Proceedings of the 3rd International Conference on Nonlinear Dynamics
ND-KhPI2010
September 21-24, 2010, Kharkov, Ukraine

CHAOS AND HYPERCHAOS IN DETERMINISTIC NONIDEAL HYDRODYNAMIC SYSTEMS

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ABSTRACT

The steady-state dynamics regimes of deterministic nonideal systems «tank with a liquid - electric motor» are considered. The atlas of maps of dynamic regimes of the given system is constructed. For the first time existence of quasiperiodic and hyperchaotic attractors is revealed.

INTRODUCTION

The study of oscillations of free surface of liquid in rigid tanks was carrying out of many works, which detailed bibliography are in monographies [1-3]. Excepting the big research interest, the given problems have wide practical application in many areas of modern technics, so long as modern machinery, mechanisms and vehicles as constructive elements, which contain varied in form tanks with liquids.

In overwhelming majority of works the oscillation of liquid in tanks are considered in, so-called, "ideal" statement of problem. At such statement of problem it is supposed that the source of excitation of oscillations of a liquid has an unlimited power. In consequence of that, probably to neglect feedback influence of oscillating system, in this case tank with liquid, on source of excitation of oscillations. The problems of global power savings demands the maximum minimisation of power of applied sources of excitation of oscillations. It leads to that the power of source of excitation becomes comparable to power consumed by oscillating system. Such situation more often takes place in real machines and mechanisms. In such cases application of "ideal" mathematical models can lead to gross errors in exposition of dynamics of systems «source of excitation of oscillations - oscillating subsystem». Thus there can be completely lost information about the deterministic chaos really existing in system [4, 5]. Because nonlinear interaction between oscillating subsystem and device of excitation of oscillations is one of reasons of origin of deterministic chaos.

The major aim of given work is a construction of atlas of maps of dynamic regimes of deterministic dynamic system «tank with a liquid - electric motor». On the basis of the constructed maps the careful study of types of steady-state regimes and detection of scenarios of transition between various types of regimes of system can be carrying out. The researches conducted in this work is prolongation and development the researches begun in [4-6].

1. MATHEMATICAL MODEL AND TECHNIQUE OF CARRYING OUT NUMERICAL CALCULATIONS

Let's consider rigid cylindrical tank partially filled with a liquid. We will assume that the electric motor of limited power excite horizontal oscillations of platform of tank. The given hydrodynamic system is typical nonideal, in sense of Kononenko [7], deterministic dynamic system. As shown in [4-6] mathematical model of system «tank with a liquid - electric motor» is described by following system of differential equations:

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$$
\begin{aligned}
\frac{dp_1}{d\tau} &= \alpha_1 p_1 - \left[\beta + \frac{A}{2}\left(p_1^2 + q_1^2 + p_2^2 + q_2^2\right)\right] q_1 + B(p_1 q_2 - p_2 q_1) p_2 \\
\frac{dq_1}{d\tau} &= \alpha_1 q_1 + \left[\beta + \frac{A}{2}\left(p_1^2 + q_1^2 + p_2^2 + q_2^2\right)\right] p_1 + B(p_1 q_2 - p_2 q_1) q_2 + 1 \\
\frac{d\beta}{d\tau} &= N_3 + N_1 \beta - \mu_1 q_1 \\
\frac{dp_2}{d\tau} &= \alpha_1 p_2 - \left[\beta + \frac{A}{2}\left(p_1^2 + q_1^2 + p_2^2 + q_2^2\right)\right] q_2 - B(p_1 q_2 - p_2 q_1) p_1 \\
\frac{dq_2}{d\tau} &= \alpha_1 q_2 + \left[\beta + \frac{A}{2}\left(p_1^2 + q_1^2 + p_2^2 + q_2^2\right)\right] p_2 - B(p_1 q_2 - p_2 q_1) q_1
\end{aligned} \tag{1}
$$

The system (1) is a nonlinear system of differential equations of fifth order. Phase variables p_1, q_1 and p_2, q_2, accordingly amplitudes of dominant modes of oscillations of free surface of liquid. The phase variable β is proportional to velocity of rotation of shaft of the electric motor. There are six parametres $A, B, \alpha, N_1, N_3, \mu_1$ of system (1), which are defined through physical and geometrical characteristics of tank with a liquid and electric motor. The detailed expositions of these parameters are presented in works [4-6].

In works [4-6] existence of the deterministic chaos in system (1) has been proved, some types of chaotic attractors are classified and shown that chaotic attractors are typical attractors of the given system. We will notice that the detailed and all-round study of chaotic dynamics of system (1) is possible only by means of a series of numerical methods and algorithms. The technique of carrying out of such researches is described in works [4-5].

The particular interest calls construction of maps of dynamic regimes of system (1). Maps of dynamic regimes represent diagrammes on plane on which axes values of arbitrary parametres of system which are called as bifurcation are put aside. Various colours on maps plot areas corresponding to various types of the steady-state dynamical regimes. The basic classification of this or that type of dynamic regimes is the analysis of its spectrum of Lyapunov's characteristic exponents (LCE) [4, 8]. The boundaries between areas of dynamic regimes of different types are especially carefully analyzed. In these cases for correct classification of type of dynamic regimes its phase portraits, Poincare sections and maps, distributions of spectral densities and invariant measures are taken in consideration.

2. CONSTRUCTION OF THE ATLAS OF MAPS OF THE STEADY-STATE DYNAMIC REGIMES

First, we shall consider parameters N_3 and α as a bifurcation ones. Let's assume that, $A = 1.12$; $B = -1.531$; $\mu_1 = 0.5$; $N_1 = -1$. In fig. 1 the sheet of atlas of maps of dynamic regimes of systems «tank with a liquid – electric motor» is shown. This map is obtained as a result of the analysis and data processing of computer experiments according to earlier stated technique.

In fig. 1 areas of existence of three various types of attractors of system (1) are plotted. By white colour plots areas of values of parametres and N_3 at α which equilibrium positions will be attractors of a system. The signature of their spectrum LCE looks like $\langle -,-,-,-,- \rangle$. The areas of grey colour correspond to limit cycles (periodic regimes) of system(1) with the signature of spectrum LCE $\langle 0,-,-,-,- \rangle$. Black colour plots areas of the deterministic chaos with the signature of spectrum LCE $\langle +,0,-,-,- \rangle$. As from fig. 1 in some parts of a map black areas of chaotic attractors "incise" into areas of periodic regimes, in other parts, on the contrary, light gleams in chaotic areas which are called as "periodicity windows" are looked through.

Let's consider examples of regular and chaotic attractors corresponding to various areas of a map. So, at values and $N_3 = -1.5$ $\alpha = -0.4$ the corresponding point in map locates in area of white colour. Position of the equilibrium which coordinates have values: $p_1 = 0.699$, $q_1(0) = 0.214$, $\beta(0) = -1.607$, $p_2 = q_2 = 0$ will be a system attractor. At $N_3 = -1$, $\alpha = -0.48$ and at $N_3 = -0.72$, $\alpha = -0.3$ corresponding points in the map locate in area of grey colour. Limit cycles will be system attractors in this case. Projections of phase portraits of the given cycles are shown in fig. 2a and 2b. Both cycles represent closed lines in a phase space, however the second of these cycles has more complicated, multistage structure. At last at $N_3 = -0.4$, $\alpha = -0.3$ the corresponding point locates in black area of a map of dynamic regimes. In this case the system (1) has a chaotic attractor. Projection of a phase portrait of given chaotic attractor are shown in fig. 2c.

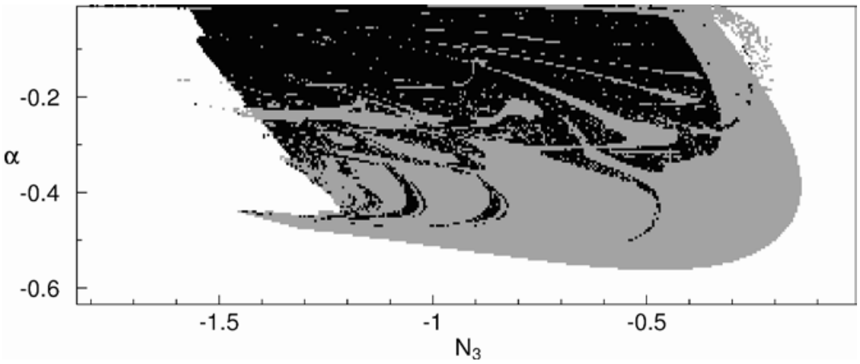


Fig. 1. Sheet of maps of dynamic regimes at changing of parametres N_3 *and* α *.*

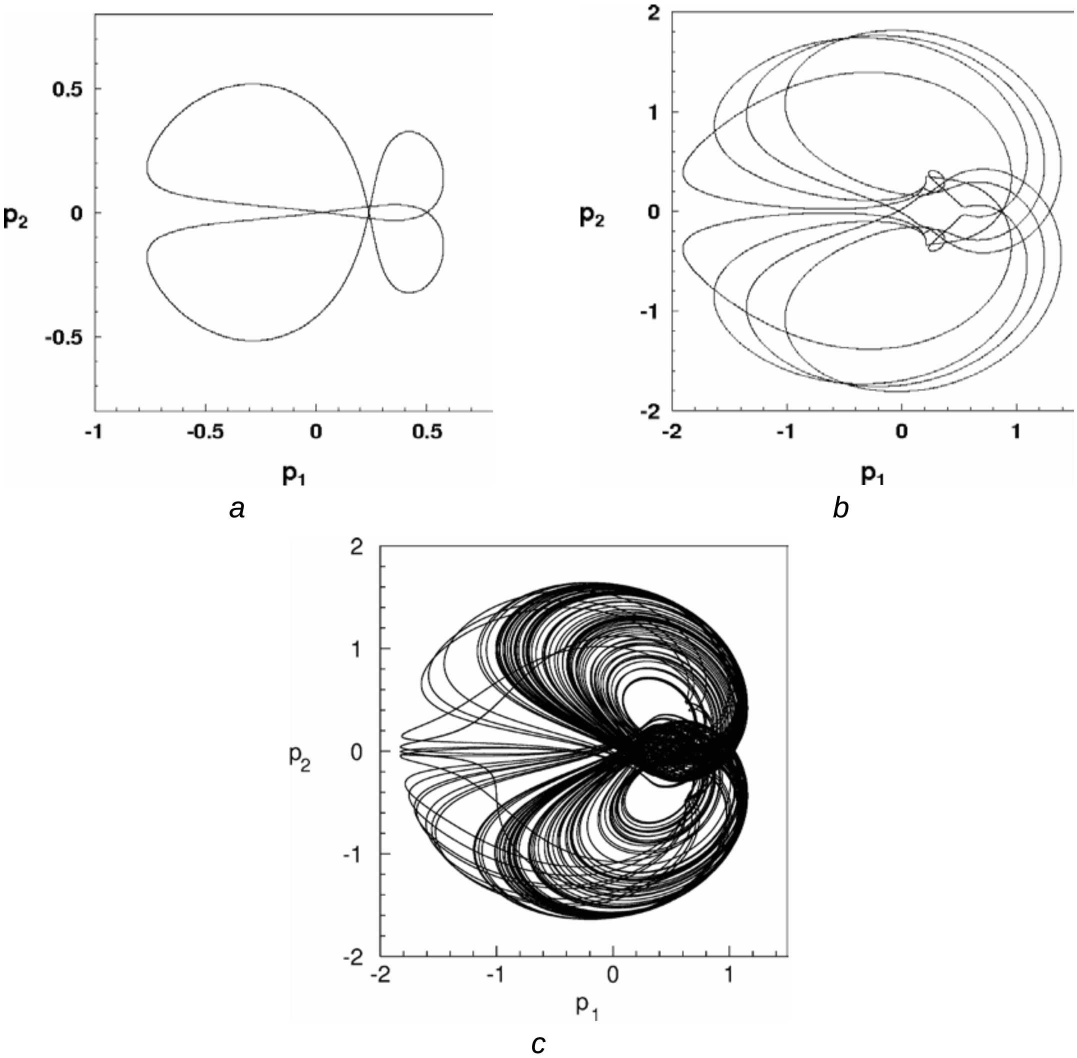
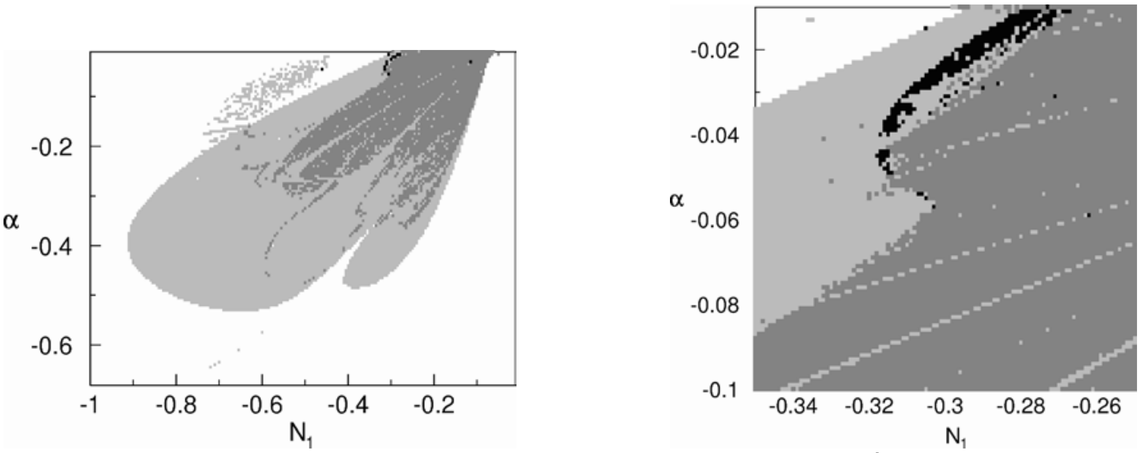


Fig. 2. Projections of limit cycles phase *portraits at* $N_3 = \text{-}1$, $\alpha = \text{-}0.48$*(a); and at* $N_3 = \text{-}0.72$, $\alpha = \text{-}0.3$*(b); of chaotic attractor at* $N_3 = \text{-}0.4$, $\alpha = \text{-}0.3$*(c).*

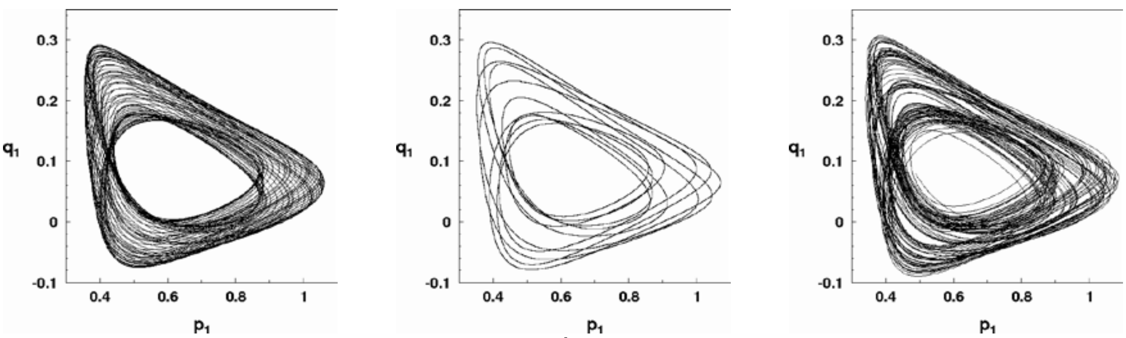
Further we will assume that $N_3 = -0.1$. Parameters N_1 and α we will choose as bifurcation parameters. The values A, B and μ_1 it is considered by the invariable. In fig. 3a the new sheet of the atlas of maps of dynamic regimes in which areas of four types of dynamic regimes are plotted. By white colour denote areas in space of parameters in which in system exist the equilibrium positions. The areas of light grey colour correspond to limit cycles of system (1). Dark grey colour areas corresponds to areas of chaotic attractors. And, at last, areas of black colour correspond to areas of quasiperiodic regimes with the signature of spectrum LCE $<0,0,-,-,->$. In fig. 3b the increased fragment of the constructed map is shown. On this increased fragment the black area of quasiperiodic attractors, which places near to boundary of areas of existence of regular and chaotic attractors, is clear visible. We will notice that areas in space of parameters of system (1) in which attractors of system are limit toruses have not been discovered in the previous researches of system (1). Thus quasiperiodic attractors are new type of attractors for the given systems.



a *b*

Fig. 3. Sheet of maps of dynamic regimes at changing of parametres N_1 and α .

Let's consider examples of attractors of system (1) which exist in various areas of a map from fig. 3. So in fig. 4a the projection of phase portrait of quasiperiodic attractor (limit torus), constructed at values $N_1 = -0.32, \alpha = -0.045$ is shown. In fig. 4b the limit cycle projection (resonance cycle in torus), constructed at values $N_1 = -0.314, \alpha = -0.045$ is shown. At last in fig. 4c the projection of one of chaotic attractors of system (1), constructed at values $N_1 = -0.3131, \alpha = -0.045$ is shown. In this case transition to chaos through destruction of a quasiperiodic attractor is realised.



a *b* *c*

Fig .4. Projections of phase *portraits of* quasiperiodic attractor *at* $N_1 = -0.32,\ \alpha = -0.045$*(a); limit cycle at* $N_1 = -0.314,\ \alpha = -0.045$*(b); chaotic attractor at* $N_3 = -0.3131,\ \alpha = -0.045$*(c).*

Now we will assume that $N_3 = N_1 = -1$. Parameters μ_1 and α we will choose as bifurcation parameters. The values A , B it is considered by the invariable. In fig. 5 (a-b) a few fragments of new sheet of the atlas of maps of dynamic regimes are plotted. The areas of dynamic regimes of five types be present in given maps. By white colour areas of existence of positions of equilibrium are plotted. The areas of light grey colour correspond to periodic regimes of system. By grey colour notes areas of existence of chaotic attractors. Areas of quasiperiodic regimes are designated by black colour. And at last, areas of existence of hyperchaotic attractors are plotted by dark grey colour. The signature of spectrum LCE of hyperchaotic attractors looks like $\langle +,+,0,-,- \rangle$. So two positive exponents are at spectrum of hyperchaotic attractors. We will notice that hyperchaotic attractors not discovered at earlier researches of system (1).

Let's consider some of hyperchaotic attractors existing in system (1). So in fig. 6a the projection of phase portrait of hyperchaotic attractor constructed at values $\mu_1 = 4.125, \alpha = -0.04$ is shown. In fig. 6b the projection hyperchaotic attractor, constructed at values $\mu_1 = 4.125, \alpha = -0.0403$ is shown. Phase portraits of these attractors noticeably differ one from another. First of all the hyperchaotic attractor presented in fig. 6b differs from a hyperchaotic attractor presented in fig. 6a appreciable increasing of volume of its area of localisation in a phase space. In fig. 6c the increased fragment of a central part of an attractor from fig. 6b. Apparently from fig. 6c in this fragment contours of a hyperchaotic attractor presented in fig. 6a are accurately looked through. Such qualitative similarity of a fragment of one attractor to other attractor has allowed to make clear existing in system the transition of type of «hyperchaos - hyperchaos» . It was possible to prove that this transition is realised under the scenario of intermittency generalising the known scenario of Pomeau and Manneville. And if in works [4, 5] it was possible to generalise the scenario of Pomeau and Manneville for type of transition of «chaos - chaos», now it succeed to be generalised and on type of transition of «hyperchaos - hyperchaos».

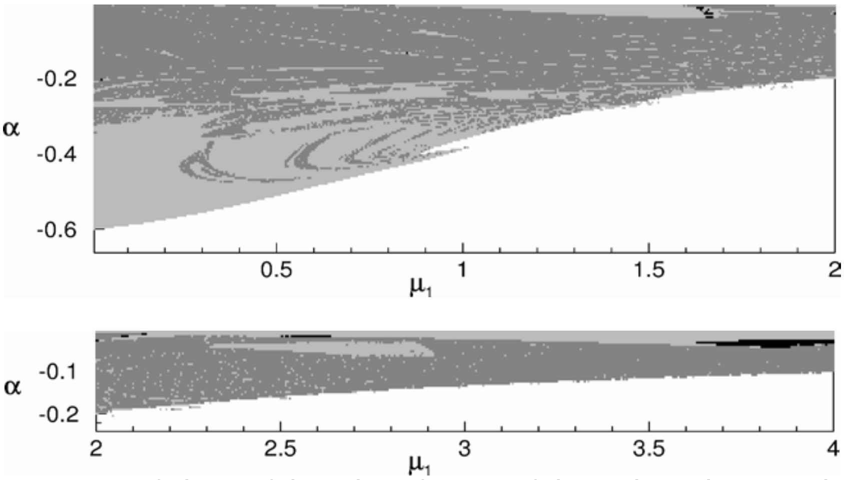


Fig. 5a Fragments of sheet of the atlas of maps of dynamic regimes at changing of parametres μ_1 and α .

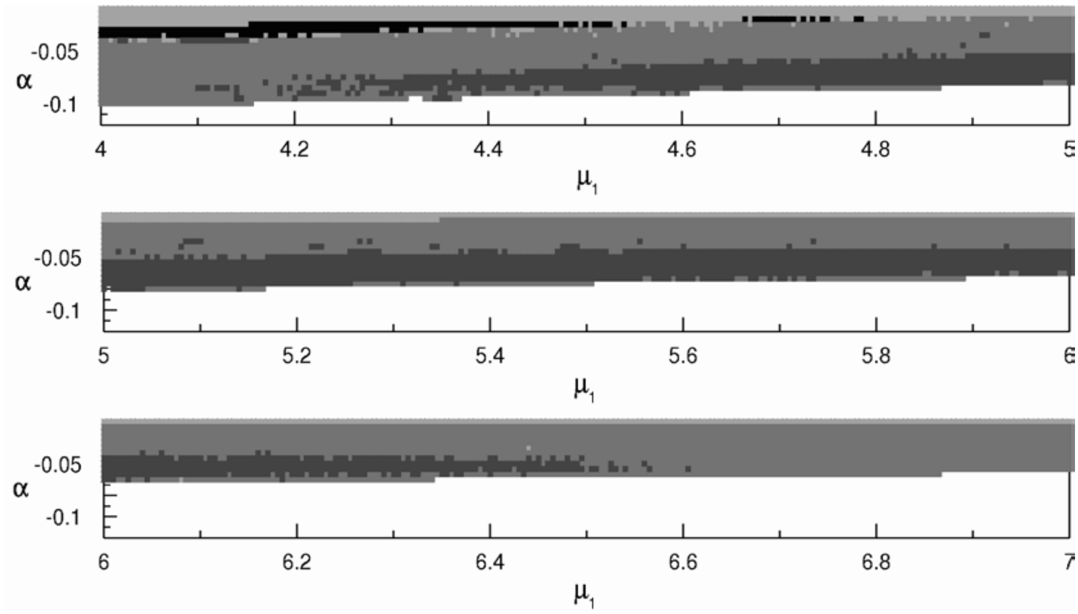


Fig. 5b. Fragments of sheet of the atlas of maps of dynamic regimes at changing of parametres μ_1 and α .

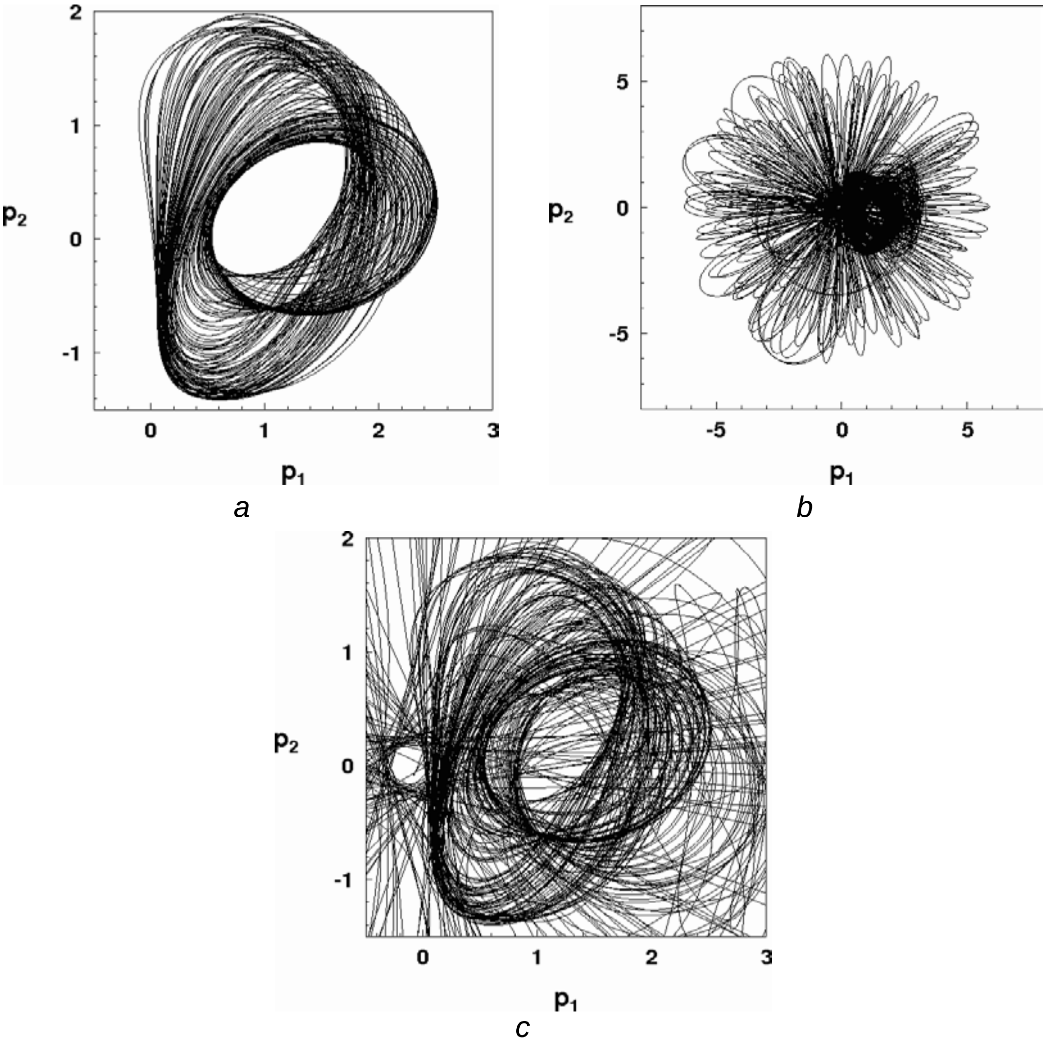


Fig 6 Projections of hyperchaotic attractors phase *portraits at* $\mu_1 = 4.125,\ \alpha = -0.04$ *(a) and at* $\mu_1 = 4.125,\ \alpha = -0.0403$ *(b-c).*

CONCLUSIONS

Thus, in this work maps of dynamic regimes of nonideal deterministic system "tank with a liquid-electric motor" for the first time are constructed. The constructed maps are of great importance for detailed research of regular and chaotic attractors of the given system. The knowledge of such maps allows essentially abridge duration of time of carrying out of natural experimental researches of dynamic systems of this kind. Also in space of parameters of system for the first time the discovered areas of existence of hyperchaotic and quasiperiodic attractors.

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