

## AN APPROACH TO IDENTIFICATION OF IMPACT INTERACTION MODEL FOR A VIBROIMPACT SYSTEM

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### ABSTRACT

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In this work a new approach to the modelling of vibroimpact systems was proposed. Verification criteria estimating the quality of the identified model of contact interaction were offered. The applicability of the approach was shown for numerical and experimental studies of a real shake-out machine.

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### INTRODUCTION

Dynamics of vibroimpact machines is quite a peculiar area of mechanical engineering. On the one hand oscillations with impact have very complex nature. On the other hand this dynamical process has to be modelled with high precision as soon as it concerns the robust design of such machines. Particularly, a good estimate of the value of the altering forces acting on their elements is really essential for the durability of the design. This motivates the development of a new model for the dynamics of vibroimpact systems proposed in this work. This model is elaborated here with a strong reference to the highly loaded shake-out machines. Such machines are used to separate the casting from its mould. For the dynamics of these machines the two specific factors are of a major significance. The first factor is the impact of the moulding upon shake-out grid. That takes place when a heavy moulding comes into contact with the oscillating grid and is characterized by high impact velocities and high impact loads. The other factor considered is the damage of the mould. At each collision some amount of sand lumps breaks off from the moulding. This separation dissipates some part of the kinetic energy.

In the developed model there's a strong emphasis on the two phenomena. We propose a new approach to their treatment. The key feature of this approach is that it enables to overcome the uncertainty that is characteristic for the considered class of mechanical systems. Generally most of the constructive elements of a shake-out machine have clearly and easily determined mechanical properties. The design parameters such as the mass of the grid frame, the stiffness of the elastic supports, the properties of the dampers are well known and are controlled by the designer. The uncertainty comes from the two factors mentioned above that govern the impact interaction of the moulding and the shake-out grid. The impact force is the key quantity that describes this interaction. It is influenced by many different factors some of which are random to a great extend. The existing models found in the literature [1 - 6] postulate only some simplistic laws expressing this dependency. Such an empirical approach can not capture in detail the important characteristics of the impact process, namely the duration of the impact, the amplitude and the time distribution of the impact force and the amount of energy dissipated during the collision at all the possible conditions.

### APPROACH DESCRIPTION

The approach is described here for two-body vibroimpact system depicted on the Fig. 1. It represents in general the shake-out machine as soon as only vertical motion of its elements is considered. Such representation captures all the peculiar features this work is focused on. The first body  $m_1$  can be viewed as the grid frame. It rests on elastic supports of total stiffness  $C_1$  and dampers of viscosity  $H_1$ . It oscillates under action of a cyclic force  $A \sin \omega t$ . In the real shake-out machine this force is produced by debalance drive. The second body  $m_2$  represents the moulding. It periodically falls onto the first body and pops back into the air. Hence there's a non-linear one-sided constraint between these two bodies that is only active when they are in contact.

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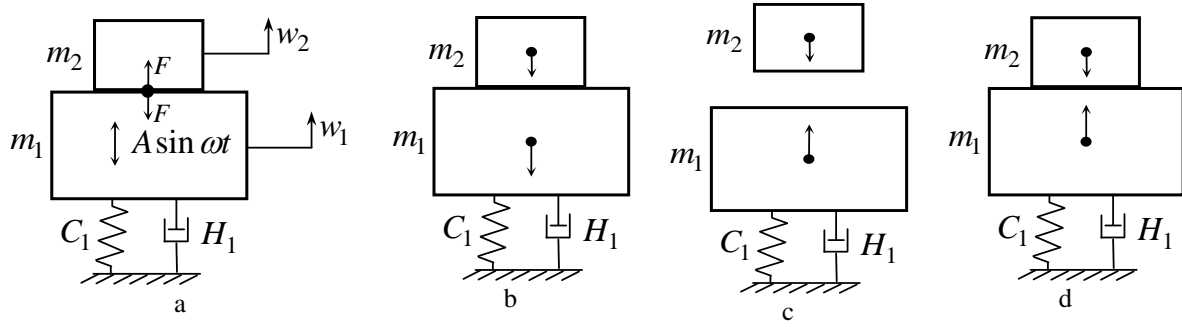


Fig. 1 Two-body vibroimpact system

According to the proposed approach the form of this impact interaction is not postulated and is initially unknown. The model for the impact force is established by an identification procedure based on the specially designed verification criteria.

### BASIC MODEL

This procedure starts with the basic model that is set up in the beginning and does not change during the verification. First of all, it describes the known part of the examined object. For the considered vibroimpact system the equation of motion are well established and can be written as

$$\begin{cases} m_1 \ddot{w}_1 + C_1 \dot{w}_1 + H_1 w_1 + A \sin \omega t + m_1 g + F = 0 \\ m_2 \ddot{w}_2 + m_2 g - F = 0. \end{cases} \quad (1)$$

Secondly, the initial setting postulates some properties the identified part of the developed model is either known or assumed to possess. In the considered case this concerns the unknown impact force  $F$  only. The main assumption about this force is that it depends on the interpenetration  $\zeta = w_1 - w_2$  of the two bodies and the penetration velocity  $\dot{\zeta} = \dot{w}_1 - \dot{w}_2$

$$F = F(\zeta, \dot{\zeta}). \quad (2)$$

As if it were produced by a viscoelastic interface layer as depicted on Fig. 2. This layer effectively represents the elastic deformations of the moulding and its dissipative damage. Based on the representation (2) of the unknown impact response some further conditions can be formulated. Thus naturally the following two conditions must hold

$$F = 0, \quad \zeta < 0, \quad (3)$$

$$F \geq 0. \quad (4)$$

The first one expresses the fact that the contact force vanishes when there's no interpenetration (no contact) of the bodies. The second follows from a natural assumption that the force between the moulding and the grid is non-adhesive. Another property of  $F$  we determine in the initial setting concerns the influence of the penetration velocity. The dependency of  $F$  on  $\dot{\zeta}$  is introduced in order to represent the dissipative damage of the moulding that we link to the viscoelastic layer. We postulate that damage and separation of sand lumps from the moulding takes only place when penetration velocity is positive. That means that the negative values of  $\dot{\zeta}$  have no effect on the impact force:

$$F(\zeta, \dot{\zeta}) = F(\zeta, 0), \quad \dot{\zeta} < 0. \quad (5)$$

We also guarantee the dissipativeness of the impact response by introducing another condition related to the penetration velocity:

$$F(\zeta, \dot{\zeta}) > F(\zeta, 0), \quad \dot{\zeta} > 0. \quad (6)$$

The effect of these conditions on the character of impact interaction is illustrated on Fig.2. Consider a single collision depicted on the penetration-force diagram on Fig.2. Point 1 on this diagram corresponds to the moment of time  $t_1$  when the bodies come into contact ( $\zeta(t_1) = 0$ ) with some positive impact velocity  $\dot{\zeta}(t_1) > 0$ . At this moment one will observe a jump of the force  $F$  from 0 value before contact to a positive finite value  $F(\zeta(t_1), \dot{\zeta}(t_1)) = F(0, \dot{\zeta}(t_1)) > F(0, 0) = 0$  after the impact. The penetration grows until the repulsive contact force stops the motion of the two bodies towards each other at time  $t_2$ , corresponding to the point 2 on the diagram. At this moment the relative velocity changes its sign from the positive to the negative, hence  $\dot{\zeta}(t_2) = 0$ . The subsequent unloading is elastic and according to (5) follows the curve  $F(\zeta, 0)$  to the point 3 at which the two bodies disengage and the contact force vanishes again. It should be noted that the viscoelastic loading branch 1-2 is always above the elastic unloading branch 2-3 in case if contact interaction law (2) satisfies the condition (6). This fact guarantees the positiveness of the hysteresis of the impact force and a priori dissipativeness of the identified model. The Fig. 2 illustrates the above discussed effect of the constraints (3-6) in the two-dimensional phase space for a single phase trajectory of a typical collision.

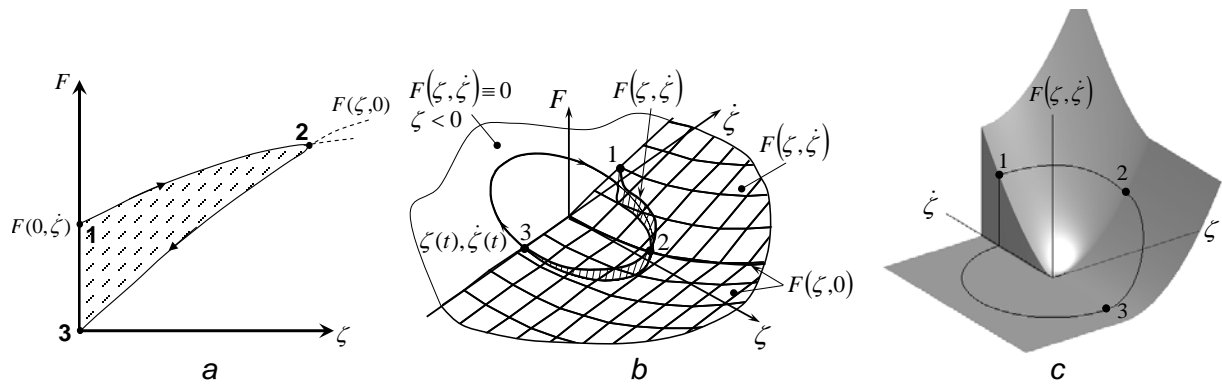


Fig. 2 The loading curve for a single collision (a) and the general form of the viscoelastic force (2) satisfying the conditions (3-6) (b, c)

## PARAMETRIC APPROXIMATION

With the initial setting at hand one can begin the identification of the unknown part, which in the considered case is the unknown impact force  $F$ . One needs to establish it as a function of the penetration  $\zeta$  and the penetration velocity  $\dot{\zeta}$  which would satisfy the conditions (3-6). In this approach this function is sought for in the form of a series expansion in the domain  $\zeta > 0, \dot{\zeta} > 0$  (for the other values of  $\zeta$  and  $\dot{\zeta}$  the impact force is determined then according to the identities (3) and (5)). Particularly, one can think of a polynomial representation

$$\begin{cases} F^{\wedge}(\zeta, \dot{\zeta}) = \alpha_1 \zeta + \alpha_2 \dot{\zeta} + \alpha_3 \zeta \dot{\zeta} + \dots, & \zeta, \dot{\zeta} > 0 \\ F^{\wedge}(\zeta, \dot{\zeta}) = F^{\wedge}(\zeta, 0), & \dot{\zeta} < 0 \\ F^{\wedge}(\zeta, \dot{\zeta}) = 0, & \zeta < 0 \end{cases}, \quad (7)$$

parameterized by unknown positive coefficients  $\alpha_i$  that need to be established.

The approximate law of the dependency of the impact force on the penetration and its velocity given by (7) has to be verified. This is done by comparison of the numerical simulation results obtained for the approximate model of the impact interaction.

## VERIFICATION CRITERIA

In order to validate the approximation (7) some definite verification criteria have to be chosen. One can think of different quantities measures that would estimate the discrepancy between the prediction provided by approximate model and the real behaviour observed in the experiment. One can take different dynamic parameters of the examined vibroimpact system for this comparison. In the proposed approach the time distribution of the identified impact force is verified towards its experimental values. Particularly we focus on the on the steady-state oscillations regime since that is

most essential for the performance of the shake-out machine. Consider two time distributions of the impact force for the steady oscillations with frequency  $\nu = 2\pi\omega$  and period  $T = 1/\nu$  plotted on the Fig. 3. One curve  $F_E$  is experimentally derived at the considered regime. The other  $F_N$  is obtained from a numerical simulation with all the known parameters set to be identical to the experiment and some approximate model for the impact interaction parameterized by coefficients  $\alpha_i$ . We put both distributions on a common time axis setting the beginning of the impact both in the experiment and the simulation to a same time point  $t^*$ . The difference between the non-negative values  $F_E(t)$  and  $F_N(t)$  as well as the durations of the impulse  $\tau_E$  and  $\tau_N$  derived from the experiment and the simulation is essential for the verification.

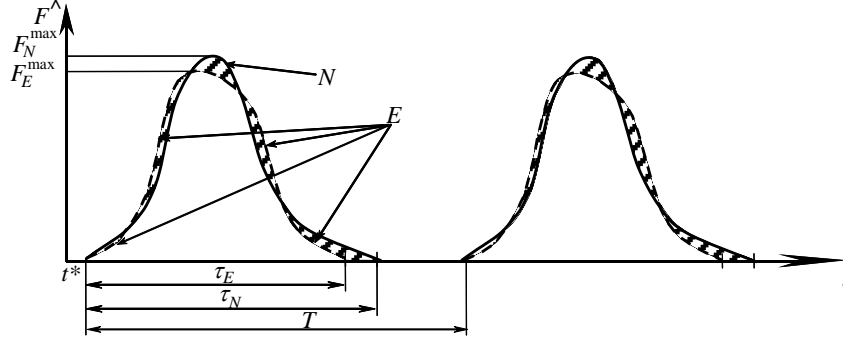


Fig. 3 Numerically predicted time distribution of the impact impulse at the steady-state regime compared to the experimental observation

In order to measure this discrepancy we introduce several functionals of the time distributions  $F_E(t)$  and  $F_N(t)$ :

$$I_1 = \frac{\max |F_N^{\hat{}} - F_E^{\hat{}}|}{\max |F_E^{\hat{}}|}; \quad I_2 = \frac{\int |F_N^{\hat{}} - F_E^{\hat{}}| dt}{\int |F_E^{\hat{}}| dt}; \quad (8)$$

$$I_3 = \sqrt{\frac{\int (F_N^{\hat{}} - F_E^{\hat{}})^2 dt}{\int (F_E^{\hat{}})^2 dt}}; \quad I_4 = \frac{\int |\text{sign}(F_N^{\hat{}}) - \text{sign}(F_E^{\hat{}})| dt}{\int \text{sign} F_E^{\hat{}} dt} = \frac{\tau_N - \tau_E}{\tau_E}.$$

This functionals have the properties of a norm

$$I_k \geq 0 \quad \forall \quad F_N^{\hat{}}, F_E^{\hat{}}, k; \quad (9)$$

$$I_k = 0 \Rightarrow F_N^{\hat{}} \equiv F_E^{\hat{}}, k = 2, 3. \quad (10)$$

The lesser is their value the closer is the simulation results to the experimental observation, and hence the better is the approximation (7) for some definite set of the parameters  $\alpha_i$  to the true impact interaction law.

## VERIFICATION

We identify the parameters  $\alpha_i$  of the approximate model by minimizing the value of one of the norms in (8). The choice of the functional turns out to be really essential. In order to illustrate this consider a model verification problem.

Assume that the impact force is really expressed by

$$F = \bar{\alpha}_1 \zeta + \bar{\alpha}_2 \dot{\zeta}, \quad (\zeta, \dot{\zeta}) > 0, \quad (11)$$

with the known  $\bar{\alpha}_1 = 2.06 \cdot 10^8 \text{ N/m}$ ,  $\bar{\alpha}_2 = 1.28 \cdot 10^7 \text{ N} \cdot \text{s/m}$ . In this artificial situation the approximation (7) with only two members of the series reproduces this “real” impact interaction law for  $\alpha_1 = \bar{\alpha}_1$  and  $\alpha_2 = \bar{\alpha}_2$ . Consider then the sensitivity of the functionals  $I_k$  to the identified parameters. It is known that  $I_k(\bar{\alpha}_1, \bar{\alpha}_2) = 0$ , since for these values of expansion coefficients  $F(\zeta, \dot{\zeta})$

coincides with  $F^{\wedge}(\zeta, \dot{\zeta})$  and hence  $F_E(t) = F_N(t)$  (the experimental curve  $F_E(t)$  is attained from the virtual numerical experiment just repeats the simulation). On the Fig. 4 one can see the values of  $I_k(\alpha_1, \alpha_2)$  in the domain  $\alpha_1 \in [1.03 \cdot 10^8, 3.09 \cdot 10^8] \times \alpha_2 \in [0.64 \cdot 10^7, 1.92 \cdot 10^7]$ . One can observe that the functionals  $I_2$  and  $I_3$  have a distinctive minimum at  $\alpha_1 = \bar{\alpha}_1, \alpha_2 = \bar{\alpha}_2$ . To the contrary the functionals  $I_1$  and  $I_4$  display bad sensitivity towards the identified parameters, which will definitely hinder the minimization procedure. The situation can be improved by introducing an alternative functional  $I_0 = (I_1 + I_4)/2$  that is their combination and has better shape as can be seen on the Fig. 5.

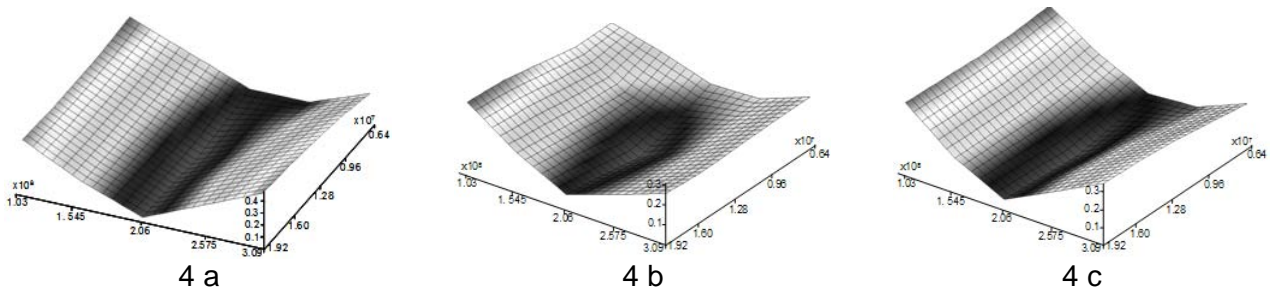


Fig. 4 Variation of verification norms with  $I_1$  (a),  $I_2$  (b),  $I_3$  (c),  $I_4$  (d)

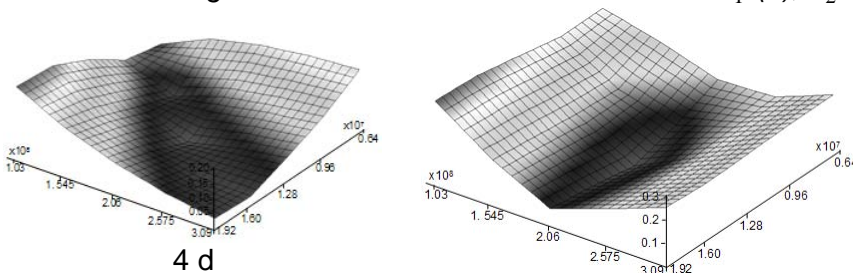


Fig. 5 Enhanced functional  $I_0 = (I_1 + I_4)/2$

With a good choice of the functional norm one obtains stable convergence of the identification process. The minimization can be performed by an accelerated coordinated descent method. Particularly the choice of  $I_2$

functional lead to the following results. The consequent iterations are shown on the Fig. 6. The curves on the Fig. 7 show the convergence of the impact force distribution  $F_N(t)$  to its “exact” value.

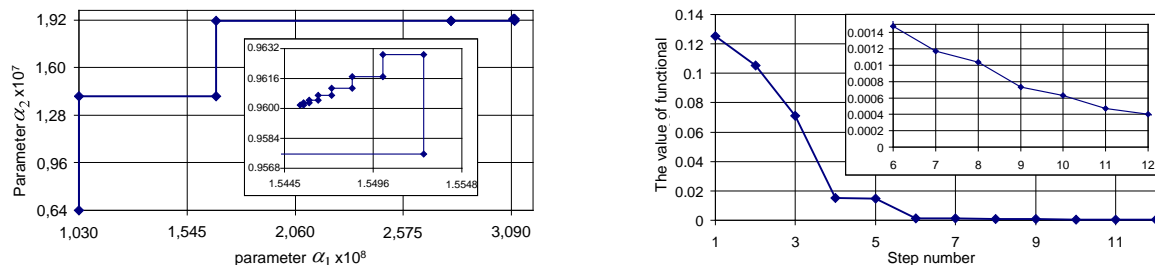


Fig. 6 Iterative minimization process for  $I_2$

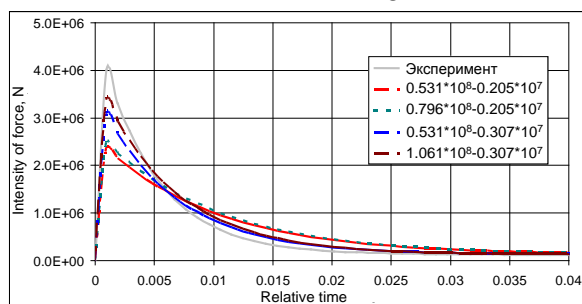


Fig. 7 Proximity of the impact force for the consequent iterations to its exact distribution

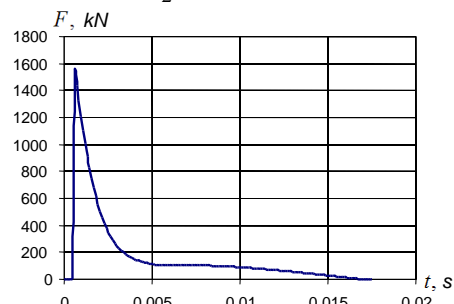


Fig. 8 The predicted character of the time distribution of the impact force by the approximate model

The verification procedure displays for this model problem the performance similar to the illustrated above for the two other functionals  $I_3$  and  $I_0$ . Thus minimization of  $I_2, I_3$  or  $I_0$  can be advised as the verification criteria for the general case.

## APPROACH PERFORMANCE

The proposed approach was employed for the identification of the model for the impact force between the grid frame and the moulding for the shake-out machine produced by Azovmash. A 4-member polynomial expansion was taken for the verification.

The proposed approximation captures most of the key characteristic features of the real impact interaction, which was proposed by the sensitivity analysis [7]. Particularly the non-linear members allow to introduce a shift of the maximum of the impact force from the beginning of the collision closer to its middle part (Fig.8).

Ultimately a good agreement of the experimental data with the simulation with the identified model was achieved. Fig.9 shows the discrepancy between the experimental and numerical values of the stresses in the shake-out grid frame controlled during the verification [7]. The result model provides a very good prediction of the parameters of the dynamics of the examined vibro-impact system such as duration of the impact impulse, accelerations and force amplitude in the shake-out machine with a precision of 11-18% [7].

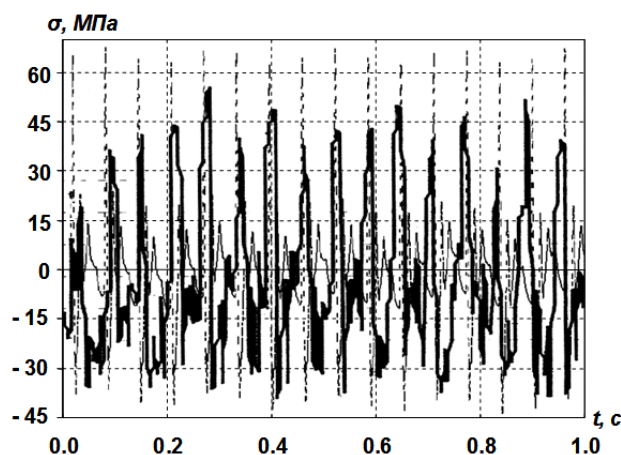


Fig. 9 Comparison of numerically predicted and experimental values of controlled stress in the shake-out grid frame character of the time distribution of the impact force by the approximate model

estimating the quality of the identified model of contact interaction were offered in this work. Their performance was illustrated for a model verification problem, based on which the final recommendations for their choice were made.

4. The applicability of the approach was shown for numerical and experimental studies of a real shake-out machine. It enabled to achieve high precision in the description of its dynamics.

The approach is extendable on a broader class of vibro-impact systems and used for their analysis and synthesis.

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## CONCLUSIONS

In this work a new approach to the modelling of vibroimpact systems was proposed.

The main distinctive features of this approach as well as the key result are summarised below.

1. The approach does not postulate any certain form of the contact interaction model, but suggest its identification through a verification procedure.

2. During this identification the physical peculiarities of the dynamical process are taken into account. Particularly, the damage of the moulding due to the applied shock is considered. It is effectively represented by a non-linear viscoelastic contact layer.

3. Different verification criteria

**AT THE VERY INSTANT WHEN THE AUTHOR CAME ACROSS  
AN INEXPERIENCED BEHAVIOR**

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**ABSTRACT**

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The appearance of the data the author accidentally came upon during hand-made analog computer (by three years his senior Mr. M. Abe using vacuum-tubes as his research project) experiments on the 27th of November, 1961 was like a broken egg with jagged edges. The original sheet of data was now kept at Brookhaven National Laboratory in New York (BNL Photography Division Negative No. 1-380-90). The data was eventually recognized as a chaotic attractor first obtained in an actual physical system. In this presentation the author would like to reproduce the unforgettable situation before the study of chaos began.

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**INTRODUCTION**

In this presentation, periodically forced oscillatory phenomena are leading as a whole. The subject matters of reflections were nothing but the author's subjective accounts. Accordingly, he presumed to write proper nouns, and each subject was restricted within the possible inspections by references and/or survived materials.

**2. SYNCHRONIZATION PHENOMENA**

When a periodic force is applied, or a periodic signal is injected to self-oscillatory systems, the behavior of the systems is synchronized with the external signal depending on a frequency and an amplitude of the external force. Such effects are well known as synchronization phenomena. And a region of (control) parameters (frequency and amplitude of external signal) is called synchronization regime. Self-oscillatory systems generate respective fixed oscillations whose (angular) frequencies and amplitudes are maintained constants which depend on system structures and parameter values of constituent elements.

When control parameters are given outside of synchronization regimes, asynchronous beat oscillations appear. It is well-known that the mechanism of synchronization is classified into two kinds, that is, frequency entrainment (pull-in) and (amplitude) quenching. Consequently, for intermediate values of external signals between the above mentioned two mechanisms, there may appear overlapped regime of both mechanisms in general, that is, coexisting attractors may be observed. The boundaries of different regimes are called bifurcation sets on the parameter plane.

Asynchronous beat oscillations observed in the periodically forced van der Pol's oscillator were represented by invariant simple closed curves of the mapping defined by using solutions of the equation. While among beat oscillations in general periodically forced self-oscillatory systems chaotic oscillations were subsisted. It was the author who first disclosed a chaotic oscillation in a periodically forced negative resistance oscillator. Since he met the data (like a broken egg), it rubbed him with the question "What are the possible steady states of a nonlinear system?" It seemed to give him intuitions that were shape of the attractor and movement of stroboscopic images on the attractor. In this section, Broken Egg (chaotic) Attractor and Local Bifurcation Sets are briefly explained. In both following Figures 1 and 2, items (a) were obtained in 1961, while items (b) were in 2006, truly 45 years was elapsed between these materials were obtained. The differential equation under study was

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