Mathematical Modelling and Optimal Design of Plate-and-Frame Heat Exchangers

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The optimal design for multi-pass plate-and-frame heat exchanger with mixed grouping of plates is considered. It is formulated as the mathematical problem of finding the minimal value for implicit nonlinear discrete/continues objective function with inequality constraints. The optimizing variables include the number of passes for both streams, the numbers of plates with different corrugation geometries in each pass, the plate type and size. To estimate the value of objective function in a space of optimizing variables the mathematical model of plate heat exchanger developed. To account for thermal and hydraulic performance of channels between plates with different corrugation patterns, the exponents and coefficients in formulas to calculate heat transfer coefficients and friction factors used as model parameters. The procedure and software for numerical experiment to identify model parameters by comparing the calculation results with those obtained with free available in web computer programs of plate manufacturers is developed. The sets of such parameters are obtained for a number of industrially manufactured plates. The described approach implemented as software for plate heat exchangers calculation. Two case studies results discussed.

1. Introduction

Plate heat exchangers (PHEs) are one of the most efficient types of heat transfer equipment. The principles of their construction and design methods are sufficiently well described elsewhere, see e.g. Wang, Sunden and Manglik (2007), Tovazshnyansky et al (2004). This equipment is much more compact, requires much less material for production, much smaller footprint, than conventional shell and tubes units. PHEs have a number of advantages, such as compactness, low total cost, less fouling, flexibility in changing the heat transfer surface area, accessibility, and what is important for energy saving, close temperature approach.

2. Mathematical modelling of PHE

Plate-and-frame PHE consists of a set of corrugated heat transfer plates clamped together, see e.g Wang, Sunden and Manglik (2007). In multi-pass PHE plates are arranged in such way, that they are forming groups of parallel channels. Such group is corresponding to one pass and the stream is going consequently through the passes, as shown on Figure 1. The temperature distributions in passes are different, and in different groups of channels both counter-current and co-current flows may occur.

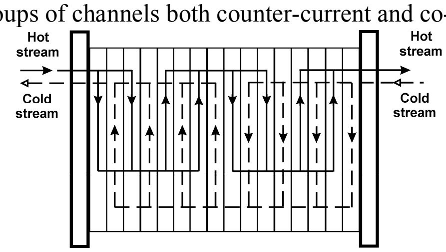


Figure 1. An example of streams flows through channels in multi-pass PHE.

We can regard PHE as a system of one-pass blocks of plates. The conditions for all channels in such block are equal.

When X_1 and X_2 numbers of passes for each stream, the total number of blocks is $n_b=X_1X_2$ and the number of heat transfer units in one block, counted for hot stream:

$$NTU_b = U_b \cdot F_b \cdot X_2 / (G_1 c_1) \qquad (1)$$

Where U_b – overall heat transfer coefficient in block, W/(m^{2}K); G_1- mass flow rate of hot stream, kg/s; c_1 – specific heat of hot stream, J/(kg·K).

When $G_1c_1/X_2<G_2c_2/X_1$, block heat exchange effectiveness ε_b for counter-current flow:

$$\varepsilon_b = [1-\exp(NTU_b \cdot R_b - NTU_b)] / [(1-R_b \cdot \exp(NTU_b \cdot R_b - NTU_b)], \qquad (2)$$

where $R_b = G_1 \cdot c_1 \cdot X_1 / (G_2 \cdot c_2 \cdot X_2)$- the ratio of going trough block heat capacities of streams; G_2 and c_2 mass flow rate [kg/s] and specific heat [J/(kg·K)] of cold stream.

If R_b=1, then $\varepsilon_b = NTU_b / (1+NTU_b)$. (3)

In case of co-current flow directions

$$\varepsilon_b = [1-\exp(-NTU_b \cdot R_b - NTU_b)] / (1+R_b) \qquad (4)$$

On the other hand heat exchange effectiveness of block *I* : $\varepsilon_{bi} = \delta t_{1i} / \Delta t_i$, (5)

where δt_{1i} - temperature drop in block *i*; Δt_i - the temperature difference of streams entering block *i*.

The temperature change of cold stream: $\delta t_{2i} = \delta t_{1i} \cdot R_b$. (6)

These relations are mathematical model of block.

It shown by Tovazshnyansy et al. (1992), for any numbers of passes, it is can be obtained the system of algebraic equations in matrix form: $[Z][\delta t_i] = [\varepsilon_{bi}\Delta]$ (7)

where $[\delta t_i]$- vector-column of temperature drops in blocks; $[\varepsilon_i \Delta]$- vector-column of the right hand parts of the system; $[Z]$- matrix of system coefficients; Δ- temperature difference of streams entering PHE.

One can easily perform the numerical solution of linear algebraic equations system (7) on PC. After that the total temperatures changes in PHE $\delta t_{\Sigma 1}$ and $\delta t_{\Sigma 2}$ are calculated as the sum of temperature changes in blocks of respective passes.

The total heat load of PHE: $Q = \delta t_{\Sigma 1} \cdot G_1 \cdot c_1$ (8)

This system must accompany the equations for calculation of overall heat transfer coefficient U, W/m^{2}K, as below.

$$U = 1/\left(1/h_1 + 1/h_2 + \delta_w/\lambda_w + R_f\right) \quad (9)$$

where h_1, h_2 - film heat transfer coefficients for hot and cold streams, respectively, W/m^{2}K; δ_w - the wall thickness, m; λ_w - heat conductivity of the wall material, W/(mK); $R_f = R_{f1} + R_{f2}$ - the sum of fouling thermal resistances, m^{2}K/W.

For plate heat exchangers the film heat transfer coefficients usually calculated by empirical correlations: $Nu = f(\mathrm{Re}, \mathrm{Pr}) = A * \mathrm{Re}^n \, \mathrm{Pr}^{0.4} (\mu / \mu_w)^{0.14}$ (10)

Here μ and μ_w dynamic viscosity at stream and at wall temperatures, respectively; Re- Reynolds number: Pr- Prandtl number.

Nusselt number is: $Nu = h \cdot d_e / \lambda$, (11)

λ - heat conductivity of the respective stream, W/(mK); d_{eq} - equivalent diameter of interplate channel, m: $d_e = 4b\delta / \left[2(b+\delta)\right] \approx 2\delta$, (12)

where f_{ch} - cross section area of inter plate channel, m^2; δ - inter plate gap, m; b - channel width, m.

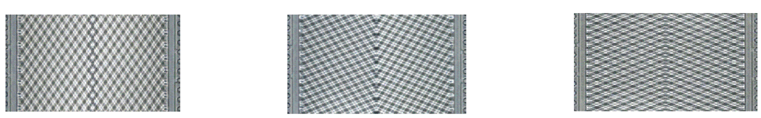
The streams velocities calculated as: $w = g / (f_{ch} \rho)$ (13)

Where g is flow rate of stream through one channel, kg/s.

The pressure drop in one PHE channel: $\Delta p = \zeta \cdot L_p \cdot \rho \cdot w^2 / (d_e \cdot 2)$ (18)

where L_p -effective plate length; ζ – friction factor, which is usually determined by empirical correlations of following form: $\zeta = B / \mathrm{Re}^m$ (14)

For multi pass PHE pressure drop in one pass multiplied by number of passes X.



Channel L formed by L-plates | *Channel M formed by L- and H- plates* | *Channel H formed by H-plates*

Figure 2. Channels formed by combining plates of different corrugation geometries

In modern PHEs plates of one type are usually made with two different corrugation angles. When assembled in PHE they can form three different channels, see Figure 2.

Plates of H type have corrugations with bigger angles (about 60^0) and form the H channels with higher intensity of heat transfer and higher hydraulic resistance. Plates of L type have a lower angle (about 30^0) and form the L channels with lower heat transfer and hydraulic resistance. Combined, these plates form channels M with intermediate characteristics (see Fig. 2).

In one PHE two groups of channels are usually used. One is of higher hydraulic resistance and heat transfer (x-channels), another of lower characteristics (y-channels). When the stream is going through set of such channels, the temperature changes in different channels are differ. After mixing in collector part of PHE block, the temperature is determined by heat balance. The heat exchange effectiveness of plates block with different channels:

$$\varepsilon_b = (g_x \cdot n_x \cdot \varepsilon_x + g_y \cdot n_y \cdot \varepsilon_y) / (g_x \cdot n_x + g_y \cdot n_y), \quad (15)$$

where n_x and n_y are the numbers of x and y channels in a block of plates, respectively; $g_{x,y} = w_{x,y} \cdot \rho \cdot f_{ch}$ - the mass flow rates through one channel of type x or y. These flow rates should satisfy equation $\Delta p_x = \Delta p_y$ and material balance:

$$g_x \cdot n_x + g_y \cdot n_y = G_b, \quad (16)$$

where G_b - flow rate of the stream through the block of plates.

When the numbers of channels are determined, we can calculate the number of plates:

$$N_{pl} = \sum_{i=1}^{X_1} \left(n_{x1i} + n_{y1i} \right) + \sum_{j=1}^{X_2} \left(n_{x2i} + n_{y2i} \right) + 1 \quad (17)$$

The total heat transfer area of PHE, m^2: $F_{PHE} = (N_{pl} - 2) \cdot F_{pl}$ (18)

where F_{pl} - heat transfer area of one plate, m^2.

The above algebraic equations (1)-(18) describe the relationship between variables, which characterize PHE and heat transfer process in it. It is a mathematical model of PHE, which solution enables to calculate pressure and temperature change of streams entering known heat exchanger. It is a problem of PHE rating (analysis).

The problem of PHE design (synthesis) require to find its characteristics (such as plate type, numbers of passes, numbers of plates with different corrugations) which will in the best way satisfy to the required process conditions. Here the optimal design with pressure drop specification considered, in a sense as described by Wang and Sunden (2003). The objective function is total heat transfer area of PHE. Also the conditions for heat load Q^0 and allowable pressure drops for both streams $\Delta p_1^{\,0}, \Delta p_2^{\,0}$ must be satisfied. These conditions are partial inequality constraints:

$$Q \geq Q^0; \qquad \Delta p_1 \leq \Delta p_1^{\,0}; \qquad \Delta p_2 \leq \Delta p_2^{\,0} \quad (19)$$

It is the mathematical problem of finding the minimal value for implicit nonlinear discrete/continues objective function with inequality constraints. It does not permit analytical solution without considerable simplifications. To solve it by numerical methods the software developed for IBM compatible PC.

3. Identification of mathematical model parameters

Based on described above mathematical model, the technique of numerical experiment is developed. It enables to identify model parameters (coefficients in correlations) by comparison with results obtained for the same conditions with the use of PHE calculation software, which is now available in internet from most of PHE manufacturers. The results for some plates manufactured by Alfa Laval are presented in Table 1. The geometrical parameters of plates are present in Table 2.

The comparison of results obtained with our software to those of Alfa Laval free available software has shown good agreement (discrepancies not more then 5%).

***Table 1**. Parameters in correlations for some Alfa Laval PHEs (Re>250)*

Plate type	Channel type	A	n	Re	B	m	Re	B	m
M6M	H	0.27	0.7	<1300	11.7	0.13	≥1300	4.55	0.0
	L	0.11	0.71	<2200	4.23	0.23	≥2200	1.88	0.12
	ML/MH	0.14	0.73	<2100	5.61	0.16	≥2100	1.41	0.0
M10B	H	0.24	0.7	<2000	11.1	0.15	≥2000	3.5	0.0
	L	0.11	0.7	<1500	12	0.36	≥1500	2.42	0.14
	ML/MH	0.12	0.74	<2700	6.2	0.2	≥2700	1.9	0.05

***Table 2**. Geometrical parameters for Alfa Laval PHE plates*

Plate type	δ, mm	d_e, mm	b, mm	F_{pl}, m^2	$D_{connection}$, mm	$f_{ch} \cdot 10^3$, m^2	L_p, mm
M6M	3.0	6.0	210	0.14	50	0.630	666
M10B	2.5	5.0	334	0.24	100	0.835	719

The obtained correlations and developed software recommended to use for preliminary calculations only, when optimizing PHEs or heat exchanger network. The final calculations, when ordering the PHE, should be performed by its manufacturer.

4. Case studies

Example 1. It is required to heat 5 m^3/h of distillery wash fluid from 28 to 90 ^oC by hot water coming with temperature 95 ^oC and flow rate 15 m^3/h. Allowable pressure drop for hot stream 1.5 bar. It is 1 bar for cold stream. The properties of wash fluid are: density – 978,4 kg/m^3; heat capacity – 3,18 kJ/(kg·K); cconductivity – 0.66 W/(kg·m). Dynamic viscosity at temperatures $t = 25;\ 60;\ 90^\circ C$ is taken as $\mu = 19,5; 16,6; 9\ cP$.

The results of calculations revealed that the global optimum (38 plates) is achieved at X_1=2 and X_2=4 with all medium channels (19 H and19 L plates in PHE) for M6M PHE. The closest other option (41 plates) is at X_1=X_2=2 with mixed channel arrangement in one pass. If we would have only one plate type in PHE, the minimal number of plates would be 44 for both H and L plates, or 15% higher than with mixed channels.

Example 2. The conditions of example presented by Wang and Sunden (2003) are considered. It is required to cool 40 kg/s of hot water from 70 ^oC down to 40 ^oC. The flow rate of cooling water is 30 kg/s, temperature 10 ^oC Allowable pressure drop on hot side ΔP_1=40 kPa, cold side 60 kPa. The results of calculations presented in Table3. First five cases (rows in Table 3) for clean plates. The minimal heat transfer area is in case #1. Here the allowable pressure drop on hot side is completely used and heat load exactly equal to specified (the margin φ=(Q-Q°)/Q° equal to zero). In example of Wang and Sunden (2003) the fouling thermal resistance is equal to $0.5 \cdot 10^{-4}$ m^2K/W for each side. The total $R_f = 1 \cdot 10^{-4}$ m^2K/W was taken in case #6. It lead to increase of surface area on 60%- to 59.04 m^2. As it shown by Gogenko et al. (2007), the excessive allowance for fouling can lead to increase of fouling in real conditions by lowering the flow velocity. In case #7 (see Table 3) the calculations made at margin to overall heat transfer coefficient 10%. The area increase is only 13%.

Table 3. *Calculations results for Example 2 (two M10B PHEs installed in parallel)*

#	Total area, m^2	Grouping of channels	ΔP_1, kPa	ΔP_2, kPa	$R_f\ 10^4$, m^2K/W	φ, %	F / F_{min}
1	36.96	(3H+35M)/ (3H+35M)	39	26	0*	0	1
2	63.92	64H / 64H	40	21	0*	68	1.73
3	38.88	40M / 40M	38	25	0*	1	1.05
4	91.68	95L / 95L	14	8	0*	0	2.50
5	83.04	2*43L/2*43L	40	24	0*	62	2.25
6	59.04	(57H+4M)/ (57H+4M)	40	23	1.0*	56	1.60
7	41.76	(15H+28M)/(15H+28M)	40	27	0.18	10*	1.13

In all cases with mixed grouping of channels the specified values of allowable pressure drop for one stream (hot in our example) are satisfied exactly, as also condition for heat load (when margin is specified, then with margin). It means that by using the mixed grouping of plates with different corrugation pattern, we can change the thermal and hydraulic characteristics of a plate pack in a way close to continues one. The level of discreteness is equal to one plate in a pack. It allows us to satisfy specified conditions very close to equality.

5. Conclusions

The mathematical model for multi-pass PHE assembled with plates of different corrugation patterns is developed. The model parameters corresponding to some industrially manufactured plates obtained. The examples of calculation results for two case studies show the possibility with such method to obtain optimal solutions with exact satisfaction of constraints for total heat load and pressure drop of one stream. It gives the considerable reduction of the PHE heat transfer area.

5. References

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* This value is specified at calculation conditions.