1968. V.55. P.904. [Sov. Phis. JETP 1968, v.28, p.469].

[89] Glauber R. In collection: Quantum Optics and Electronics. - New York, Gordon and Breach, Science Publishers, 1965, - in lecture 16.

[90] Sciamanda R.J. Am. J. Phys. - 1970. V.37. P.1128.

[91] Paul H. Rev. Mod. Phys. - 1986. V.58, N 1. P.209-231.

[92] Spasskii B.I. and Moskovskii A.V. Usp. Fiz. Nauk. - 1984. V.142. P.599. [Sov. Phys. Usp., 1984, v.27, p.273].

[93] Akhiezer A.I. and Polovin R.V. Usp. Fiz. Nauk. - 1972. V.107. P.463. [Sov. Phys. Usp., 1972, v.15, p. 500].

[94] Grib A.A. Usp. Fiz. Nauk. - 1984. V.142. P.619. [Sov. Phys. Usp., 1984, v.27, p.284].

[58] Polovin R.V. Applied Theory of Random Processes - Kharkov: Vyshcha Shkola. - 1982. (in Russian).

[59] Wittaker E.T. Aristotle, Newton, Einstein Philos. Mag. - 1943. - V.34. P.266.

[60] L. de Broglie Will Quantum Mechanics Remain Indeterministic? in: Problems of causality in Quantum Mechanics - I.L.-M.: - 1955. - P.11-33. (in Russian).

[61] Heisenberg W. "Developments in the interpretation of quantum mechanics", in: Niels Bohr and the Development of Physics -I.L.-M.: - 1958. - P.23ñ45. [Russian transl.].

[62] Markov M.A. About three interpretations of quantum mechanics - M.: Nauka. - 1991. (in Russian).

[63] Zel'dovitch Ya.B. Uspekhi fizicheskih nauk. - 1975. - V.115, N 2. - P.161-195. (in Russian).

[64] Caldirola P. Nuovo cim. - 1959. - V.14, N 1. - P.260.

[65] Borel E. A Chance - Moskwa-Petrograd: Gosizdat. - 1923. (in Russian).

[66] Mayer J.E. J. Chem. Phys. - 1961. - V.34. P.1207.

[67] Bergman P.G., Leibowitz J.L. Phys. Rev. - 1955. - V.99. P.578.

[68] Khinchin A.Ya. Usp. Fiz. Nauk. - V.9, N 2. P.164. (in Russian).

[69] Blatt J.M. Prog. Theor. Phys. - 1959. - V.22, N 6. P.745.

[95] D.N. Klyshko Usp. Fiz. Nauk. - 1988. V. 154. P.133. [Sov. Phys. Usp., 1988, v.31, p.74].

[96] Molchanov Yu.B. The Einstein-Podolsky-Rosen paradox and the principle of causality Voprosy Filosofii. - 1983. N 3. P.30-39. (in Russian).

[97] Aharonov Y. and Bohm D. Phys. Rev. - 1959. - V. 155. - P. 485ñ491.

[98] Feinberg E.L. Usp. Fiz. Nauk. - 1962. V.78. P.53. [Sov. Phys. Usp., 1963, v.5, p.753].

[99] Bohm D., "Possible interpretation of quantum theory in terms of hidden variables", in Problems of Causality in Quantum mechanics - I.L.M.: 1955. P. 54-94. (in Russian).

[100] Bell J.S. Rev. Mod. Phys. - 1966. V.38. P.447.

[101] Kochen S. and Specker E.P. J. Math. phys. - 1967. V.16. P.59.

[102] Turner J.E. J. Math. Phys. - 1968. V.9. P.1411.

[103] Ballentine L.E. Rev. Mod. Phys. - 1970.\ V.42. - P.358.
[104] Kholevo A.S. Probabilistic and Statistical Aspects of Quantum Theory - M.: Nauka. 1980. Chap.1. (in Russian).

[105] Nelson E. Quantum Fluctuations - Princeton: Princeton University Press. - 1985.

[106] Weizel W. Z. Phys. - 1953. V.134. P.264.

[107] Adirovich E.I. and Podgoretskii M.I. Zh. Eksp. Teor. Fiz. - 1954. - V.26. P.150-152.

[108] Tyapkin A.A. "Development of the satistical interpretation of quantum mechanics based on modern coordinate-momentum representation" in Filosofskie Voprosy Kvantovoi Fiziki - M.: Nauka. - 1970. [in Russian].

[109] Kershaw D. Phys. Rev. B. - 1964. - V.136. P.1850.

[110] Geilikman B.T. Zh. Eksp. Teor. Fiz. - 1947. V.17. P.830.

[111] Aron J.C. Prog. Theor. Phys. - 1965. V.33. P.726.

[112] Bohm D. and Hiley B.J. Phys. Rep. - 1989. V.172. P.93.

[113] Gardener C.S., Greene J.M., Kruskal M.D., and Miura R.M. Phys. Rev. Lett. - 1967. V.19. P.1095-1097.

[114] Wiener N. and Siegel A. Nuovo Cimento Suppl. - 1955. V.2. P.982.

[115] Wigner E. Phys. Rev. - 1932. - V.40. P.749.

[116] Blokhintsev D. and Nemirovskii P. Zh. Eksp. Teor. Fiz. - 1940. - V.10. P.1263.

[117] Dishkant G.P. Zh. Eksp. Teor. Fiz. - 1955. - V.28. P.117. [Sov. Phys. JETP, 1955, v.1, p.166].

[118] Stratonovich R.L. Zh. Eksp. Teor. Fiz. - 1956. V.31. P.1012. [Sov. Phys. JETP, 1957, v.4, p.891].

[119] Stratonovich R.L. Zh. Eksp. Teor. Fiz. - 1957. V.32. P.1483. . [Sov. Phys. JETP, 1957, v.5, p.1206].

[120] Ryazanov G.V. Quantum-Mechanical probabilities as path sums Zh. Eksp. Teor. Fiz. - 1958. V.35. P.121. [Sov. Phys. JETP, 1959, v.8, p.85].

[121] Blokhintsev D.J. J. Phys. USSR. - 1940. V.2. P.71.

[122] Takabayashi T. Prog. Theor. Phys. - 1954. V.11. P.341.

[123] Baker G.A. Phys. Rev. - 1958. - V.109. P.21-98.

[124] Margenau H. and Hill R.N. Prog. Theor. Phys. - 1961. - V.26. P.722.

[125] Klimontovich Yu. L. and Silin V.P. Usp. Fiz. Nauk. - 1960. V.70 P.247. [Sov. Phys. USP., 1960, v.3, p.84].

[126] Springbord M. Phys. Rev. A. - 1987. V.36, N 3. P.1051.

[127] Carrutheres P., Zachariasen F. Rev. Mod. Phys. - 1983. V.55, N 1. P.245.

[128] Groenewold H.J. Phys. Rep. - 1985. V.127. P.379.
[129] Muüchkenheim W. Phys. Rep. - 1986. V.133. P.337.

[130] Hilbert D. and Achermann W. Principles of Mathematical Logic - New York: Chelsea Publishing. - 1950. Chap. 1. [Russ. transl. Of earlier ed., IL., M., 1947].

[131] Novikov P.S. Elements of Mathematical Logic - M.: Fizmatgiz. – 1959ю - Chap.1. (in Russian).

[132] Birkhoff H. Theory of Structures - I.L. - M.: - 1952. Russian transl.].

[133] Varadarajan V.S. - Comm. Pure Appl. Math. - 1962. V.15. P.189.

[134] Piron C. Helv. Phys. Acta. - 1964. V.37. P.439.

[135] Bunge M. Causality - I.L. - M.: - 1962. [Russian transl.].

[136] Leggett V.S. Contemp. Phys. (UK). - 1984. V.25. P.583.

[137] Laplace P. Philosophical Aspects of the Theory of Probability - M.: I.N. Kushnerev and Co. - 1908. - P. 10-11. [Russian transl.].

[138] Cited in Maxwell J.C. Articles and Lectures - M.: Nauka. - 1968. [Russian transl.].

[139] Bogolyubov N.N. and Shirkov D.V. Introduction To The Theory of Quanticzed Fields - New York: Wiley. - 1976. [Russ. original, Fizmatlit, M., 1984].

[140] Euler L. Dissertatio de principio minimae actionis una cum examine objectionum Cl. prof. Koenigii contra hoc. principium - 1953. - Berlin. (Russian. in the collection "Variational principles of mechanics", ed. by Polak L.S., - M.: Fizmatlit. - 1959. P.96.)

[141] Polak L.S. Variational principles of mechanics, in collection "Variational principles of mechanics" - M.: Fizmatlit. - 1959. P.780.

[142] Plank M. Das Prinzip der Kleinsten Wircung, Die Kultur der Gegenwart - 1915. - Physik. - V. 1.

[143] Rozental' I.L. Usp. Fiz. Nauk. - 1977. - V.121, N 2. - P.319-329. (in Russian).

[144] Davies P.C.W. The accidental Universe - London: Cambridge University press. - 1982.

[145] Weinberg C.S. Rev. Mod. Phys. - 1989. - V.61. P.1.

[146]Sivukhin D.V. General Course of Physics M.: Nauka. - 1975. V.2. (in Russian).

[147] Akhiezer A.I. Voprosy Filosofii. - 1970. - N 6. P.37-47. (in Russian).

[148] Rindler W. Amer. Journ. Phys. - 1966. - V.34, N 10. P.937-942.

[149] Lanzheven P. "Atoms and corpusculars", In Selected Works, - M.: In.lit. - 1949. (in Russian).

[150] Bryusov V. The world of electron.

[151] Heisenberg W. "Development of Quantum Mechanics", in Contemporary Quantum Mechanics - L.-M.: Gostekhizdat. - 1934. P.11-36. [Russ. transl.]

[152] Great Soviet Encyclopedia, 3rd edition.

[153] Einstein A. "Maxwell's influence on the development of ideas about physical reality", in J.C. Maxwell: Articles and Lectures - M.: Nauka. - 1968. (in Russian).

[154] Polovin R.V. "Physics and the pictures of the World", in Philosophical Problems of Science - Kiev: Izd. KhGU. - 1986. - N 60. P.62-73.

[155] Frank P. Rev. Mod. Phys. - 1941. - V.13. - P.171-175.

[156] Heisenberg W. Usp. Fiz. Nauk. - 1968. - V.94, N 1. - P.155. (in Russian).

[157] Treder G.Yu. Evolution of Fundamental Physical Ideas - Kiev: Naukova Dumka. - 1968. (in Russian).

[158] Vorontsov-Vel'yaminov B.A. Laplace - M.: Nauka. - 1985. (in Russian).

[159] Tarasov L.V. Osnovy Kvantovoi Mekhaniki - M.: Vysshaya Shkola. - 1978. (in Russian).

[160] Gramer J.G. Rev. Mod. Phys. - 1986. - V.58, N 3. P.647-687.

[161] Einstein A., The Meaning of Relativity - Princeton: Princeton Univ. Press. 3-rd ed. - 1950. - P.107.

162. Yu.P. Stepanovskii Nobelevskie premii po fizike 2000 i sto let kvantovoj fiziki Electromagnetic pnenomena. - 2001. - V. 2, N 1 (5). - P. 131-148. (in Russian).

163. M.B. Mensky. Kvantovaya mehanica: novue experimentu, novue prilozeniya i novue formulirovki starukh voprosov.- Yspekhi fizicheskih nauk. - 2000. - V. 170, N 6. - P. 631-647.

164. Pis'ma v redactsiy. - Yspekhi fizicheskih nauk. - 2001. - V. 170, N 4. - P. 437-458.

165. Yu.A. Berezhnoy, I.R. Biletsky. Ontologicheskie aspecty printsipa neopredelennosti i kvantovoi interferentsii - Visnuk KhNU. - 2001. - N 501/2. - P. 62-68.

166. L.de Broglie. Sootnosheniya neopredelennostey Heisenberga i veroaytnostnaya interpretatsiya volnovoi mechaniky. (in Russian) - M, Mir, 1986.

167. I. Tsekhmistro. Holistic Philosophy of Science. - University book, Sumy, 2002.

168. A.A. Grib. – Uspekhi fizicheskih nauk. – 1984. – V.142. - №4.

169. Lamehi – Rachti M., Mittig W. – Phys. Rev., Ser D., - 1976. – V.14, - p.2543.

170. Bell I.S. – Physica. -1965. – V.1. – p. 1951.

QUANTUM MECHANICS, COMMON SENSE AND PHILOSOPHICAL PROBLEMS

Viktor P. Demutsky and Revol'd V. Polovin, Department of Physics and Technology, Kharkov National University, Oleg M. Pignasty, National Aerospace University, Ukraine.

КВАНТОВАЯ МЕХАНИКА, ЗДРАВЫЙ СМЫСЛ И ФИЛОСОФСКИЕ ПРОБЛЕМЫ

Виктор Петрович Демуцкий, Револьд Васильевич Половин, Харьковский Национальный университет, Олег Михайлович Пигнастый, Национальный Аэрокосмический университет, Украина.

Підписано до друку 12.01.2003 р.
Формат 60×84 1/16, Папір офсетний. Друк офсетний.
Умов. друк. аркуш. 5,4. Ум. вид. арк.. 11,16. Тираж 1000 прим.
Замовлення № 03-0112

Надруковано в друкарні ПП "ШТРІХ"
м.Харків, вул. Бакуліна, 33
тел. (0572) 456-037
Свідоцтво про реєстрацію: серія ДК №290 від 21.12.2001